On Semi-Continuous and Semi-Irresolute Soft Multifunctions

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***Abstract*— In this paper, we introduce the concept of upper (lower) semi continuous and upper (lower) semi irresolute soft multifunctions that defined sense [9] and obtain some characterizations and several properties concerning these soft multifunctions. Also, the relationship between these soft multifunctions are investigated.**

***Keywords*— Soft semi open sets, soft topology, soft multifunctions, soft continuity.**

I. INTRODUCTION

Soft set theory is a new mathematical tool for dealing with uncertainties in the complicated problems in the field of sciences such as engineering, medical science etc. Molodtsov [1] introduced the concept of soft set theory as a new approach for coping with uncertainties and these problems. Then Molodtsov et al. [2], Maji and Roy [3], Shabir and Naz [4], Çağman et al. [5], Kharal and Ahmad [6], Maji et al. [7], Ali et al. [8], Zorlutuna et al. [10], Chen [11], Akdağ et al. [12] studied in the soft set theory.

In this paper, we define the semi upper (lower) continuous multifunction from a soft topological spaces (X,τ,E) to a soft topological spaces (Y,σ,K) and study several properties of these continuous multifunctions. Also we we define the semi upper (lower) irresolute multifunctions and investigate some properties.

II. PRELIMINARIES

Definition 1. [9] The soft set (G,E) over X is called a soft point in X, denoted by $E_e{}^x$ if for $e\in E$ there exist $x\in X$ such that $G(e)=\{x\}$ and $G(e')=\emptyset$ for all $e'\in E-\{e\}$.

Definition 2. [9] The soft point $E_e{}^x$ is said to be in the soft set (H,E), denoted by $E_e{}^x \tilde{\in} (H,E)$, if $x\in H(e)$.

Definition 3. [11] Let (X,τ,E) be a soft topological space and (F,E) be a soft set over X. Then,
(a) (F,E) is soft semi open set if and only if $(F,E)\tilde{\subseteq} cl(int(F,E))$.
(b) (F,E) is soft semi closed set if and only if $int(cl(F,E))\tilde{\subseteq}(F,E)$.

Definition 4. [11] Let (X,τ,E) be a soft topological space and (F,E) be a soft set over X. The soft semi interior and soft semi closure of (F,E) is denoted by $sint(F,E)$, $scl(F,E)$ and defined as follows respectively.
(a) $sint(F,E)=\tilde{\cup}\{(G,E):(G,E)$ is soft semi open and $(G,E)\tilde{\subseteq}(F,E)\}$.
(b) $scl(F,E)=\tilde{\cup}\{(G,E):(G,E)$ is soft semi closed and $(F,E)\tilde{\subseteq}(G,E)\}$.

Lemma 1. [12] Let (F,E) be a soft set in a soft topological space (X,τ,E). Then
(i) $scl(F,E)=(F,E)\,\tilde{\cup}\, int(cl(F,E))$ and
(ii) $sint(F,E)=(F,E)\,\tilde{\cap}\, cl(int(F,E))$.

Proposition 1. [11] Let (X,τ,E) be a soft topological space and (F,E) be a soft set over X. Then the following statements are true;
(a) $scl(\tilde{X}-F,E)=\tilde{X}-sint(F,E)$,
(b) $sint(\tilde{X}-F,E)=\tilde{X}-scl(F,E)$.

Definition 5. [9] Let $S(X,E)$ and $S(Y,K)$ be two soft classes. Let $u:X\to Y$ be multifunction and $p:E\to K$ be mapping. Then a soft multifunction $F:S(X,E)\to S(Y,K)$ is defined as follows:
For a soft set (G,E) in (X,E), $(F(G,E),K)$ is a soft set in (Y,K) given by $F(G,E)(k)=\cup(u(G(e)))$ for $k\in K$. $(F(G,E),K)$ is called a soft image of a soft set (G,E).
Moreover, $F(G,E)=\tilde{\cup}\{F(E_e{}^x):E_e{}^x\,\tilde{\in}\,(G,E)\}$, for a soft subset (G,E) of X .

Definition 6. [9] Let $F:(X,E) \to (Y,K)$ be a soft multifunction. The soft upper inverse image and soft lower inverse image of (H,K) denoted by $F^{+}(H,K)$, $F^{-}(H,K)$ and defined as follows respectively;
$F^{+}(H,K) = \{E_e{}^x \tilde{\in} X: F(E_e{}^x) \tilde{\subseteq} (H,K)\}$,
$F^{-}(H,K) = \{E_e{}^x \tilde{\in} X: F(E_e{}^x) \tilde{\cap} (H,K) \neq \Phi\}$.

Definition 7. [9] Let $F:S(X,E) \to S(Y,K)$ and $G:S(X,E) \to S(Y,K)$ be two soft multifunctions. Then, F equal to G if $F(E_e{}^x) = G(E_e{}^x)$, for each $E_e{}^x \tilde{\in} X$.

Definition 8. [9] The soft multifunction $F:S(X,E) \to S(Y,K)$ is called surjective if p and u are surjective.

Theorem 1. [9] Let $F:S(X,E) \to S(Y,K)$ be a soft multifunction Then, for soft sets (F,E), (G,E) and for a family of soft sets $(G_i,E)_{i\in I}$ in the soft class $S(X,E)$, the following statements are hold:
(a) $F(\Phi) = \Phi$
(b) $F(\tilde{X}) \tilde{\subseteq} \tilde{Y}$
(c) $F((G,A) \tilde{\cup} (H,B)) = F(G,A) \tilde{\cup} F(H,B)$ in general $F(\tilde{\cup}_{i\in I}\,(G_i,E)) = \tilde{\cup}_{i\in I}\,(F(G_i,E))$
(d) $F((G,A) \tilde{\cap} (H,B)) \tilde{\subseteq} F(G,A) \tilde{\cap} F(H,B)$ in general $F(\tilde{\cap}_{i\in I}\,(G_i,E)) \tilde{\subseteq} \tilde{\cap}_{i\in I}\,(F(G_i,E))$
(e) If $(G,E) \tilde{\subseteq} (H,E)$, then $F(G,E) \tilde{\subseteq} F(H,E)$.

Theorem 2. [9] Let $F:S(X,E) \to S(Y,K)$ be a soft multifunction Then, for soft sets (G,K), (H,K) in the soft class $S(X,E)$ the following statements are hold:
(a) $F^{-}(\Phi) = \Phi$ and $F^{+}(\Phi) = \Phi$
(b) $F^{-}(\tilde{Y}) = \tilde{X}$ and $F^{+}(\tilde{Y}) = \tilde{X}$
(c) $F^{-}((G,K) \tilde{\cup} (H,K)) = F^{-}(G,K) \tilde{\cup} F^{-}(H,K)$
(d) $F^{+}(G,K) \tilde{\cup} F^{+}(H,K) \tilde{\subseteq} F^{+}((G,K) \tilde{\cup} (H,K))$
(e) $F^{-}((G,K) \tilde{\cap} (H,K)) \tilde{\subseteq} F^{-}(G,K) \tilde{\cap} F^{-}(H,K)$
(f) $F^{+}(G,K) \tilde{\cap} F^{+}(H,K) = F^{+}((G,K) \tilde{\cap} (H,K))$
(g) If $(G,K) \tilde{\subseteq} (H,K)$, then $F^{-}(G,K) \tilde{\subseteq} F^{-}(H,K)$ and $F^{+}(G,K) \tilde{\subseteq} F^{+}(H,K)$.

Proposition 2. [9] Let (G_i,K) be soft sets over Y for each $i \in I$. Then the following statements are true for a soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$;
(a) $F^{-}(\tilde{\cup}_{i\in I}\,(G_i,K)) = \tilde{\cup}_{i\in I}\,(F^{-}(G_i,K))$,
(b) $\tilde{\cap}_{i\in I}\,(F^{+}(G_i,K)) = F^{+}(\tilde{\cap}_{i\in I}\,(G_i,K))$,
(c) $\tilde{\cup}_{i\in I}\,(F^{+}(G_i,K)) \subset F^{+}(\tilde{\cup}_{i\in I}\,(G_i,K))$,
(d) $F^{-}(\tilde{\cap}_{i\in I}\,(G_i,K)) \subset \tilde{\cap}_{i\in I}\,(F^{-}(G_i,K))$.

Proposition 3. [9] Let $F:(X,\tau,E) \to (Y,\sigma,K)$ be a soft multifunction. Then the following statements are true:
(a) $(G,A) \tilde{\subseteq} F^{+}(F(G,A)) \tilde{\subseteq} F^{-}(F(G,A))$ for a soft subset (G,A) in X. If F is surjectice then $(G,A) = F^{+}(F(G,A)) = F^{-}(F(G,A))$.
(b) $F(F^{+}(H,B)) \tilde{\subseteq} (H,B) \tilde{\subseteq} F(F^{-}(H,B))$ for a soft subset (H,B) in Y.
(c) For two soft subsets (H,B) and (U,C) in Y such that $(H,B) \tilde{\cap} (U,C) = \Phi$ then $F^{+}(H,B) \tilde{\cap} F^{-}(U,C) = \Phi$.

Proposition 4. [9] Let $F:(X,\tau,E) \to (Y,\sigma,K)$ and $G:(Y,\sigma,K) \to (Z,\eta,L)$ be two soft multifunction. Then the following statements are true:
(a) $(F^{-})^{-} = F$
(b) For a soft subset (T,C) in Z, $(GoF)^{-}(T,C) = F^{-}(G^{-}(T,C))$ and $(GoF)^{+}(T,C) = F^{+}(G^{+}(T,C))$.

Proposition 5. [9] Let (G,K) be a soft set over Y. Then the following statements are true for a soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$:
(a) $F^{+}(\tilde{Y} - (G,K)) = \tilde{X} - F^{-}(G,K)$
(b) $F^{-}(\tilde{Y} - (G,K)) = \tilde{X} - F^{+}(G,K)$.

III. Upper and Lower Semi Continuity of Soft Multifunctions

Definition 9. Let (X,τ,E) and (Y,σ,K) be two soft topological spaces. Then a soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ is said to be;

(a) soft upper semi continuous at a soft point ${E_e}^x \tilde{\in} \tilde{X}$ if for every soft open set (G,K) such that $F({E_e}^x) \tilde{\subseteq} (G,K)$, there exists a soft semi open neighborhood (P,E) of ${E_e}^x$such that $F({E_e}^z) \tilde{\subseteq} (G,K)$ for all ${E_e}^z \tilde{\in} (P,E)$.

(b) soft lower semi continuous at a point ${E_e}^x \tilde{\in} \tilde{X}$ if for every soft open set (G,K) such that $F({E_e}^x) \tilde{\cap} (G,K) \neq \Phi$, there exists a soft semi open neighborhood (P,E) of ${E_e}^x$such that $F({E_e}^z) \tilde{\cap} (G,K) \neq \Phi$ for all ${E_e}^z \tilde{\in} (P,E)$.

(c) soft upper(lower) semi continuous if F has this property at every soft point of X.

Proposition 6. $F:(X,\tau,E) \to (Y,\sigma,K)$ is soft lower semi continuous multifunction if and only if $F^-(G,K)$ is soft semi open set in X, for every soft open set (G,K).

Proof. First assume that F is soft lower semi continuous. Let (G,K) be any soft open set over Y and let ${E_e}^x \tilde{\in} F^-(G,K)$. Then there exists a soft semi open neighborhood (P,E) of ${E_e}^x$ such that $F({E_e}^z) \tilde{\cap} (G,K) \neq \Phi$ for all ${E_e}^z \tilde{\in} (P,E)$. So, $(P,E) \tilde{\subseteq} F^-(G,K)$ and ${E_e}^x \tilde{\in} (P,E) = sint(P,E) \tilde{\subseteq} sint(F^-(G,K))$. Thus, $F^-(G,K) \tilde{\subseteq} sint(F^-(G,K))$ which implies that $F^-(G,K)$ is soft semi-open in X.

Now suppose that $F^-(G,K)$ is soft semi-open. Let ${E_e}^x \tilde{\in} F^-(G,K)$. Then $F^-(G,K)$ is an soft semi-open neighborhood of ${E_e}^x$and for all ${E_e}^z \tilde{\in} F^-(G,K)$ we have $F({E_e}^z) \tilde{\cap} (G,K) \neq \Phi$. So, F is soft lower semi continuous.

Proposition 7. A soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ is soft upper semi continuous if and only if $F^+(G,K)$ is soft semi open in X, for every soft open set (G,K).

Proof. The proof is similar to previous proposition.

Theorem 3. For a soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ the following are equivalent;

(a) F is soft lower semi continuous.

(b) $F^+(G,K)$ is soft semi closed set in X, for every soft closed set (G,K) in Y.

(c) $int(cl(F^+(H,K))) \tilde{\subseteq} F^+(H,K)$, for every soft closed set (H,K) in Y.

(d) $F^-(G,K) \tilde{\subseteq} cl(int(F^-(G,K)))$, for every soft open set (G,K) in Y.

(e) $scl(F^+(B,K))) \tilde{\subseteq} F^+(cl(B,K))$, for every soft set (B,K) in Y.

(f) $F^-(int(N,K)) \tilde{\subseteq} sint(F^-(N,K))$, for every soft set (N,K) in Y.

Proof. (a)⇒(b) Let (G,K) be soft closed set in Y. Then $\tilde{Y} - (G,K)$ is soft open set in Y. By proposition 8, $F^-(\tilde{Y} - (G,K)) = \tilde{X} - F^+(G,K)$ is soft open set in X. Thus $F^+(G,K)$ is soft semi closed set in X.

(b)⇒(c) Let (H,K) be soft closed set in Y. By (b), $F^+(H,K)$ is soft semi closed set in X. Then $cl(F^+(H,K)) \tilde{\subseteq} F^+(H,K)$ and thus $int(cl(F^+(H,K))) \tilde{\subseteq} int(F^+(H,K)) \tilde{\subseteq} F^+(H,K)$.

(c)⇒(d) Let (G,K) be soft open set in Y, then $\tilde{Y} - (G,K)$ is soft closed set in Y. By (c), $int(cl(F^+(\tilde{Y} - (G,K)))) \tilde{\subseteq} F^+(\tilde{Y} - (G,K))$. Thus, $int(cl(\tilde{X} - F^-(G,K))) = int(\tilde{X} - int(F^-(G,K))) = \tilde{X} - cl(int(F^-(G,K))) \tilde{\subseteq} \tilde{X} - F^-(G,K))$. Thus, $F^-(G,K) \tilde{\subseteq} cl(int(F^-(G,K)))$.

(d)⇒(e) For every soft set (B,K) in Y, $\tilde{Y} - cl(B,K)$ is soft open set. By (d), $F^-(\tilde{Y} - cl(B,K)) \tilde{\subseteq} cl(int(F^-(\tilde{Y} - cl(B,K)))$. Then $\tilde{X} - cl(int(F^-(\tilde{Y} - cl(B,K))) \tilde{\subseteq} \tilde{X} - F^-(\tilde{Y} - cl(B,K))$ and $int(cl(\tilde{X} - F^-(\tilde{Y} - cl(B,K)))) \tilde{\subseteq} F^+(\tilde{Y} - (\tilde{Y} - cl(B,K)))$. Thus $int(cl(F^+(cl(B,K)))) \tilde{\subseteq} F^+(cl(B,K))$. By lemma 1, $scl(F^+(cl(B,K)) \tilde{\subseteq} F^+(cl(B,K))$. Therefore, $scl(F^+(B,K)) \tilde{\subseteq} F^+(cl(B,K))$.

(e)⇒(f) For every soft set (N,K) in Y, $\tilde{Y} - (N,K)$ is soft set and by (e), $scl(F^+(\tilde{Y} - (N,K))) \tilde{\subseteq} F^+(cl(\tilde{Y} - (N,K)))$. Then, $X - F^+(cl(\tilde{Y} - (N,K))) \tilde{\subseteq} \tilde{X} - scl(F^+(\tilde{Y} - (N,K)))$. Then, $F^-(\tilde{Y} - cl(\tilde{Y} - (N,K))) \tilde{\subseteq} sint(\tilde{X} - F^+(\tilde{Y} - (N,K)))$. Thus, $F^-(int(\tilde{Y} - (\tilde{Y} - (N,K)))) \tilde{\subseteq} sint(F^-(\tilde{Y} - (\tilde{Y} - (N,K))))$. Therefore, $F^-(int(N,K)) \tilde{\subseteq} sint(F^-(N,K))$.

(f)⇒(a) Let (G,K) be soft open set in Y such that ${E_e}^x \tilde{\in} F^-(G,K)$. By (f), $F^-(int(G,K)) \tilde{\subseteq} sint(F^-(G,K))$. Then, $F^-(G,K) \tilde{\subseteq} sint(F^-(G,K))$. Therefore, $F^-(G,K) = (U,E)$ is soft semi open set containing ${E_e}^x$. Thus, F is soft lower semi continuous.

Theorem 4. For a soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ the following are equivalent;

(a) F is soft upper semi continuous

(b) $F^-(G,K)$ is soft semi closed set in X, for every soft closed set (G,K) in Y.

(c) $int(cl(F^-(H,K))) \tilde{\subseteq} F^-(H,K)$, for every soft closed set (H,K) in Y.

(d) $F^+(G,K) \tilde{\subseteq} cl(int(F^+(G,K)))$, for every soft open set (G,K) in Y.

(e) $scl(F^-(B,K)) \tilde{\subseteq} F^-(cl(B,K))$, for every soft set (B,K) in Y.

(f) $F^+(int(N,K)) \tilde{\subseteq} sint(F^+(N,K))$, for every soft set (N,K) in Y.

Proof. The proof is similar to previous theorem.

Definition 10. [9] Let (X,τ,E) and (Y,σ,E) be two soft topological spaces. Then a soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ is said to be;

(a) soft upper continuous at a point $E_e{}^x \tilde{\in} X$ if for each soft open (G,K) such that $F(E_e{}^x) \tilde{\subset} (G,K)$, there exists an soft open neighborhood $P(E_e{}^x)\ of\ E_e{}^x$ such that $F(E_e{}^z) \tilde{\subset} (G,K)$ for all $E_e{}^z \tilde{\in} P(E_e{}^x)$.

(b) soft lower continuous at a point $E_e{}^x \tilde{\in} X$ if for each soft open (G,K) such that $F(E_e{}^x) \tilde{\cap} (G,K) \neq \Phi$, there exists an soft open neighborhood $P(E_e{}^x)$ of $E_e{}^x$ such that $F(E_e{}^z) \tilde{\cap} (G,K) \neq \Phi$ for all $E_e{}^z\} \tilde{\in} P(E_e{}^x)$.

(c) soft upper(lower) continuous if F has this property at every soft point of X.

Proposition 8. [9] A soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ is;

(a) soft upper continuous if and only if $F^{+}(G,K)$ is soft open in X, for every soft open set (G,K).

(b) soft lower continuous if and only if $F^{-}(G,K)$ is soft open set in X, for every soft set (G,K).

Remark 1. Every soft upper (lower) continuous functions is soft upper (lower) semi continuous. But the converse is not true as shown by the following examples.

Example 1. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$. $\tau = \{\Phi, X, (G,E)\}$, where $(G,E) = \{(e_2, \{x_2\})\}$ and $\sigma = \{\Phi, Y, (H,K)\}$, where $(H,K) = \{(k_1, \{y_2\}), (k_2, Y)\}$. Let $u: X \to Y$ be multifunction defined as: $u(x_1) = \{y_1, y_2\}$, $u(x_2) = \{y_2\}$ and $p: E \to K$ be mapping defined as: $p(e_1) = \{k_1\}$, $p(e_2) = \{k_2\}$. Then the soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ is soft upper semi continuous but is not soft upper continuous. Because for a soft open set (H,K) in Y, $F^{+}(H,K) = \{(e_1, \{x_2\}), (e_2, X)\}$ is soft semi open set but is not soft open set in X.

Example 2. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$. $\tau = \{\Phi, X, (G,E)\}$, where $(G,E) = \{(e_2, \{x_2\})\}$ and $\sigma = \{\Phi, Y, (H,K)\}$, where $(H,K) = \{(k_1, \{y_1\}), (k_2, \{y_2\})\}$. Let $u: X \to Y$ be multifunction defined as: $u(x_1) = \{y_1, y_2\}$, $u(x_2) = \{y_2\}$ and $p: E \to$K be mapping defined as: $p(e_1) = \{k_1\}$, $p(e_2) = \{k_2\}$. Then the soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ is soft lower semi continuous but is not soft lower continuous . Because for a soft open set (H,K) in Y, $F^{+}(H,K) = \{(e_1, \{x_2\}), (e_2, X)\}$ is soft semi open set but is not soft open in X.

UPPER AND LOWER SEMI-IRRESOLUTE SOFT MULTIFUNCTIONS

Definition 11. Let (X,τ,E) and (Y,σ,K) be two soft topological spaces. Then a soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ is said to be;

(a) soft upper semi irresolute at a soft point $E_e{}^x \tilde{\in} X$ if for every soft semi open set (G,K) such that $F(E_e{}^x) \tilde{\subset} (G,K)$, there exists a soft semi open neighborhood (P,E) of $E_e{}^x$ such that $F(E_e{}^z) \tilde{\subset} (G,K)$ for all $E_e{}^z \tilde{\in} (P,E)$.

(b) soft lower semi irresolute at a point $E_e{}^x \tilde{\in} X$ if for every soft semi open set (G,K) such that $F(E_e{}^x) \tilde{\cap} (G,K) \neq \Phi$, there exists a soft semi open neighborhood (P,E) of $E_e{}^x$ such that $F(E_e{}^z) \tilde{\cap} (G,K) \neq \Phi$ for all $E_e{}^z \tilde{\in} (P,E)$.

(c) soft upper(lower) semi irresolute if F has this property at every soft point of X.

Proposition 9. $F:(X,\tau,E) \to (Y,\sigma,K)$ is soft lower semi continuous multifunction if and only if $F^{+}(G,K)$ is soft semi open set in X, for every soft semi open set (G,K).

Proof. First assume that F is soft upper semi irresolute. Let (G,K) be any soft semi open set over Y and let $E_e{}^x \tilde{\in} F^{+}(G,K)$. Then by soft upper semi irresolute of F, there exists a soft semi open neighbourhood (U,E) of $E_e{}^x$ such that $F(E_e{}^z) \tilde{\subset} (G,K)$ for all $E_e{}^z \tilde{\in} (U,E)$. So, $F(U,E) \tilde{\subset} (G,K)$ and $(U,E) \tilde{\subset} F^{+}(G,K)$. Since (U,E) is soft semi open then $(U,E) = sint(U,E) \tilde{\subset} sint(F^{+}(G,K))$. Thus $E_e{}^x \tilde{\in} sint(F^{+}(G,K))$ and $F^{+}(G,K) \tilde{\subset} sint(F^{+}(G,K))$. Therefore we have $F^{+}(G,K)$ is soft semi-open in X.

Conversely, suppose that $F^{+}(G,K)$ is soft semi open set for every soft semi open set (G,K). Let $E_e{}^x \tilde{\in} F^{+}(G,K)$. Put $(U,E) = F^{+}(G,K)$, then (U,E) is a soft semi-open neighborhood of $E_e{}^x$ and $F(E_e{}^z) \tilde{\in} (G,K)$ for all $E_e{}^z \tilde{\in} (U,E)$. Thus, F is soft upper semi irresolute.

Proposition 10. A soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ is soft lower semi irresolute if and only if $F^{-}(G,K)$ is soft semi open in X, for every soft semi open set (G,K).

Proof. The proof is similar to previous proposition.

Example 3. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$. $\tau = \{\Phi, X, (G,E)\}$, where $(G,E) = \{(e_2, \{x_2\})\}$ and $\sigma = \{\Phi, Y, (H,K)\}$, where $(H,K) = \{(k_1, \{y_1\}), (k_2, \{y_2\})\}$. Let $u: X \to Y$ be multifunction defined as: $u(x_1) = \{y_1, y_2\}$,

$u(x_2) = \{y_2\}$ and $p: E \to K$ be mapping defined as: $p(e_1) = \{k_1\}$, $p(e_2) = \{k_2\}$. Then the soft multifunction F:(X,τ,E)→(Y,σ,K) is soft upper semi irresolute. Because for the soft semi open sets $\{(k_2,\{y_2\})\}$, $\{(k_1,\{y_1\}),(k_2,\{y_2\})\}$ and $\{(k_1,\{y_1\}),(k_2,Y)\}$ the upper inverses $F^+(\{(k_2,\{y_2\})\}) = \{(e_2,\{x_2\})\}$, $F^+(\{(k_1,\{y_1\}),(k_2,\{y_2\})\}) = \{(e_2,\{x_2\})\}$ and $F^+(\{(k_1,\{y_1\}),(k_2,Y)\}) = \{(e_2,X)\}$ are soft semi open sets.

Example 4. Let $X = \{x_1,x_2\}$, $Y = \{y_1,y_2\}$, $E = \{e_1,e_2\}$, $K = \{k_1,k_2\}$. $\tau = \{\Phi, X, (G,E)\}$, where $(G,E) = \{(e_2,\{x_2\})\}$ and $\sigma = \{\Phi, Y, (H,K)\}$,where $(H,K) = \{(k_1,\{y_1\}),(k_2,\{y_2\})\}$. Let $u: X \to Y$ be multifunction defined as: $u(x_1) = \{y_1,y_2\}$, $u(x_2) = \{y_2\}$ and $p: E \to K$ be mapping defined as: $p(e_1) = \{k_1\}$, $p(e_2) = \{k_2\}$. Then the soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ is soft lower semi irresolute. Because for the soft semi open sets $\{(k_2,\{y_2\})\}$, $\{(k_1,\{y_1\}),(k_2,\{y_2\})\}$ and $\{(k_1,\{y_1\}),(k_2,Y)\}$ the lower inverses $F^-(\{(k_2,\{y_2\})\}) = \{(e_2,X)\}$, $F^-(\{(k_1,\{y_1\}),(k_2,\{y_2\})\}) = \{(e_1,\{x_1\}),(e_2,X)\}$ and $F^-(\{(k_1,\{y_1\}),(k_2,Y)\}) = \{(e_1,\{x_1\}),(e_2,X)\}$ are soft semi open sets.

Theorem 5. For a soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ the following properties are equivalent;

(a) F is soft upper semi irresolute

(b) $F^-(H,K)$ is soft semi closed set in X, for every soft semi closed set (H,K) in Y.

(c) $sint(scl(F^-(M,K))) \tilde{\subseteq} F^-(M,K)$, for every soft semi closed set (M,K) in Y.

(d) $F^+(G,K) \tilde{\subseteq} scl(sint(F^+(G,K)))$, for every soft semi open set (G,K) in Y.

(e) $scl(F^-(B,K))) \tilde{\subseteq} F^-(scl(B,K))$, for every soft set (B,K) in Y.

(f) $F^+(sint(N,K)) \tilde{\subseteq} sint(F^+(N,K))$, for every soft set (N,K) in Y.

Proof. (a)⇒(b) Let (H,K) be any soft semi closed set in Y. Then $\tilde{Y} - (H,K)$ is soft semi open set in Y. Since F is soft upper semi irresolute, by proposition 11, $F^+(\tilde{Y} - (H,K)) = \tilde{X} - F^-(H,K)$ is soft semi open set in X. Thus $F^-(H,K)$ is soft semi closed set in X.

(b)⇒(c) Let (M,K) be any soft semi closed set in Y. Then $scl(M,K) = (M,K)$ and by (b), $F^-(M,K)$ is soft semi closed set in X. Then, $scl(F^-(M,K)) = F^-(M,K)$ and thus $sint(scl(F^-(M,K))) \tilde{\subseteq} F^-(M,K)$.

(c)⇒(d) Let (G,K) be any soft semi open set in Y. Then $\tilde{Y} - (G,K)$ is soft semi closed set in Y. By (c), $sint(scl(F^-(\tilde{Y} - (G,K)))) \tilde{\subseteq} F^-(\tilde{Y} - (G,K))$. Then, $\tilde{X} - F^-(\tilde{Y} - (G,K)) \tilde{\subseteq} \tilde{X} - sint(scl(F^-(\tilde{Y} - (G,K))))$ and $F^+(\tilde{Y} - (\tilde{Y} - (G,K))) \tilde{\subseteq} scl(sint(\tilde{X} - (F^-(\tilde{Y} - (G,K)))))$ and $F^+(G,K) \tilde{\subseteq} scl(sint(F^+(\tilde{Y} - (\tilde{Y} - (G,K))))) = scl(sint(F^+(G,K)))$.

(d)⇒(e) Obvious.

(e)⇒(f) Let (N,K) be any soft set in Y. Then $\tilde{Y} - (N,K)$ is soft set and by (e) $scl(F^-(\tilde{Y} - (N,K))) \tilde{\subseteq} F^-(scl(\tilde{Y} - (N,K))$ and $\tilde{X} - F^-(scl(\tilde{Y} - (N,K))) \tilde{\subseteq} \tilde{X} - scl(F^-(\tilde{Y} - (N,K)))$. Then $F^+(\tilde{Y} - scl(\tilde{Y} - (N,K))) \tilde{\subseteq} sint(\tilde{X} - F^-(\tilde{Y} - (N,K)))$ and hence we have $F^+(sint(N,K)) \tilde{\subseteq} sint(F^+(N,K))$.

(f)⇒(a) Let (G,K) be any soft semi open set in Y. By (f), $F^+(sint(G,K)) \tilde{\subseteq} sint(F^+(G,K))$ and $F^+(G,K) \tilde{\subseteq} sint(F^+(G,K))$. Thus $F^+(G,K)$ is soft semi open set in X. Then by proposition 11, F is soft upper semi irresolute.

Theorem 6. For a soft multifunction $F:(X,\tau,E) \to (Y,\sigma,K)$ the following properties are equivalent;

(a) F is soft lower semi irresolute

(b) $F^+(H,K)$ is soft semi closed set in X, for every soft semi closed set (H,K) in Y.

(c) $sint(scl(F^+(M,K))) \tilde{\subseteq} F^+(M,K)$, for every soft semi closed set (M,K) in Y.

(d) $F^-(G,K) \tilde{\subseteq} scl(sint(F^-(G,K)))$, for every soft semi open set (G,K) in Y.

(e) $scl(F^+(B,K))) \tilde{\subseteq} F^+(scl(B,K))$, for every soft set (B,K) in Y.

(f) $F^-(sint(N,K)) \tilde{\subseteq} sint(F^-(N,K))$, for every soft set (N,K) in Y.

Proof. Similar to previous theorem.

IV. An Application For Soft Multifunctions

One of the important task of a guiding service is to provide for students with a way to solve problems in their courses. An example relation of this is given below.

Let S(X,E) and S(Y,K) be two soft set families. Where S(X,E) shows that the courses which students take and degrees of failure in these courses and S(Y,K) shows that the cause of failure in these courses and the operation method for the elimination of these causes.

x_1 = Algebra, x_2 = Topology, x_3 = Analysis, x_4 = Differantial Equations, x_5 = Geometry,
e_1 = high importance, e_2 = medium importance, e_3 = low importance, e_4 = very low importance,
y_1 = do not solve example, y_2 = do not preparation before course, y_3 = do not repetition after course,

y_4 = absent in the course, y_5 = motivationlessness,
k_1 = frequent and very effective studying, k_2 = infrequent and very effective studying,
k_3 = frequent and normal effective studying, k_4 = infrequent and normal effective studying.

The courses of a students which taked and the degree of failure in these courses, have been encoded with a soft set as follows;

$$(G,E) = \{(e_1,\{x_5\}),(e_2,\{x_1,x_4\}),(e_3,\{x_2\}),(e_4,\{x_3\})\}.$$

As a first of the guiding service, is to apply this case using the stored knowledge and is to transform failure causes and study suggestion for student.

Let the multifunction u:X→Y and the function p:E→K defined as follows:

$$u(x_1) = \{y_2,y_5\},\quad u(x_2) = \{y_1\},\quad u(x_3) = \{y_3\},\quad u(x_4) = \{y_5\},\quad u(x_5) = \{y_4\},$$
$$p(e_1) = k_4,\quad p(e_2) = k_3,\quad p(e_3) = k_2,\quad p(e_4) = k_1.$$

Then calculations give us the represents causes and study preference for student as follows:

$$F(G,E)=\{(k_1,\{y_3\}),(k_2,\{y_1\}),(k_3,\{y_2,y_5\}),(k_4,\{y_4\})\}.$$

V. Conclusions

Since the soft sets is very useful in information systems, we hoped that these multifunctions will be useful for the many sciences, such as medical, engineering, economy, social science, etc. In this paper, firstly we define the upper (lower) semi continuity of soft multifunctions and several properties of these multifunction have been given. Then we define the upper (lower) semi irresolute soft multifunctions and by examples and counter examples several properties of these multifunction have been given. Finally we give the application for soft multifunction.

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