Soft g* -closed sets in Soft Topological Spaces

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Abstract

The focus of this paper is to introduce Soft g*-closed sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. Moreover we investigate the relationship of soft g*- closed sets with other soft closed sets. Further the new separation axioms namely soft $T^*_{\frac{1}{2}}-$space and soft ${}^*_{\frac{1}{2}}T-$space are introduced and its basic properties are discussed.

Keywords: soft g*-closed , soft g*-open, soft $T^*_{\frac{1}{2}}-$space , soft ${}^*_{\frac{1}{2}}T-$space

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1.INTRODUCTION

Molodstov[6] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainty problems. Soft systems provide a general framework with the involvement of parameters. In recent years the development in the field of soft set theory and its application has been taking place in a rapid pace. Muhammad Shabir and Munazza Naz [7]introduce the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters.

Levine [4] introduced g-closed sets in general topology. Kannan [3] introduced soft g-closed sets in soft topological spaces. In the present study, we introduce some new concepts in soft topological such as soft g*-closed sets and soft g*-open sets and derive some of its properties. soft $T^*_{\frac{1}{2}}-$space , soft ${}^*_{\frac{1}{2}}T-$space

2.PRELIMINARIES

Definition :2.1[6] Let U be the initial universe and P(U) denote the power set of U .Let E denote the set of all parameters .Let A be a non-empty subset of E.A pair (F,A) is called a soft set over U, where F is a mapping given by $F:A\to P(U)$.In other words ,a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon\in A, F(\varepsilon)$ may be considered as the set $\varepsilon-$ approximate elements of the soft set (F,A).

Definition :2.2[6]For two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B) if (1) $A \tilde{\subset} B$ and (2)for all $e \in A$,F(e) and G(e) are identical approximations .We write $(F,A) \tilde{\subset} (G,B)$. (F,A) is said to be a soft super set of (G,B) ,if (G,B) is a soft subset of (F,A).We denote it by $(F,A) \tilde{\supset} (G,B)$.

Definition :2.3[5] Two soft sets (F,A) and (G,B) over a common universe U are said to be soft equal if (F,A) is soft subset of (G,B) and (G,B) is a soft subset of (F,A)

Definition :2.4[5] The union of two soft sets of (F,A) and (G,B) over the common universe U is the soft set (H,C) where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

We write $(F,A) \cup (G,B) = (H,C)$

Definition :2.5[6] The intersection (H,C) of two soft sets of (F,A) and (G,B) over the common universe U denoted $(F,A) \cap (G,B)$, is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$

Definition :2.6[7] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if(1) Φ ,X belong to τ ,(2)the union of any number of soft sets in τ belongs to τ ,(3)the intersection of any two soft sets in τ belongs to τ .The triplet (X, τ ,E) is called a soft topological space over X. Let(X, τ ,E) be a soft space over X, then the members of τ are said to be soft open sets in X

Definition :2.7[8]A subset (A,E) of a topological space X is called soft regular closed (soft r-closed),if cl(int(A,E)) = (U,E).The complement of soft regular closed is soft regular set

Definition :2.8[9] The finite union of soft regular open sets is said to be soft π-open .The complement of soft π-open is said to be soft π-closed.

Definition :2.9[3]A subset (A,E) of a topological space X is called a soft generalized **closed**(soft g-closed) if $cl(A,E) \tilde{\subset} (U,E)$ whenever $(A,E) \tilde{\subset} (U,E)$ and (U,E)is soft open in X .

Definition :2.10[1]A subset (A,E) of a topological space X is called a soft regular closed(soft r-closed) if cl(int((A,E)) = (A,E) .The complement of soft regular closed set is soft regular open set.

Definition :2.11[9]A subset (A,E) of a topological space X is called a soft πg-closed in a soft topological space (X, τ,E) if cl(A,E)$\tilde{\subset}$ (U,E) whenever (A,E)$\tilde{\subset}$ (U,E) and (U,E)is soft π-open in X .

Definition :2.12[9]A subset (A,E) of a topological space X is called a soft πsg-closed in a soft topological space (X, τ,E) if scl(A,E)$\tilde{\subset}$ (U,E) whenever (A,E)$\tilde{\subset}$ (U,E) and (U,E) is soft π-open in X .

Definition :2.13[9]A subset (A,E) of a topological space X is called a soft gpr-closed in a soft topological space (X, τ,E) if pcl(A,E)$\tilde{\subset}$ (U,E) whenever (A,E)$\tilde{\subset}$ (U,E) and (U,E) is soft regular open in X .

Definition :2.14[8]A subset (A,E) of a topological space X is called a soft regular generalised-closed(soft rg-closed) in a soft topological space (X, τ,E) , if cl(A,E)$\tilde{\subset}$ (U,E) whenever (A,E) $\tilde{\subset}$ (U,E) and (U,E) is soft regular open in X .

Definition :2.15[1]A subset (A,E) of a topological space X is called a soft generalised-semi closed(soft gs-closed) in a soft topological space (X, τ,E) , if scl(A,E)$\tilde{\subset}$ (U,E) whenever (A,E) $\tilde{\subset}$ (U,E) and (U,E) is soft open in X .

Definition :2.16[1]A subset (A,E) of a topological space X is called a soft α generalised-closed(soft αg-closed) in a soft topological space (X, τ,E) , if αcl(A,E)$\tilde{\subset}$ (U,E) whenever (A,E)$\tilde{\subset}$ (U,E) and (U,E) is soft open in X .

Definition :2.17[1]A subset (A,E) of a topological space X is called a soft generalized β closed(soft gβ-closed) in a soft topological space (X, τ,E) , if βcl(A,E)$\tilde{\subset}$ (U,E) whenever (A,E)$\tilde{\subset}$ (U,E) and (U,E) is soft open in X .

3.SOFT g^*-CLOSED SET

Definition :3.1A subset (F,E) of a soft topological space (X, τ,E) is called a soft g^*- closed set , if cl(F,E)$\tilde{\subset}$ (U,E) whenever (F,E)$\tilde{\subset}$ (U,E) and (U,E) is g-open in X .

Proposition :3.2

i) Every soft closed set is soft g*-closed.
ii) Every soft g^*- closed set is soft g-closed.
iii) Every soft g^*- closed set is soft gs-closed.

iv) Every soft g^*- closed set is soft πg -closed.
v) Every soft g^*- closed set is soft g β -closed.
vi) Every soft g^*- closed set is soft rg-closed.
vii) Every soft g^*- closed set is soft α g-closed.
viii) Every soft g^*- closed set is soft πsg -closed.
ix) Every soft g^*- closed set is soft gpr-closed.
x) Every soft g^*- closed set is soft πgb -closed.

Proof:

i)Let (F,E) be soft closed set in (X, τ ,E) and (U,E) be soft g- open in X, such that (F,E)$\tilde{\subset}$ (U,E).Then cl(F,E)=(F,E)$\tilde{\subset}$ (U,E).Hence (F,E) is soft g*-closed

ii)Let (F,E) be soft g*- closed set in (X, τ ,E) .Let (U,E) be soft open in X such that (F,E)$\tilde{\subset}$ (U,E).Since every soft open set is soft g-open, we have cl(F,E)$\tilde{\subset}$ (U,E).Hence (F,E) is soft g-closed

iii) Let (F,E) be soft g*- closed set in (X, τ ,E) .Let (U,E) be soft open in X such that (F,E)$\tilde{\subset}$ (U,E).Since every soft open set is soft g-open, we have cl(F,E)$\tilde{\subset}$ (U,E).But scl(F,E)$\tilde{\subset}$ cl(F,E)$\tilde{\subset}$ (U,E). Hence (F,E) is soft gs-closed

iv)Proof is obvious and straight forward.

v) Proof is obvious and straight forward

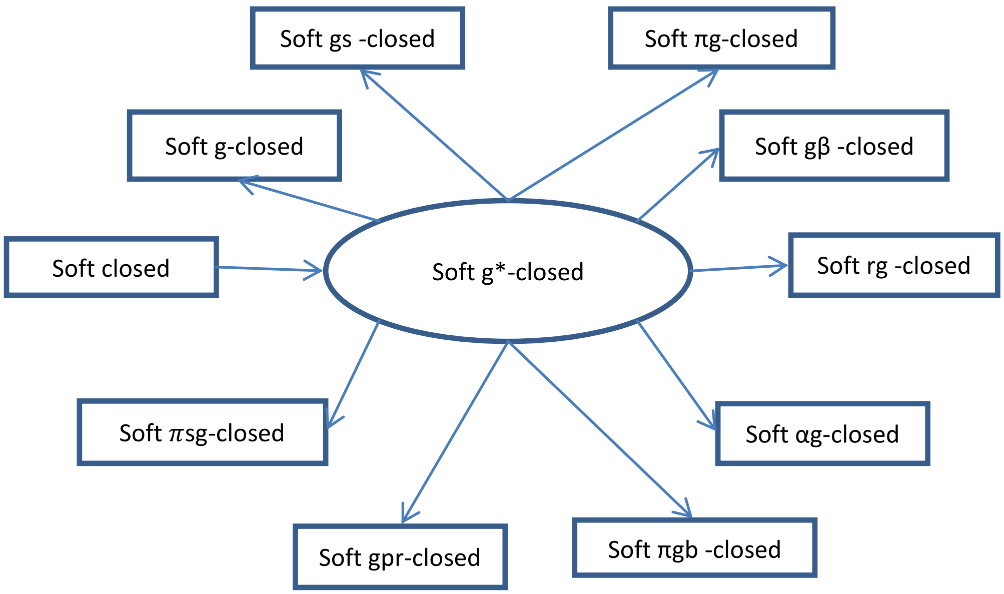
vi) Let (F,E) be soft g*- closed set in (X, τ ,E) .Let (U,E) be soft regular open in X such that (F,E)$\tilde{\subset}$ (U,E).Since every soft regular open set is soft g-open, we have cl(F,E)$\tilde{\subset}$ (U,E). Hence (F,E) is soft rg-closed

vii) Let (F,E) be soft g*- closed set in (X, τ ,E) .Let (U,E) be soft open in X such that (F,E)$\tilde{\subset}$ (U,E).Since every soft open set is soft α g-open, we have cl(F,E)$\tilde{\subset}$ (U,E).But α cl(F,E)$\tilde{\subset}$ cl(F,E)$\tilde{\subset}$ (U,E). Hence (F,E) is soft α g-closed

viii) Let (F,E) be soft g*- closed set in (X, τ ,E) .Let (U,E) be soft π -open in X such that (F,E) $\tilde{\subset}$ (U,E).Since every soft π - open set is soft g-open, we have cl(F,E)$\tilde{\subset}$ (U,E).But scl(F,E)$\tilde{\subset}$ cl(F,E)$\tilde{\subset}$ (U,E). Hence (F,E) is soft π sg-closed

ix) Let (F,E) be soft g*- closed set in (X, τ ,E) .Let (U,E) be soft regular open in X such that (F,E)$\tilde{\subset}$ (U,E).Since every soft regular open set is soft g-open, we have cl(F,E)$\tilde{\subset}$ (U,E).But pcl(F,E)$\tilde{\subset}$ cl(F,E)$\tilde{\subset}$ (U,E). Hence (F,E) is soft gpr-closed

x) Let (F,E) be soft g*- closed set in (X, τ ,E) .Let (U,E) be soft π -open in X such that (F,E)$\tilde{\subset}$ (U,E).Since every soft π - open set is soft g-open, we have cl(F,E)$\tilde{\subset}$ (U,E).But sbcl(F,E)$\tilde{\subset}$ cl(F,E)$\tilde{\subset}$ (U,E). Hence (F,E) is soft π gb-closed



Theorem :3.3 Every finite union of soft g*-closed set is soft g*-closed

Proof: Let (F,A) and (G,B) be two soft g*-closed subset of X. Let (U,E) be a soft g-open set in (X, τ ,E),such that (F,A)$\tilde{\cup}$(G,B)$\tilde{\subseteq}$ (U,E).Then cl(F,A) $\tilde{\subseteq}$ (U,E) and cl(G,B) $\tilde{\subseteq}$ (U,E).Therefore cl((F,A)$\tilde{\cup}$(G,B)) $\tilde{\subseteq}$ cl(F,A)$\tilde{\cup}$cl(G,B)$\tilde{\subseteq}$ (U,E).This implies cl((F,A)$\tilde{\cup}$(G,B)) $\tilde{\subseteq}$ (U,E). Hence (F,A)$\tilde{\cup}$(G,B) is soft g*- closed.

Theorem :3.4 If (F,E) is a soft g*-closed set of X such that (F,E)$\tilde{\subset}$ (G,E) $\tilde{\subset}$ cl(F,E),then (G,E) is **a** soft g*-closed.

Proof:

Let (G,E)$\tilde{\subset}$ (U,E) where (U,E) is soft g-open. Then (F,E) $\tilde{\subset}$ (G,E) implies (F,E) $\tilde{\subset}$ (U,E).Since (F,E) is a soft g*-closed set,cl(F,E)$\tilde{\subset}$ (U,E) .Given (G,E)$\tilde{\subset}$ cl(U,E).Hence cl(G,E)$\tilde{\subset}$ cl(cl(F,E)) $\tilde{\subset}$ cl(F,E)$\tilde{\subset}$(U,E) which implies cl(G,E)$\tilde{\subset}$(U,E).Therefore (G,E) is a soft g*-closed set.

Theorem :3.5 A soft set (G,E) is soft g*-closed if and only if cl(G,E) $-$ (G,E) contains only null soft g-closed set.

Proof: Necessity part Let (G,E) be a soft g*-closed set. Let (F,E) be soft g-closed such that (F,E) $\tilde{\subset}$ cl(G,E) $-$ (G,E).Then (F,E) $\tilde{\subset}$ cl(G,E) and (F,E)$\tilde{\subset}$(G,E)^C. This implies(G,E)$\tilde{\subset}$(F,E)^C.Then cl(G,E)$\tilde{\subset}$(F,E)^C as (F,E)^C is a soft g-open set .This implies (F,E)$\tilde{\subset}$(cl(G,E))^C . Therefore (F,E)$\tilde{\subset}$ cl(G,E) $\cap$(cl(G,E))^C. Hence (F,E) is a null soft g-closed set.

Theorem :3.6 If (A,E) is soft g-open and soft g*-closed ,then (A,E) is soft closed.

Proof: Obvious

4.SOFT g*-OPEN SET

Definition :4.1 A subset (A,E) of a topological space X is called soft g*-open in a soft topological space (X, τ ,E) ,if (F,E) $\tilde{\subset}$ int(A,E) whenever (F,E) $\tilde{\subset}$ (A,E) and (F,E) is soft g-closed in X.

Theorem :4.2 If (A,E) is a soft g*-open set of X and int(A,E)$\tilde{\subset}$ (B,E) $\tilde{\subset}$ (A,E),then (B,E) is also soft g*-open set of X.

Proof:Let (A,E) be soft g*-open in X. Suppose (G,E) is soft g-closed set such that (G,E) $\tilde{\subset}$ (B,E).By assumption, (B,E) $\tilde{\subset}$ (A,E) ,we have (G,E) $\tilde{\subset}$ (A,E) .Since (A,E) be soft g*-open set, (G,E) $\tilde{\subset}$ int(A,E).Then int(int(A,E)) $\tilde{\subset}$ (B,E) implies that int(A,E) $\tilde{\subset}$ int(B,E).Hence (G,E) $\tilde{\subset}$ int(A,E) $\tilde{\subset}$ int(B,E) implies that (G,E) $\tilde{\subset}$ int(B,E) . Then (B,E) is soft g*-open set of X.

Theorem :4.3 If (F,A) and (G,B) are soft g*-open sets,then ((F,A)$\tilde{\cap}$(G,B)) is also soft g*-open set

Proof: Let (F,A) and (G,B) be soft g*-open sets.Suppose (H,E) is soft g-closed set such that (H,E) $\tilde{\subset}$ ((F,A)$\tilde{\cap}$(G,B)).Then (H,E)$\tilde{\subset}$ (F,A) and (H,E)$\tilde{\subset}$ (G,B).Since (F,A) and (G,B) are soft g*-open sets , (H,E) $\tilde{\subset}$ int(F,A) and (H,E) $\tilde{\subset}$ int(G,B) . Therefore (H,E)$\tilde{\subset}$ int(F,A) $\cap$ int(G,B).Thus (H,E)$\tilde{\subset}$ int((F,A)$\tilde{\cap}$(G,B)). Hence ((F,A)$\tilde{\cap}$(G,B))is a soft g*- open set.

5.SOFT $T^*_{\frac{1}{2}}$-SPACE AND SOFT $^*_{\frac{1}{2}}T$-SPACE

Definition:5.1 A Soft topological space (X, τ ,E) is called a soft $T^*_{\frac{1}{2}}$ space if every g*-closed set is soft closed

Theorem :5.2 Every soft $T_{\frac{1}{2}}$ space is soft $T^*_{\frac{1}{2}}$ space**.**

Proof : Let (X, τ ,E) be a soft $T_{\frac{1}{2}}$ space and let (A,E) be soft g*-closed set in (X, τ ,E).By proposition 3.2(ii),(A,E) is soft g-closed.Since (X, τ ,E) is soft $T_{\frac{1}{2}}$ space,(A,E) is soft closed in

Theorem :5.3 For a space (X, τ ,E) ,the following conditions are equivalent

1) (X, τ ,E) is a soft $T^*_{\frac{1}{2}}$ space
2) Every singleton of X is either soft g-closed set or soft open.

Proof :(1)$\Rightarrow$ (2) Let x$\in$ X and suppose $\{x\}$is not soft g-closed set of (X, τ ,E) .Then $X-\{x\}$is not soft g-open.This implies X is the only soft set containing $X-\{x\}$.So $X-\{x\}$ is a soft g*-closed set of (X, τ ,E)..Since (X, τ ,E) is a soft $T^*_{\frac{1}{2}}$ space ,then $X-\{x\}$ is soft closed or equivalently $\{x\}$is soft open in (X, τ ,E) .

(2)$\Rightarrow$ (1) Let A be a soft g*-closed set of (X, τ ,E) .Trivially (A,E) $\tilde{\subset}$ cl(A,E).Let x $\in$ cl(A,E).By (2), $\{x\}$ is either soft g-closed or soft open.

Case (i) **:** Suppose $\{x\}$ is soft g-closed.If x$\notin$ A ,then cl(A,E)$-$ (A,E) contains a non – empty soft g-closed set .But this is not possible according to the theorem 3.5 as A is a soft g*-closed set.Therefore x$\in$ A.

Case (ii) :Suppose $\{x\}$ is soft open.Since x $\in$cl(A,E) ,then $\{x\}\cap A\neq\phi$.so x $\in$ (A,E)

Therefore x $\in$(A,E) .So in any case cl(A,E)$\tilde{\subset}$ (A,E) .Thus cl(A,E)$=$ (A,E) or equivalently (A,E) is a soft closed set of (X, τ ,E)

Definition:5.4 A Soft topological space (X, τ ,E) is called a soft ${}_{\frac{1}{2}}^{*}T$ space if every soft g*-closed set is soft g-closed

Theorem :5.5 Every soft $T_{\frac{1}{2}}$ space is soft ${}_{\frac{1}{2}}^{*}T$ space**.**

Proof : Let (X, τ ,E) be a soft $T_{\frac{1}{2}}$ space .Let (A,E) be soft g-closed set of (X, τ ,E).Since (X, τ ,E) is soft $T_{\frac{1}{2}}$ space,(A,E) is soft closed in (X, τ ,E). By proposition 3.2(i),(A,E) is soft g^{*}-closed. Therefore (X, τ ,E) is a soft ${}_{\frac{1}{2}}^{*}T$ space.

Theorem :5.6 If (X, τ ,E) is a soft ${}_{\frac{1}{2}}^{*}T$ space **,**then x$\in$ X , $\{x\}$is either soft closed or soft g*-open.

Proof : Suppose (X, τ ,E) is a soft ${}_{\frac{1}{2}}^{*}T$ space **.** x$\in$ X and assume that $\{x\}$is not soft closed. Then $X-\{x\}$ is not a soft open set .This implies $X-\{x\}$ is a g-closed set since X is the only soft open set which contains $X-\{x\}$ .Since (X, τ ,E) ,then $X-\{x\}$ is a soft g*-closed or equivalently $\{x\}$ is a soft g*-open.

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