Confidence Interval for Standard Deviation of Normal Distribution with Known Coefficients of Variation

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***Abstract*--Motivated by the recent work of Herbert, Hayen, Macaskill and Walter [Interval estimation for the difference of two independent variances. Communications in Statistics, Simulation and Computation, 40: 744-758, 2011.], we investigate, in this paper, the new confidence interval for the difference between two normal population standard deviations based on the simple confidence interval of Donner and Zou [Closed-form confidence intervals for functions of the normal mean and standard deviation, 1-13, 2010.]. For a single confidence interval for a standard deviation, we derived analytic expressions to find the coverage probability and its expected length compared with the standard confidence interval. Monte Carlo simulation results for the difference of standard deviations are given to compare proposed confidence intervals.**

***Keywords*-- Coverage probability, expected length, variances**

I. INTRODUCTION

Recently, Herbert et al. [2] have argued that methods and analyzes for constructing the confidence interval estimation of the difference between variances have not been described. They proposed a simple analytical method to construct a confidence interval for the difference between two independent samples. It is seen that their proposed confidence interval works well when observations are highly skewed and leptokurtic. Cojbasica and Tomovica [1] showed that the nonparametric confidence interval of the population variance of two-sample problem based on *t*-statistic combined with bootstrap techniques performs very well when data are from exponential families. Phonyiem and Niwitpong [5] proposed the new generalized confidence interval for the difference between the normal variances.Their proposed interval performs very well compared to the existing confidence interval**.** Related works of the confidence interval for the difference between two variances, the reader is referred to the references cited in the above papers. In this paper, we emphasize only data are from the normal distribution and we propose the new confidence interval for the single and the difference of two normal population standard deviations, based on the simple confidence interval of [3]. For the confidence interval for a standard deviation, we derived analytic expressions to find coverage probability and expected length. We also present the Monte Carlo simulation results for the confidence intervals for the difference of standard deviations.

II. CONFIDENCE INTERVALS FOR THE DIFFERENCE OF TWO NORMAL POPULATION STANDARD DEVIATIONS

Let $X_1, \dots, X_n$ and $Y_1, \dots, Y_m$ be random samples from two independent normal distributions with means μ_1, μ_2 and standard deviations σ_1, σ_2, respectively. The sample means and sample variances for X and Y are, respectively, denoted as $\bar{X}, \bar{Y}, S_1^2$ and S_2^2 when $\bar{X} = n^{-1}\sum_{i=1}^{n} X_i, \bar{Y} = m^{-1}\sum_{i=1}^{m} Y_i,\ S_1^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(X_i - \bar{X})^2$ and $S_2^2 = \frac{1}{(m-1)}\sum_{i=1}^{m}(Y_i - \bar{Y})^2$. We are interested in $100(1-\alpha)\%$ confidence intervals for σ_1 and $\sigma_1 - \sigma_2$.

A. Confidence Interval for The Single Standard Deviation

It is well known that for a given sample variance X which is $S_1^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(X_i - \bar{X})^2$ then $\chi^2 = \frac{(n-1)S_1^2}{\sigma_1^2}$ is the Chi-square distributed with $(n-1)$ degrees of freedom. For given a real number α, where $\alpha \in (0,1)$. We can construct the confidence interval for a standard deviation:

$$\begin{aligned}
1-\alpha &= P\left(\chi^2_{(n-1),(\alpha/2)} \le \chi^2 \le \chi^2_{(n-1),(1-\alpha/2)}\right) \\
&= P\left(\chi^2_{(n-1),(\alpha/2)} \le \frac{(n-1)S_1^2}{\sigma_1^2} \le \chi^2_{(n-1),(1-\alpha/2)}\right) \\
&= P\left(\frac{(n-1)S_1^2}{\chi^2_{(n-1),(1-\alpha/2)}} \le \sigma_1^2 \le \frac{(n-1)S_1^2}{\chi^2_{(n-1),(\alpha/2)}}\right).
\end{aligned}$$

A $100(1-\alpha)\%$ confidence interval for σ_1^2 is

$$CI_{\sigma_1^2} = \left[\frac{(n-1)S_1^2}{\chi^2_{(n-1),(1-\alpha/2)}}, \frac{(n-1)S_1^2}{\chi^2_{(n-1),(\alpha/2)}}\right]. \quad (1)$$

From (1), a $100(1-\alpha)\%$ confidence interval for σ_1 is therefore

$$CI_1 = \left[\sqrt{\frac{(n-1)S_1^2}{\chi^2_{(n-1),(1-\alpha/2)}}}, \sqrt{\frac{(n-1)S_1^2}{\chi^2_{(n-1),(\alpha/2)}}}\right]. \quad (2)$$

B. Confidence Interval for The Standard Deviation with Known a Coefficient of Variation

Consider $\tau_1 = \sigma_1 / \mu_1$ where τ_1 is a coefficient of variation, we then have $\sigma_1 = \tau_1 * \mu_1$. For a $100(1-\alpha)\%$ confidence interval for σ_1 , when a coefficient of variation is known, can be easily solved which is

$$CI_2 = \left[\tau_1\left(\bar{X} - c_0 S_1/\sqrt{n}\right),\ \tau_1\left(\bar{X} + c_0 S_1/\sqrt{n}\right)\right] \quad (3)$$

where c_0 is a $100(1-\alpha/2)\%$ percentile of a standard *t*-distribution with $n-1$ degrees of freedom.

We now derive coverage probabilities and expected lengths of confidence intervals CI_1 compared to CI_2.

Theorem 1 The coverage probability and the expected length of are respectively $E[\Phi(W_1) - \Phi(-W_1)]$ and $(c_2 - c_1)\sigma_1\sqrt{2/(\mathrm{n}-1)}(\Gamma(\mathrm{n}/2)/\Gamma((\mathrm{n}-1)/2)$ where

$W_1 = \frac{S_1 - c_2 S_1}{\sigma_1\sqrt{c_3}}, W_2 = \frac{S_1 - c_1 S_1}{\sigma_1\sqrt{c_3}}, c_1 = \sqrt{\frac{(n-1)}{\chi^2_{(n-1),(1-\alpha/2)}}}, c_2 = \sqrt{\frac{(n-1)}{\chi^2_{(n-1),(\alpha/2)}}}$ and
$c_3 = 1 - 2(\mathrm{n}-1)^{-1}(\Gamma(\mathrm{n}/2)/\Gamma((\mathrm{n}-1)/2))^2$.

Proof: Following [4], the coverage probability of CI_1 is

$$
\begin{aligned}
1-\alpha &= P\left(\chi^2_{(n-1),(\alpha/2)} \le \chi^2 \le \chi^2_{(n-1),(1-\alpha/2)}\right)\\
&= P\left(\chi^2_{(n-1),(\alpha/2)} \le \frac{(n-1)S_1^2}{\sigma_1^2} \le \chi^2_{(n-1),(1-\alpha/2)}\right)\\
&= P\left(\frac{(n-1)S_1^2}{\chi^2_{(n-1),(1-\alpha/2)}} \le \sigma_1^2 \le \frac{(n-1)S_1^2}{\chi^2_{(n-1),(\alpha/2)}}\right)\\
&= P\left(\sqrt{\frac{(n-1)S_1^2}{\chi^2_{(n-1),(1-\alpha/2)}}} \le \sqrt{\sigma_1^2} \le \sqrt{\frac{(n-1)S_1^2}{\chi^2_{(n-1),(\alpha/2)}}}\right)\\
&= P\left(c_1 S_1 \le \sigma_1 \le c_2 S_1\right), c_1 = \sqrt{\frac{(n-1)}{\chi^2_{(n-1),(1-\alpha/2)}}}, c_2 = \sqrt{\frac{(n-1)}{\chi^2_{(n-1),(\alpha/2)}}}\\
&= P\left(S_1 - c_2 S_1 \le S_1 - \sigma_1 \le S_1 - c_1 S_1\right)\\
&= P\left(\frac{S_1 - c_2 S_1}{\sigma_1\sqrt{c_3}} \le \frac{S_1 - \sigma_1}{\sigma_1\sqrt{c_3}} \le \frac{S_1 - c_1 S_1}{\sigma_1\sqrt{c_3}}\right), c_3 = 1 - 2(\mathrm{n}-1)^{-1}(\Gamma(\mathrm{n}/2)/\Gamma((\mathrm{n}-1)/2))^2\\
&= P\left(\frac{S_1 - c_2 S_1}{\sigma_1\sqrt{c_3}} \le Z \le \frac{S_1 - c_1 S_1}{\sigma_1\sqrt{c_3}}\right)\\
&= E[I_{\{-W_1<Z<W_1\}}(\tau)], I_{\{-W_1<Z<W_1\}}(\tau) = \begin{cases} 1, if\, \tau \in \{-W_1 < Z < W_1\} \\ 0, otherwise \end{cases}\\
&= E[E[I_{\{-W_1<Z<W_1\}}(\tau)] \mid S^2]\\
&= E[\Phi(W_1) - \Phi(-W_1)]
\end{aligned}
$$

where $Z \sim N(0; 1)$.

Note that,

$$
\begin{aligned}
Var(\mathrm{S}_1) &= \mathrm{E}(\mathrm{S}_1^2) - (\mathrm{E}(\mathrm{S}_1))^2\\
&= \sigma_1^2 - (\sigma_1\sqrt{2/(\mathrm{n}-1)}\Gamma(\mathrm{n}/2)/\Gamma((\mathrm{n}-1)/2)^2\\
&= \sigma_1^2(1-(2/(\mathrm{n}-1))(\Gamma(\mathrm{n}/2)/\Gamma((\mathrm{n}-1)/2)^2).
\end{aligned}
$$

The expected length for CI_1 is therefore

$$
\begin{aligned}
E(CI_1) &= (c_2 - c_1)\mathrm{E}(\mathrm{S}_1)\\
&= (c_2 - c_1)\sigma_1\sqrt{2/(\mathrm{n}-1)}(\Gamma(\mathrm{n}/2)/\Gamma((\mathrm{n}-1)/2).
\end{aligned}
$$

Theorem 2 The coverage probability and the expected length of are respectively, $E[(W_2)-\Phi(W_3)]$ and $\frac{2c_0}{\sqrt{n}}\tau\sigma\sqrt{2/(n-1)}(\Gamma(n/2)/\Gamma((n-1)/2)$.

Proof: Following [4], the coverage probability of CI_2 is

$$1-\alpha = P\left(\tau_1\left(\bar{X}-c_0S_1/\sqrt{n}\right)\le\sigma_1\le\ \tau_1\left(\bar{X}+c_0S_1/\sqrt{n}\right)\right)$$

$$= P\left(-\tau_1\left(\bar{X}+c_0S_1/\sqrt{n}\right)\le-\sigma_1\le-\tau_1\left(\bar{X}-c_0S_1/\sqrt{n}\right)\right)$$

$$= P\left(S_1-\tau_1\left(\bar{X}+c_0S_1/\sqrt{n}\right)\le S_1-\sigma_1\le S_1-\tau_1\left(\bar{X}-c_0S_1/\sqrt{n}\right)\right)$$

$$= P\left(\frac{S_1-\tau_1\left(\bar{X}+c_0S_1/\sqrt{n}\right)}{\sigma_1\sqrt{c_3}}\le\frac{S_1-\sigma_1}{\sigma_1\sqrt{c_3}}\le\frac{S_1-\tau_1\left(\bar{X}-c_0S_1/\sqrt{n}\right)}{\sigma_1\sqrt{c_3}}\right)$$

$$= P\left(\frac{S_1-\tau_1\left(\bar{X}+c_0S_1/\sqrt{n}\right)}{\sigma_1\sqrt{c_3}}\le Z\le\frac{S_1-\tau_1\left(\bar{X}-c_0S_1/\sqrt{n}\right)}{\sigma_1\sqrt{c_3}}\right), c_3 = 1-(2/(n-1))(\Gamma(n/2)/\Gamma((n-1)/2)^2$$

$$= E[I_{\{W_2<Z<W_3\}}(\tau)], I_{\{W_2<Z<W_3\}}(\tau) = \begin{cases}1, if\ \tau\in\{W_2<Z<W_3\}\\ 0, otherwise\end{cases}, W_2 = \frac{S_1-\tau_1\left(\bar{X}+c_0S_1/\sqrt{n}\right)}{\sigma_1\sqrt{c_3}},$$

$$= E[E[I_{\{W_2<Z<W_3\}}(\tau)]\,|\,S^2], W_3 = \frac{S_1-\tau_1\left(\bar{X}-c_0S_1/\sqrt{n}\right)}{\sigma_1\sqrt{c_3}}$$

$$= E[(W_2)-\Phi(W_3)]$$

The expected length of CI_2 is therefore

$$E(CI_2) = E\left(\tau_1\left(\bar{X}+c_0S_1/\sqrt{n}\right)-\tau_1\left(\bar{X}-c_0S_1/\sqrt{n}\right)\right)$$

$$= E\left(2c_0\tau_1S_1/\sqrt{n}\right)$$

$$= \frac{2c_0\tau_1}{\sqrt{n}}E(S_1)$$

$$= \frac{2c_0}{\sqrt{n}}\tau_1\sigma_1\sqrt{2/(n-1)}(\Gamma(n/2)/\Gamma((n-1)/2).$$

C. Confidence Intervals for the Difference between Normal Standard Deviations

Donner and Zou [3] proposed the confidence interval for the difference between $\theta_1-\theta_2$ which is $CI_3 = [L,U]$ where $L = \hat{\theta}_1-\hat{\theta}_2-\sqrt{\left(\hat{\theta}_1-l_1\right)^2+\left(u_2-\hat{\theta}_2\right)^2}$, $U = \hat{\theta}_1+\hat{\theta}_2-\sqrt{\left(u_1-\hat{\theta}_1\right)^2+\left(\hat{\theta}_2-l_2\right)^2}$. Plug in $\theta_1-\theta_2 = \sigma_1-\sigma_2, \hat{\theta}_1 = c_4S_1, \hat{\theta}_2 = c_5S_2$, $c_4 = \sqrt{2/(n-1)}(\Gamma(n/2)/\Gamma((n-1)/2), c_5 = \sqrt{2/(m-1)}(\Gamma(m/2)/\Gamma((m-1)/2)$,

$(l_1,u_1)=\left(\sqrt{\frac{(n-1)S_1^2}{\chi^2_{1-\alpha/2,(n-1)}}},\sqrt{\frac{(n-1)S_1^2}{\chi^2_{\alpha/2,(n-1)}}}\right)$, $(l_2,u_2)=\left(\sqrt{\frac{(m-1)S_2^2}{\chi^2_{1-\alpha/2,(m-1)}}},\sqrt{\frac{(m-1)S_2^2}{\chi^2_{\alpha/2,(m-1)}}}\right)$ in to $CI_3=[L,U]$, we have a new confidence interval for $\sigma_1-\sigma_2$.

Another confidence interval for $\sigma_1-\sigma_2$ is $CI_4=[L_1,U_1]$, where

$$L_1=\hat{\theta}_1-\hat{\theta}_2-\sqrt{\left(\hat{\theta}_1-l_3\right)^2+\left(u_4-\hat{\theta}_2\right)^2},\ U_1=\hat{\theta}_1+\hat{\theta}_2-\sqrt{\left(u_3-\hat{\theta}_1\right)^2+\left(\hat{\theta}_2-l_4\right)^2},$$

$$(l_3,u_3)=\left(\tau_1\left(\bar{X}-c_1S_1/\sqrt{n}\right),\ \tau_1\left(\bar{X}+c_1S_1/\sqrt{n}\right)\right),$$

$$(l_4,u_4)=\left(\tau_2\left(\bar{Y}-c_1S_2/\sqrt{m}\right),\ \tau_2\left(\bar{Y}+c_1S_2/\sqrt{m}\right)\right).$$

III. SIMULATION RESULTS

In this section, using the R program (version 3.01), we compare our proposed confidence intervals via Monte Carlo simulation. We set the sample $n=m=10,30,50,100,200,(\tau_1,\tau_2)=(0.05,0.05)$, $(0.05,0.10),(0.10,0.10),(0.10,0.20),$
$(0.20,0.20),(0.20,0.30),(0.30,0.30),(0.30,0.40),(0.40,0.40),(0.40,0.50),(0.50,0.50),(0.50,0.60),$
$(0.60,0.60),(0.60,0.70),(0.70,0.70),(0.70,0.80),(0.80,0.80),(0.80,0.90),(0.90,0.90),(0.90,1.00).$
The coverage probabilities and the expected lengths for confidence intervals $CI_3=[L,U]$ compared to $CI_4=[L_1,U_1]$ are reported in Table 1 with 10, 000 simulation runs.

TABLE 1 COVERAGE PROBABILITY AND EXPECTED LENGTH FOR CONFIDENCE INTERVALS CI_3, CI_4 WITH $1-\alpha=0.95$.

n	M	τ_1	τ_2	COV1	COV2	LENGH1	LENGTH2
10	10	0.05	0.05	0.9571	0.6108	0.0864	0.0303
		0.05	0.10	0.9544	0.6353	0.1331	0.0487
		0.10	0.10	0.9566	0.6943	0.1731	0.0628
		0.10	0.20	0.9547	0.6991	0.2665	0.1090
		0.20	0.20	0.9559	0.8310	0.3459	0.1436
		0.20	0.30	0.9582	0.7680	0.4363	0.2063
		0.30	0.30	0.9554	0.8894	0.5165	0.2539
		0.30	0.40	0.9545	0.8366	0.6053	0.3330
		0.40	0.40	0.9565	0.9231	0.6910	0.4043
		0.40	0.50	0.9548	0.8909	0.7806	0.5101
		0.50	0.50	0.9534	**0.9519**	0.8659	0.5985
		0.50	0.60	0.9567	**0.9330**	0.9521	0.7169
		0.60	0.60	0.9550	**0.9652**	1.0379	0.8298
		0.60	0.70	0.9533	**0.9523**	1.1249	0.9710
		0.70	0.70	0.9533	**0.9750**	1.2084	1.0951
		0.70	0.80	0.9543	**0.9717**	1.2996	1.2720
		0.80	0.80	0.9565	**0.9817**	1.3868	1.4258
		0.80	0.90	0.9554	**0.9796**	1.4759	1.6119
		0.90	0.90	0.9581	**0.9833**	1.5514	1.7685
		0.90	1.00	0.9540	**0.9830**	1.6430	1.9809

30	30	0.05	0.05	0.9500	0.5887	0.0397	0.0166
		0.05	0.10	0.9522	0.6168	0.0622	0.0266
		0.10	0.10	0.9510	0.6819	0.0795	0.0349
		0.10	0.20	0.9526	0.6704	0.1243	0.0589
		0.20	0.20	0.9536	0.8099	0.1589	0.0786
		0.20	0.30	0.9497	0.7595	0.2020	0.1118
		0.30	0.30	0.9506	0.8632	0.2383	0.1379
		0.30	0.40	0.9509	0.8352	0.2813	0.1831
		0.40	0.40	0.9529	0.9145	0.3178	0.2175
		0.40	0.50	0.9482	0.8800	0.3601	0.2739
		0.50	0.50	0.9506	0.9449	0.3977	0.3214
		0.50	0.60	0.9492	0.9453	0.4381	0.3800
		0.60	0.60	0.9534	**0.9516**	0.4769	0.4402
		0.60	0.70	0.9483	**0.9445**	0.5186	0.5209
		0.70	0.70	0.9522	**0.9638**	0.5564	0.5864
		0.70	0.80	0.9532	**0.9619**	0.5976	0.6764
		0.80	0.80	0.9550	**0.9750**	0.6360	0.7536
		0.80	0.90	0.9515	**0.9733**	0.6763	0.8552
		0.90	0.90	0.9535	**0.9818**	0.7159	0.9434
		0.90	1.00	0.9538	**0.9820**	0.7578	1.0556

COV1 stands for the coverage probability for CI_3
COV2 stands for the coverage probability for CI_4
LENGTH1 stands for the expected length of CI_3
LENGTH2 stands for the expected length of CI_4

TABLE 1 (CONTINUE)

n	m	τ_1	τ_2	COV1	COV2	LENGH1	LENGTH2
50	50	0.05	0.05	0.9522	0.5859	0.0295	0.0128
		0.05	0.10	0.9510	0.6143	0.0463	0.0203
		0.10	0.10	0.9516	0.6886	0.0590	0.0264
		0.10	0.20	0.9507	0.6770	0.0927	0.0451
		0.20	0.20	0.9496	0.7976	0.1181	0.0602
		0.20	0.30	0.9523	0.7547	0.1499	0.0842
		0.30	0.30	0.9466	0.8629	0.1772	0.1067
		0.30	0.40	0.9511	0.8250	0.2084	0.1385
		0.40	0.40	0.9482	0.9013	0.2363	0.1669
		0.40	0.50	0.9500	0.8730	0.2672	0.2081
		0.50	0.50	0.9532	**0.9448**	0.2953	0.2448
		0.50	0.60	0.9557	**0.9481**	0.3259	0.2933
		0.60	0.60	0.9512	**0.9535**	0.3546	0.3389
		0.60	0.70	0.9487	**0.9469**	0.3846	0.3942
		0.70	0.70	0.9532	**0.9629**	0.4127	0.4483
		0.70	0.80	0.9495	**0.9586**	0.4429	0.5136
		0.80	0.80	0.9485	**0.9754**	0.4722	0.5763
		0.80	0.90	0.9500	**0.9681**	0.5030	0.6529
		0.90	0.90	0.9513	**0.9773**	0.5317	0.7186
		0.90	1.00	0.9541	**0.9780**	0.5614	0.7992

100	100	0.05	0.05	0.9523	0.5844.	0.0202	0.0090
		0.05	0.10	0.9493	0.6044	0.0318	0.0142
		0.10	0.10	0.9526	0.6778	0.0404	0.0187
		0.10	0.20	0.9521	0.6780	0.0637	0.0317
		0.20	0.20	0.9503	0.7937	0.0809	0.0422
		0.20	0.30	0.9513	0.7544	0.1029	0.0595
		0.30	0.30	0.9504	0.8620	0.1214	0.0738
		0.30	0.40	0.9528	0.8239	0.1429	0.0975
		0.40	0.40	0.9533	0.9032	0.1616	0.1164
		0.40	0.50	0.9465	0.8736	0.1831	0.1462
		0.50	0.50	0.9520	0.9383	0.2022	0.1701
		0.50	0.60	0.9526	**0.9452**	0.2234	0.2056
		0.60	0.60	0.9450	**0.9506**	0.2428	0.2366
		0.60	0.70	0.9529	**0.9405**	0.2636	0.2772
		0.70	0.70	0.9504	**0.9605**	0.2831	0.3147
		0.70	0.80	0.9489	**0.9604**	0.3040	0.3598
		0.80	0.80	0.9509	**0.9739**	0.3238	0.4061
		0.80	0.90	0.9497	**0.9699**	0.3447	0.4583
		0.90	0.90	0.9540	**0.9790**	0.3640	0.5032
		0.90	1.00	0.9482	**0.9799**	0.3848	0.5648

TABLE 1 (CONTINUE)

n	m	τ_1	τ_2	COV1	COV2	LENGH1	LENGTH2
200	200	0.05	0.05	0.9512	0.5880	0.0140	0.0063
		0.05	0.10	0.9520	0.6074	0.0222	0.0100
		0.10	0.10	0.9504	0.6737	0.0281	0.0131
		0.10	0.20	0.9487	0.6735	0.0444	0.0222
		0.20	0.20	0.9523	0.7951	0.0563	0.0297
		0.20	0.30	0.9517	0.7584	0.0716	0.0419
		0.30	0.30	0.9526	0.8644	0.0844	0.0524
		0.30	0.40	0.9456	0.8191	0.0994	0.0675
		0.40	0.40	0.9500	0.8950	0.1125	0.0820
		0.40	0.50	0.9511	0.8726	0.1274	0.1029
		0.50	0.50	0.9502	0.9397	0.1407	0.1203
		0.50	0.60	0.9517	**0.9433**	0.1554	0.1442
		0.60	0.60	0.9470	**0.9490**	0.1690	0.1670
		0.60	0.70	0.9496	**0.9367**	0.1835	0.1939
		0.70	0.70	0.9520	**0.9638**	0.1971	0.2214
		0.70	0.80	0.9518	**0.9602**	0.2116	0.2547
		0.80	0.80	0.9487	**0.9718**	0.2252	0.2839
		0.80	0.90	0.9516	**0.9723**	0.2398	0.3197
		0.90	0.90	0.9508	**0.9788**	0.2533	0.3559
		0.90	1.00	0.9523	**0.9783**	0.2679	0.3972

COV1 stands for the coverage probability for CI_3
COV2 stands for the coverage probability for CI_4
LENGTH1 stands for the expected length of CI_3
LENGTH2 stands for the expected length of CI_4

To compare confidence intervals, we prefer confidence interval which has coverage probability at least 0.95 and has shortest expected length. From Table 1, we prefer the confidence interval
$CI_3 = [L, U]$ to the confidence interval $CI_4 = [L_1, U_1]$ when n=10, 30 ,50 , 100, 200 and small values of coefficients of variation i.e. $\tau_1, \tau_2 \le 0.5$.
otherwise we choose the confidence interval $CI_4 = [L_1, U_1]$ which has a coverage probability better than the confidence interval $CI_3 = [L, U]$.

IV. CONCLUSIONS

In this paper, we drive analytics expressions to find the coverage probability and the expected length for the confidence interval for a standard deviation CI_1 compared to CI_2. For the confidence interval for the difference between standard deviations, we compared CI_3 to CI_4,
Monte Carlo simulations are carried out. It turned out that CI_3 performed better than CI_4 for small and moderate values of coefficients of variation, i.e. $\tau_1, \tau_2 \le 0.5$. The large values of the quantities τ_1, τ_2, resulting to the better coverage probabilities of CI_4 and also a larger of expected lengths of CI_4.

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