The Split Edge Domination in Fuzzy Graphs

C.Y.Ponnappan[1], S. Basheer Ahamed[2], P.Surulinathan[3],

[1] *Department of Mathematics, Government Arts College, Paramakudi, Tamilnadu, India*
[2]*Department of Mathematics, P.S.N.A. College of Engineering and Technology, Dindigul, Tamilnadu, India.*
[3] *Department of Mathematics, Lathamathavan Engineering college, Kidaripatti, Alagarkovil, Madurai-625301,Tamilnadu,India.*

Abstract - An edge dominating set D of a fuzzy graph G= (σ, μ) is a split edge dominating set if the induced fuzzy sub graph H= (<E-D>, σ′, μ′) is disconnected. The split edge domination number $\gamma'_s(G)$ or γ'_s is the minimum fuzzy cardinality of a split edge dominating set. In this paper we study a split edge dominating set of fuzzy graphs and investigate the relationship of $\gamma'_s(G)$ with other known parameter of G.

Keywords - Fuzzy graphs, fuzzy domination, fuzzy edge domination, fuzzy split edge domination number.

AMS Classification: 05C

1. Introduction

The study of domination set in graphs was begun by Ore and Berge. Kulli V.R. et.al introduced the concept of split domination and non-split domination in graphs. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness. A.Somasundram and S.Somasundram discussed domination in Fuzzy graphs. They defined domination using effective edges in fuzzy graph. In this paper we discuss the split edge domination number of fuzzy graph and establish the relationship with other parameter which is also investigated.

2. Preliminaries

Definition 2.1

Let V be a finite non empty set. Let E be the collection of all two element subsets of V. A fuzzy graph $G=(\sigma,\mu)$ is a set with two functions $\sigma: V\to[0,1]$ and $\mu: E\to[0,1]$ such that $\mu(uv)\le\sigma(u)\wedge\sigma(v)$ for all $u,v\in V$.

Definition 2.2

Let $G=(\sigma,\mu)$ be a fuzzy graph on V and $V_1\subseteq V$. Define σ_1 on V_1 by $\sigma_1(u)=\sigma(u)$ for all $u\in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(u\,v)=\mu(u\,v)$ for all $u,v\in V_1$, then (σ_1,μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $\langle V_1\rangle$.

Definition 2.3

The order p and size q of a fuzzy graph $G=(\sigma,\mu)$ are defined to be $p=\sum_{u\in V}\sigma(u)$ and $q=\sum_{uv\in E}\mu(uv)$.

Definition 2.4

Let $G=(\sigma,\mu)$ be a fuzzy graph on V and $D\subseteq V$ then the fuzzy cardinality of D is defined to be $\sum_{u\in D}\sigma(u)$.

Definition 2.5

Let $G=(\sigma,\mu)$ be a fuzzy graph on E and $D\subseteq E$ then the fuzzy edge cardinality of D is defined to be $\sum_{e\in D}\mu(e)$

Definition 2.6

An edge e= uv of a fuzzy graph is called an effective edge if $\mu(uv) = \sigma(u) \wedge \sigma(v)$.

$N(u) = \{ v \in V / \mu(uv) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u) \cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u). $\sum_{v \in N(u)} \sigma(v)$ is called the neighborhood degree of u and is denoted by dN(u). The minimum effective degree $\delta_E(G)=\min\{dE(u)|u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u)|u \in V(G)\}$.

Definition 2.7

The effective edge degree of an edge e = uv, is defined to be $d_E(e) = dE(u)+dE(v)$. The minimum edge effective degree and the maximum edge effective degree are $\delta'_E(G) = \min\{d_E(e) \mid e \in X\}$ and $\Delta'_E(G) = \max\{d_E(e) \mid e \in X\}$ respectively. N(e) is the set of all effective edges incident with the vertices of e. In a similar way minimum neighbourhood degree and the maximum neighbourhood degree denoted by δ'_N and Δ'_N respectively can also be defined.

Definition 2.8

The complement of a fuzzy graph G denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma, \bar{\mu})$ where $\bar{\mu}(uv) = \sigma(u) \wedge \sigma(v) - \mu(uv)$.

Definition 2.9

Let $\sigma : V \rightarrow [0,1]$ be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ,μ) where $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all uv∈E and is denoted by K_σ.

Definition 2.10

A fuzzy graph G= (σ,μ) is said to be connected if any two vertices in G are connected.

Definition 2.11

Let G = (σ,μ) be a fuzzy graph on (V, E). A subset S of E is said to be an edge dominating set in G if for every edge in E-S is adjacent to atleast one effective edge in S. The minimum fuzzy cardinality of an edge dominating set is G is called the edge domination number of G and is denoted by $\gamma'(G)$ or γ'.

Definition 2.12

An edge Dominating set D of a graph G=(V,E) is a fuzzy split edge dominating set if the induced subgraph <E-D> is disconnected. The split edge domination number γ'_s is the minimum fuzzy cardinality of a split edge dominating set.

Definition 2.13

A fuzzy graph G=(σ,μ) is said to be bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2 \in V_1$ or $v_1,v_2 \in V_2$. Further if $\mu(u,v)=\sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are, respectively, the restrictions of σ to V_1 and V_2.

Remark 2 .14

It is clear that if G has atleast one edge, then $0 \leq \gamma'(G) \leq q$. However, if a graph G has no effective edges, then γ' (G) = 0.

Definition 2.15

An edge dominating set S of a fuzzy graph G is said to be minimal edge dominating set if no proper subset S is an edge dominating set of G.

Definition 2.16

An edge e of a fuzzy graph G is said to be an isolated edge if no effective edges incident with the vertices of e. Thus an isolated edge does not dominate any other edge in G.

Definition 2.17

A set D of edges of a fuzzy graph is said to be independent if for every edge e $\in$D, no effective edge of D is incident with the vertices of e.

An edge dominating set D is a said to be an edge independent edge dominating set if <D> is independent.

The minimum fuzzy cardinality of an independent edge dominating set is called the independent edge domination number of G. It is denoted by γ_i'.

Definition 2.18

An edge covering of a fuzzy graph G is a subset F of E such that each vertex of G is an end of some edge in F minimum of edge.

The minimum fuzzy edge covering cardinality of G is called the fuzzy edge covering number of G and it is denoted by $\beta'(G)$

Definition 2.19

An edge independent set of a fuzzy graph G is a subset F of E such that no two edges of F are adjacent. The maximal edge independent number $\alpha'(G)$ is defined of the fuzzy cardinality of maximal edge independent set of G.

3. Main Results

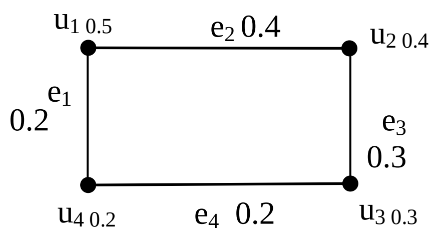
Theorem 3.1

An edge dominating set D of a fuzzy graph G is a split edge dominating set if and only if there exist two fuzzy edge e,f$\in$ E-D such that the path joining e and f contains a fuzzy edge of D.

Proof

Let G he a fuzzy graph and D_s' be the split edge domination set of G and hence for every two edges in E- D_s' such that the path joining the edge e and f contains an edge g$\in$ D_s'

Example:



$D'_s = \{e_1, e_3\}$

$E- D'_s = \{e_2, e_4\}$

$\gamma'_s(G) = 0.5$

The fuzzy graph of G induced by <E-D> is disconnected, hence D is a split edge dominating set of G with split edge domination number $\gamma'_s(G)$ = 0.3+0.2 = 0.5. We can see that there exist $e_2e_4 \in$E-D such that the path joining e_1 and e_3.

Theorem 3.2

For any fuzzy graph G=(σ,μ), $\gamma'(G) \leq \gamma'_s(G)$

Proof

By definition of $\gamma'(G), \gamma'_s(G)$ the result is obvious.

Theorem 3.3

For any fuzzy graph G= (σ, μ) $\gamma'_s(G) \leq \beta'(G)$ where β′(G) is a fuzzy edge covering number of G.

Proof

Let D be a minimal independent fuzzy edge set in G, then D has at least two fuzzy edges and every fuzzy edge in D is incident to some edge in E-D. This implies that E-D is a split edge dominating set of G thus result holds.

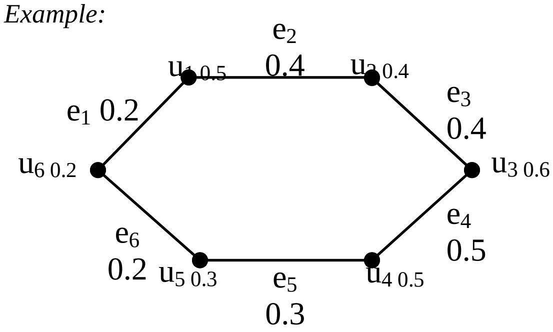
Theorem 3.4

For any fuzzy graph G= (σ, μ) $\gamma'_s(G) \leq \alpha'(G)$ where α′(G) is a fuzzy edge independent number of G.

Proof.

Let D be a maximal independent fuzzy edge set in G, then D has at least two fuzzy edges and every fuzzy edge in D is incident to some edge in E-D. This implies that E-D is a split edge dominating set of G thus result holds.

Example:



Edge covering set = {e_1, e_6, e_3, e_4}

$\beta'(G) = 1.3$

$\gamma'_s = 0.6$

$D'_s = \{e_3, e_6\}$

$\therefore \gamma'_s(G) \leq \beta'(G)$

$F = \{e_2, e_4, e_6\}$

$\alpha'(G) = 1.1$

$\gamma'_s(G) \leq \alpha'(G) = 0.6$

Theorem 3.5

A split edge dominating set D of G is minimal if and only if for each edge e∈D one of the following conditions holds.

(i) There exists an edge f∈E-D such that N(f)∩D={e}.
(ii) <E-D> is connected.

Proof.

(i) Suppose that D is minimal and there exists an edge e∈D such that e does not satisfy any of the above conditions. Then by conditions (i) and (ii), D′=D-{e} is a dominating set of G, also by (ii), <E-D′> is disconnected. This implies that D′ is split edge dominating set of G, which is contradiction. Hence <E-D> is connected.

Theorem 3.6

For any fuzzy graph G=(σ,μ),

$$\gamma'_s(G) \leq q.\Delta'(G)/(\Delta'(G)+1)$$

Proof

Let D be a split edge dominating set. Since D is minimal, by theorem (3.5) it follows that for each e∈D there exist f∈E-D such that 0<μ(u,ν)=σ(u)∧σ(ν) (ν is adjacent to u) this implies that E-D is a edge dominating set of G.

Thus $\gamma'(G) \leq |E - D| \leq q - \gamma'_s(G)$, since any fuzzy graph G =(σ,μ), $\gamma'(G) \geq q/(\Delta(G) + 1)$, hence result holds.

Theorem: 3.7

If $\gamma'_s(G) \leq \gamma'_c(G)$, then for any split edge dominating set D of G, E-D is also a split edge dominating set of G.

Proof

Since D is minimal, by theorem (3.6), E-D is edge dominating set of G and furthermore it is a split edge dominating set since <D> is disconnected.

Theorem 3.8

Let G=(σ,μ) be a fuzzy graph such that both G and $\bar{G}$ are connected, then $\gamma'_s(G) + \gamma'_s(\bar{G}) \leq 2q$

Proof

By Theorem (3.3) $\gamma'_s(G) \leq \beta'(G)$. since both G and $\bar{G}$ are connected, then $\Delta'(G), \Delta'(\bar{G}) \leq q$ this implies $\alpha'_0(G)$, $\alpha'_0(\bar{G}) \geq 0$. Hence $\gamma'_s(G) \leq q$. Similarly $\gamma'_s(\bar{G}) \leq q$. Thus, $\gamma'_s(G) + \gamma'_s(\bar{G}) \leq q + q = 2q$.

Theorem 3.9

If G=(σ,μ) has one fuzzy cut edge e and at least two fuzzy blocks H_1 and H_2 with e incident to all other edges of H_1 and H_2, then e is in every split edge dominating set of G.

Proof

Let D be a split edge dominating set of G, suppose $e \in E\text{-}D$, then each of H_1 and H_2 contributes atleast one edge to D, say f and g respectively.

This implies that D′=D-{f,g} ∪ {e} is a split edge dominating set of G.

Which is contradiction.

Hence e is in every split edge dominating set of G.

Theorem 3.10

Let e be a fuzzy cut edge of G, if there is a block H in G such that e is the only cut edge of H and e is incident with all edges of H, then there is a split edge dominating set of G containing e.

Proof

If there exist at least two blocks in G satisfying the given condition, then by theorem(3.8), e is in every split edge dominating set of G and hence the result.

ACKNOWLEDGEMENT

Thanks are due to the referees for their valuable comments and suggestions.

REFERENCES

1. Harary, E., 1969. Graph Theory. Addison Wesley, Reading, MA. McAlester, M.L.N., 1988. Fuzzy intersection graphs. Comp. Math. Appl. 15(10), 871-886.
2. Haynes, T.W., Hedetniemi S.T. and Slater P.J. (1998). Domination in Graphs: Advanced Topics, Marcel Dekker Inc. New York, U.S.A.
3. Haynes, T.W., Hedetniemi S.T. and Slater P.J. (1998). Fundamentals of domination in graphs, Marcel Dekker Inc. New York, U.S.A.
4. Kulli, V.R. and Janakiram B. (1997). The split domination number of graph. Graph Theory notes of New York. New York Academy of Sciences, XXXII, pp. 16-19.
5. Mahioub.Q.M. and Soner.N.D (2007), "The split domination number of fuzzy graph "Accepted for publication in Far East Journal of Applied Mathematics".
6. Ore, O. (1962). Theory of Graphs. American Mathematical Society Colloq. Publi., Providence, RI, 38.
7. Ponnappan.C.Y.,.Basheer Ahamed.S and Surulinathan.P.,Edge domination in fuzzy graph – new approach, International journal of IT,Engg&Applied sciences Research.Vol 4,No.1,Jan (2015)
8. Rosenfeld, A., 1975. Fuzzy graphs. In : Zadeh, L.A., Fu, K.S., Shimura, M. (Eds.), Fuzzy Sets and Their Applications. Academic Press, New York.
9. Somasundaram, A., and Somasundaram, S., Domination in fuzzy graphs-I, Pattern Recognit. Lett. 19(9) 1998), 787-791.