

# On Soft Contra $\pi g$ -continuous Functions in Soft Topological Spaces

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**Abstract-** *In this paper we apply the notion of soft  $\pi g$ -open sets in soft topological spaces to study soft contra  $\pi g$ -continuity which is weaker than soft contra continuity. We also obtain some properties of soft contra  $\pi g$ -continuous functions and discuss the relationships between soft contra  $\pi g$ -continuity and other related functions.*

**Keywords-** *soft contra  $\pi g$ -continuity, soft  $\pi g$ -compactness, soft  $\pi g$ -connectedness, soft  $\pi g$ -graph.*

## I. INTRODUCTION

The soft set theory is a rapidly processing field of mathematics. Molodtsov's [7] soft set theory was originally proposed as general mathematical tool for dealing with uncertainty problems. He proposed soft set theory, which contains sufficient parameters such that it is free from the corresponding difficulties, and a series of interesting applications of the theory instability and regularization, Game Theory, Operations Research, Probability and Statistics. Topological structure of soft sets was initiated by Shabir and Naz[9] and studied the concepts of soft open set, soft interior point, soft neighborhood of a point, soft separation axioms and subspace of a soft topological space. Many researchers extended the results of generalization of various soft closed sets in many directions. Athar Kharal and B. Ahmad[3] defined the notion of a mapping on soft classes and studied several properties of images and inverse images of soft sets. In this paper we present a new generalization of soft contra continuity called soft contra  $\pi g$ -continuity. The notion of soft contra  $\pi g$ -continuity is weaker form of soft contra continuity. Also we investigate fundamental properties of soft contra  $\pi g$ -continuous functions.

## II. PRELIMINARIES

**Definition: 2.1[7]**

Let  $U$  be the initial universe and  $P(U)$  denote the power set of  $U$ . Let  $E$  denote the set of all parameters. Let  $A$  be a non-empty subset of  $E$ . A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ .

**Definition: 2.2[5]**

A subset  $(A, E)$  of a topological space  $X$  is called soft generalized-closed (soft  $g$ -closed), if  $cl(A, E) \subseteq (U, E)$  whenever  $(A, E) \subseteq (U, E)$  and  $(U, E)$  is soft open in  $X$ .

**Definition: 2.3[2]**

A subset  $(A, E)$  of a topological space  $X$  is called soft regular closed, if  $cl(int(A, E)) = (A, E)$ . The complement of soft regular closed set is soft regular open set.

**Definition: 2.4[2]**

The finite union of soft regular open sets is said to be soft  $\pi$ -open. The complement of soft  $\pi$ -open is said to be soft  $\pi$ -closed.

**Definition: 2.5[2]**

A subset  $(A, E)$  of a topological space  $X$  is called soft  $\pi g$ -closed in a soft topological space  $(X, \tau, E)$ , if  $\text{cl}(A, E) \subseteq (U, E)$  whenever  $(A, E) \subseteq (U, E)$  and  $(U, E)$  is soft  $\pi$ -open in  $X$ .

**Definition: 2.6[1]**

Let  $(F, E)$  be a soft set over  $X$ . The soft set  $(F, E)$  is called soft point, denoted by  $(x_e, E)$ , if for element  $e \in E$ ,  $F(e) = \{x\}$  and  $F(e') = \emptyset$  for all  $e' \in E - \{e\}$ .

**Definition: 2.7[10]**

Let  $(X, \tau, E)$  and  $(Y, \tau', E)$  be two topological spaces. A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be Soft Semi continuous (Soft pre-continuous, Soft  $\alpha$ -continuous, Soft  $\beta$ -continuous), if  $f^{-1}(G, E)$  is soft semi open (soft pre-open, soft  $\alpha$ -open, soft  $\beta$ -open) in  $(X, \tau, E)$  for every soft open set  $(G, E)$  of  $(Y, \tau', E)$ .

**Definition: 2.8[3]**

Let  $(X, \tau, E)$  and  $(Y, \tau', E)$  be two topological spaces. A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be Soft regular continuous (Soft  $\pi$ -continuous, Soft  $g$ -continuous, Soft  $\pi g$ -continuous), if  $f^{-1}(G, E)$  is soft regular open (soft  $\pi$ -open, soft  $g$ -open, soft  $\pi g$ -open) in  $(X, \tau, E)$  for every soft open set  $(G, E)$  of  $(Y, \tau', E)$ .

**Definition: 2.9[3]**

A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft  $\pi g$ -irresolute, if  $f^{-1}(G, E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$  for every soft  $\pi g$ -open set  $(G, E)$  of  $(Y, \tau', E)$ .

**Definition: 2.10[2]**

A soft topological space  $(X, \tau, E)$  is a soft  $\pi g - T_{\frac{1}{2}}$ -space if every soft  $\pi g$ -closed set is soft closed.

**Definition: 2.11[9]**

A soft topological space  $(X, \tau, E)$  is a soft  $\pi g - T_0$  space, if for each pair of distinct soft points  $x$  and  $y$  in  $X$ , there exist soft open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$ .

**Definition: 2.12[3]**

A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is called  $\tilde{S}\pi g$ -open, if image of each  $\tilde{S}\pi g$ -open set is  $\tilde{S}\pi g$ -open.

**III. CONTRA  $\pi g$ -CONTINUOUS FUNCTIONS**

**Definition: 3.1**

A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is called soft contra  $\pi g$ -continuous, if  $f^{-1}(F, E)$  is soft  $\pi g$ -closed in  $X$  for every soft open set  $(F, E)$  of  $Y$ .

**Lemma: 3.2**

Let  $(A, E)$  be a subset of a space  $(X, \tau, E)$ . Then

- (1)  $\tilde{S}\pi g\text{-cl}(X \setminus (A, E)) = X \setminus \tilde{S}\pi g\text{-int}(A, E)$
- (2)  $x \in \tilde{S}\pi g\text{-cl}(A, E)$  if and only if  $(A, E) \cap (U, E) \neq \emptyset$  for each  $(U, E) \in \tilde{S}\pi GO(X, x)$ .

**Remark: 3.3**

If a subset  $(A, E)$  is  $\tilde{S}\pi g$ -closed in a space  $X$ , then  $(A, E) = \tilde{S}\pi g\text{-cl}(A, E)$ . The converse of this implication is not true in general.

**Definition: 3.4**

Let  $(A, E)$  be a subset of a space  $(X, \tau, E)$ . The set  $\cap \{(U, E) \in \tau : (A, E) \subseteq (U, E)\}$  is called the kernel of  $(A, E)$  and is denoted by  $\text{ker}(A, E)$ .

**Lemma: 3.5**

The following properties hold for subsets  $(A, E), (B, E)$  of a space  $(X, \tau, E)$ :

- (1)  $x \in \ker(A, E)$  if and only if  $(A, E) \cap (F, E) \neq \emptyset$  for any soft closed set  $(F, E)$  containing  $x$ .
- (2)  $(A, E) \tilde{\subset} \ker(A, E)$  and  $(A, E) = \ker(A, E)$  if  $(A, E)$  is soft open in  $X$ .
- (3)  $(A, E) \tilde{\subset} (B, E)$  then  $\ker(A, E) \tilde{\subset} \ker(B, E)$ .

**Theorem: 3.6**

The following are equivalent for a function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ :

- (1)  $f$  is soft contra  $\pi g$ -continuous,
- (2) the inverse image of every soft closed set of  $Y$  is soft  $\pi g$ -open.

**Theorem: 3.7**

Suppose that  $\tilde{S}\pi GC(X)$  is closed under arbitrary intersections. Then the following are equivalent for a function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ :

- (1)  $f$  is soft contra  $\pi g$ -continuous
- (2) the inverse image of every soft closed set of  $Y$  is  $\tilde{S}\pi g$ -open .
- (3) for each  $x \in X$  and each soft closed set  $(B, E)$  in  $Y$  with  $f(x) \in (B, E)$  there exists a  $\tilde{S}\pi g$ -open set  $(A, E)$  in  $X$  such that  $x \in (A, E)$  and  $f(A, E) \tilde{\subset} (B, E)$
- (4)  $f(\tilde{S}\pi g\text{-cl}(A, E)) \tilde{\subset} \ker(f(A, E))$  for every subset  $(A, E)$  of  $X$ .
- (5)  $\tilde{S}\pi g\text{ cl}(f^{-1}(B, E)) \tilde{\subset} f^{-1}(\ker(B, E))$  for every subset  $(B, E)$  of  $Y$

**Definition: 3.8**

A function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be

- (1) Soft perfectly continuous, if  $f^{-1}(A, E)$  is soft clopen in  $X$  for every soft open set  $(A, E)$  of  $Y$ .
- (2) Soft RC-continuous, if  $f^{-1}(A, E)$  is soft regular closed in  $X$  for each open set  $(A, E)$  of  $Y$ .
- (3) Soft strongly continuous, if  $f(\text{cl}(A, E)) \tilde{\subset} f(A, E)$  for every subset  $(A, E)$  of  $X$  or equivalently if the inverse image of every soft set in  $Y$  is soft clopen in  $X$ .
- (4) Soft contra-continuous, if  $f^{-1}(A, E)$  is soft closed in  $X$  for every soft open set  $(A, E)$  of  $Y$ .
- (5) Soft contra R-map, if  $f^{-1}(A, E)$  is soft regular closed in  $X$  for every soft regular open set  $(A, E)$  of  $Y$ .
- (6) Soft contra g-continuous, if  $f^{-1}(A, E)$  is soft g-closed in  $X$  for every soft open set  $(A, E)$  of  $Y$ .
- (7) Soft contra  $\pi$ -continuous, if  $f^{-1}(A, E)$  is soft  $\pi$ -closed in  $X$  for every soft open set  $(A, E)$  of  $Y$ .

**Theorem: 3.9**

- 1) Every soft contra  $\pi$ -continuous function is soft contra continuous.
- 2) Every soft contra continuous function is soft contra  $\pi g$ -continuous.
- 3) Every soft contra  $\pi$ -continuous function is soft contra  $\pi g$ -continuous.
- 4) Every soft contra g-continuous function is soft contra  $\pi g$ -continuous.

**Remark : 3.10**

None of these implications is reversible as shown in the following examples.

**Example: 3.11**

Let  $X = Y = \{a, b, c, d\}$ ,  $E = \{e_1, e_2\}$ . Let  $F_1, F_2, F_3, F_4, F_5, F_6$  are functions from  $E$  to  $P(X)$  and are defined as follows:

$$\begin{array}{ll} F_1(e_1) = \{c\} & F_1(e_2) = \{a\} \\ F_2(e_1) = \{d\} & F_2(e_2) = \{b\} \end{array}$$

$$\begin{array}{ll}
 F_3(e_1) = \{ c,d\} & F_3(e_2) = \{ a,b\} \\
 F_4(e_1) = \{ a,b\} & F_4(e_2) = \{ b,d\} \\
 F_5(e_1) = \{ b,c,d\} & F_5(e_2) = \{ a,b,c\} \\
 F_6(e_1) = \{ a,c,d\} & F_6(e_2) = \{ a,b,d\}
 \end{array}$$

Then  $\tau = \{ \tilde{\emptyset}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E) \}$  is a soft topological space over X.

Let  $G_1, G_2$  are functions from E to P(Y) and are defined as follows:

$$\begin{array}{ll}
 G_1(e_1) = \{ b\} & G_1(e_2) = \{ c\} \\
 G_2(e_1) = \{ a,b,c\} & G_2(e_2) = \{ a,c,d\}
 \end{array}$$

Then  $\tau = \{ \tilde{\emptyset}, \tilde{Y}, (G_1, E), (G_2, E) \}$  be a soft topological space over Y.

If the function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  be a function defined as  $f(a) = b, f(b) = a, f(c) = d, f(d) = c$  then f is soft contra  $\pi g$ -continuous and soft contra continuous, but not soft contra  $\pi$ -continuous. Thus every soft contra  $\pi g$ -continuous need not be soft contra  $\pi$ -continuous and every soft contra continuous need not be soft contra  $\pi$ -continuous.

**Example: 3.12**

Let  $X=Y= \{ a,b,c,d \}, E= \{ e_1, e_2 \}$ . Let  $F_1, F_2, F_3, F_4, F_5, F_6$  are functions from E to P(X) and are defined as follows:

$$\begin{array}{ll}
 F_1(e_1) = \{ c\} & F_1(e_2) = \{ a\} \\
 F_2(e_1) = \{ d\} & F_2(e_2) = \{ b\} \\
 F_3(e_1) = \{ c,d\} & F_3(e_2) = \{ a,b\} \\
 F_4(e_1) = \{ a,b\} & F_4(e_2) = \{ b,d\} \\
 F_5(e_1) = \{ b,c,d\} & F_5(e_2) = \{ a,b,c\} \\
 F_6(e_1) = \{ a,c,d\} & F_6(e_2) = \{ a,b,d\}
 \end{array}$$

Then  $\tau = \{ \tilde{\emptyset}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E) \}$  is a soft topological space over X.

Let  $G_1, G_2$  are functions from E to P(Y) and are defined as follows:

$$\begin{array}{ll}
 G_1(e_1) = \{ a\} & G_1(e_2) = \{ d\} \\
 G_2(e_1) = \{ a,c,d\} & G_2(e_2) = \{ a,b,d\}
 \end{array}$$

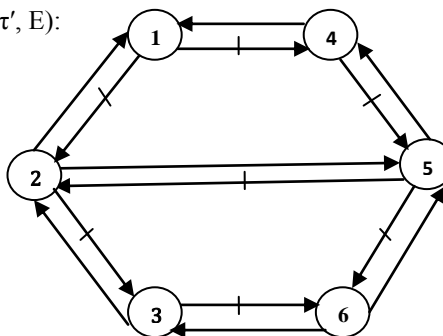
Then  $\tau = \{ \tilde{\emptyset}, \tilde{Y}, (G_1, E), (G_2, E) \}$  be a soft topological space over Y.

If the function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  be a function defined as  $f(a) = b, f(b) = a, f(c) = d, f(d) = c$  then f is soft contra  $\pi g$ -continuous, but not soft contra g-continuous. Thus every soft contra  $\pi g$ -continuous need not be soft contra g-continuous.

**Remark: 3.13**

The following diagram holds for a function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ :

- (1) Soft RC-continuous
- (2) Soft contra R-map
- (3) Soft contra continuous
- (4) Soft contra  $\pi$ -continuous
- (5) Soft contra g-continuous
- (6) Soft contra  $\pi g$ -continuous



**Lemma: 3.14**

If  $(A, E)$  is soft  $\pi$ -open and soft  $\pi g$ -closed in a space  $(X, \tau, E)$  then  $(A, E)$  is soft closed.

**Theorem: 3.15**

If a function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft contra  $\pi g$ -continuous and soft  $\pi$ -continuous, then  $f$  is soft contra continuous.

**Proof:**

Let  $(A, E)$  be a soft open set in  $Y$ . Since  $f$  is soft contra  $\pi g$ -continuous and soft  $\pi$ -continuous,  $f^{-1}(A, E)$  is soft  $\pi g$ -closed and soft  $\pi$ -open. By the previous lemma,  $f^{-1}(A, E)$  is soft closed. Hence,  $f$  is soft contra-continuous.

**Theorem: 3.16**

Suppose that  $X$  and  $Y$  are soft spaces and  $\tilde{S}\pi GO(X)$  is soft closed under arbitrary unions. If a function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft contra  $\pi g$ -continuous and  $Y$  is soft regular, then  $f$  is soft  $\pi g$ -continuous.

**Proof:**

Let  $x$  be an arbitrary point of  $X$  and  $(A, E)$  be a soft open set of  $Y$  containing  $f(x)$ . Since  $Y$  is soft regular, there exists a soft open set  $(G, E)$  in  $Y$  containing  $f(x)$  such that  $\tilde{S}cl(G, E) \subseteq (A, E)$ . Since  $f$  is soft contra  $\pi g$ -continuous, there exists  $(F, E) \in \tilde{S}\pi GO(X)$  containing  $x$  such that  $f(F, E) \subseteq \tilde{S}cl(G, E)$ . Then  $f(F, E) \subseteq \tilde{S}cl(G, E) \subseteq (A, E)$ . Hence  $f$  is  $\tilde{S}\pi g$ -continuous.

**Theorem: 3.17**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be a function. Suppose that  $(X, \tau, E)$  is a soft  $\pi g-T_{\frac{1}{2}}$  space. Then the following are equivalent:

- (1)  $f$  is soft contra  $\pi g$ -continuous.
- (2)  $f$  is soft contra  $g$ -continuous.
- (3)  $f$  is soft contra-continuous.

**Definition: 3.18**

A soft space  $(X, \tau, E)$  is said to be soft locally  $\pi g$ -indiscrete if every soft  $\pi g$ -open set of  $X$  is soft closed in  $X$ .

**Theorem: 3.19**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be a function. If  $f$  is soft contra  $\pi g$ -continuous and  $(X, \tau, E)$  is soft locally  $\pi g$ -indiscrete, then  $f$  is soft continuous.

**Definition: 3.20**

A space  $(X, \tau, E)$  is said to be soft submaximal, if each soft dense subset of  $X$  is soft open and extremally soft disconnected, if the closure of each soft open set of  $X$  is soft open in  $X$ .

**Lemma: 3.21**

Let  $(X, \tau, E)$  be a soft topological space. Then  $\tilde{S}\pi g-\tau = \{(U, E) \subseteq X: \tilde{S}\pi g-cl(X \setminus (U, E)) = X \setminus (U, E)\}$  is a soft topology for  $X$ .

**Lemma: 3.22**

For a space  $(X, \tau, E)$ ,  $X$  is extremally soft disconnected if and only if every subset of  $X$  is  $\tilde{S}\pi g$ -closed.

**Theorem: 3.23**

Every function which is defined on an extremally soft disconnected space is soft contra  $\pi g$ -continuous and  $\tilde{S}\pi g$ -continuous.

**Theorem: 3.24**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  and  $g: (Y, \tau', E) \rightarrow (Z, \tau'', E)$  be functions. Then the following properties hold:

- (1) If  $f$  is  $\tilde{S}\pi g$ -irresolute and  $g$  is soft contra  $\pi g$ -continuous, then  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is soft contra  $\pi g$ -continuous.
- (2) If  $f$  is soft contra  $\pi g$ -continuous and  $g$  is soft continuous, then  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is soft contra  $\pi g$ -continuous.

- (3) If  $f$  is soft contra  $\pi g$ -continuous and  $g$  is  $\tilde{S}\pi$ RC-continuous, then  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is  $\tilde{S}\pi g$ -continuous.  
 (4) If  $f$  is  $\tilde{S}\pi g$ -continuous and  $g$  is soft contra continuous, then  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is soft contra  $\pi g$ -continuous.

**Theorem: 3.25**

Suppose that  $\tilde{S}\pi$ GC(Y) is soft closed under arbitrary intersections. If  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is a surjective  $\tilde{S}\pi g$ -open function and  $g: (Y, \tau', E) \rightarrow (Z, \tau'', E)$  is a function such that  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is soft contra  $\pi g$ -continuous, then  $g$  is soft contra  $\pi g$ -continuous.

**Proof:**

Suppose that  $x$  and  $y$  are two soft points in  $X$  and  $Y$ , respectively, such that  $f(x) = y$ . Let  $(B, E) \in \tilde{S}C(Z, (g \circ f)(x))$ . Then there exists a  $\tilde{S}\pi g$ -open set  $(A, E)$  in  $X$  containing  $x$  such that  $g(f(A, E)) \tilde{C} (B, E)$ . Since  $f$  is  $\tilde{S}\pi g$ -open,  $f(A, E)$  is a  $\tilde{S}\pi g$ -open set in  $Y$  containing  $y$  such that  $g(f(A, E)) \tilde{C} (B, E)$ . This implies that  $g$  is soft contra  $\pi g$ -continuous.

**Corollary: 3.26**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be a surjective  $\tilde{S}\pi g$ -irresolute and  $\tilde{S}\pi g$ -open function and let  $g: (Y, \tau', E) \rightarrow (Z, \tau'', E)$  be a function. Suppose that  $\tilde{S}\pi$ GC(Y) is closed under arbitrary intersections. Then  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is soft contra  $\pi g$ -continuous if and only if  $g$  is soft contra  $\pi g$ -continuous.

**Definition: 3.27**

The  $\tilde{S}\pi g$ -frontier of a subset  $(A, E)$  of a space  $(X, \tau, E)$  is given by  $\tilde{S}\pi g\text{-fr}(A, E) = \tilde{S}\pi g\text{-cl}(A, E) \cap \tilde{S}\pi g\text{-cl}(X \setminus (A, E))$ .

**Theorem: 3.28**

Let the collection of all  $\tilde{S}\pi g$ -closed sets of a space  $(X, \tau, E)$  be closed under arbitrary intersections. The set of all points  $x \in X$  at which a function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is not soft contra  $\pi g$ -continuous is identical with the union of  $\tilde{S}\pi g$ -frontier of the inverse images of soft closed sets containing  $f(x)$ .

**Proof:**

Suppose that  $f$  is not soft contra  $\pi g$ -continuous at  $x \in X$ . Then there exists a soft closed set  $(A, E)$  of  $Y$  containing  $f(x)$  such that  $f(B, E)$  is not contained in  $(A, E)$  for every  $(B, E) \in \tilde{S}\pi$ GO(X) containing  $x$ . Then  $(B, E) \setminus (X \setminus f^{-1}(A, E)) \neq \emptyset$  for every  $(B, E) \in \tilde{S}\pi$ GO(X) containing  $x$  and hence  $x \in \tilde{S}\pi g\text{-cl}(X \setminus f^{-1}(A, E))$ . On the other hand, we have  $x \in f^{-1}(A, E) \tilde{C} \tilde{S}\pi g\text{-cl}(f^{-1}(A, E))$  and hence  $x \in \tilde{S}\pi g\text{-fr}(f^{-1}(A, E))$ .

Conversely, suppose that  $f$  is soft contra  $\pi g$ -continuous at  $x \in X$  and let  $(A, E)$  be a soft closed set of  $Y$  containing  $f(x)$ . Then there exists  $(B, E) \in \tilde{S}\pi$ GO(X) containing  $x$  such that  $(B, E) \tilde{C} f^{-1}(A, E)$ . Hence  $x \in \tilde{S}\pi g\text{-int}(f^{-1}(A, E))$ . Therefore,  $x \notin \tilde{S}\pi g\text{-fr}(f^{-1}(A, E))$  for each soft closed set  $(A, E)$  of  $Y$  containing  $f(x)$ . This completes the proof.

**IV. SEPARATION AXIOMS**

**Definition: 4.1**

A space  $(X, \tau, E)$  is said to be  $\tilde{S}\pi g\text{-}T_1$  if for each pair of distinct soft points  $x$  and  $y$  in  $X$ , there exist  $\tilde{S}\pi g$ -open sets  $(F, E)$  and  $(G, E)$  containing  $x$  and  $y$  respectively, such that  $y \notin (F, E)$  and  $x \notin (G, E)$ .

**Definition: 4.2**

A space  $(X, \tau, E)$  is said to be  $\tilde{S}\pi g\text{-}T_2$  if for each pair of distinct soft points  $x$  and  $y$  in  $X$ , there exist  $(F, E) \in \tilde{S}\pi$ GO(X,  $x$ ) and  $(G, E) \in \tilde{S}\pi$ GO(X,  $y$ ) such that  $(F, E) \cap (G, E) = \emptyset$ .

**Definition: 4.3**

A space  $(X, \tau, E)$  is said to be soft Urysohn space, if each pair of distinct soft points  $x$  and  $y$  of  $X$ , there exist two soft open sets  $(U, E)$  and  $(V, E)$  such that  $x \in (U, E)$  and  $y \in (V, E)$  and  $\tilde{\text{Scl}}(U, E) \cap \tilde{\text{Scl}}(V, E) = \emptyset$ .

**Theorem: 4.4**

Let  $(X, \tau, E)$  and  $(Y, \tau', E)$  be soft spaces. If

- (1) for each pair of distinct soft points  $x$  and  $y$  in  $X$ , there exist a function  $f$  of  $X$  into  $Y$  such that  $f(x) \neq f(y)$ .
- (2)  $Y$  is an soft Urysohn space and
- (3)  $f$  is soft contra  $\pi g$ -continuous at  $x$  and  $y$  then  $X$  is  $\tilde{\text{S}}\pi g - T_2$ .

**Proof:**

Let  $x$  and  $y$  be any distinct soft points in  $X$ . Then, there exists a soft Urysohn space  $Y$  and a function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  such that  $f(x) \neq f(y)$  and  $f$  is soft contra  $\pi g$ -continuous at  $x$  and  $y$ . Let  $z = f(x)$  and  $v = f(y)$ . Then  $z \neq v$ . Since  $Y$  is soft Urysohn, there exist soft open sets  $(A, E)$  and  $(B, E)$  containing  $z$  and  $v$ , respectively such that  $\tilde{\text{Scl}}(A, E) \cap \tilde{\text{Scl}}(B, E) = \emptyset$ . Since  $f$  is soft contra  $\pi g$ -continuous at  $x$  and  $y$ , then there exist  $\tilde{\text{S}}\pi g$ -open sets  $(F, E)$  and  $(G, E)$  containing  $x$  and  $y$  respectively such that  $f(F, E) \subseteq \tilde{\text{Scl}}(A, E)$  and  $f(G, E) \subseteq \tilde{\text{Scl}}(B, E)$ . Since  $\tilde{\text{Scl}}(A, E) \cap \tilde{\text{Scl}}(B, E) = \emptyset$  we have  $(F, E) \cap (G, E) = \emptyset$ . Hence  $X$  is  $\tilde{\text{S}}\pi g - T_2$ .

**Definition: 4.5**

A space  $(X, \tau, E)$  is called

- (1)  $\tilde{\text{S}}\pi g$  -connected, if  $X$  is not the union of two disjoint nonempty  $\tilde{\text{S}}\pi g$  -open sets.
- (2)  $\tilde{\text{S}}g$ -connected, if  $X$  is not the union of two disjoint nonempty  $\tilde{\text{S}}g$ -open sets.

**Remark: 4.6**

Every  $\tilde{\text{S}}\pi g$ -connected space is  $\tilde{\text{S}}g$ -connected. The reverse of this implication is not true in general.

**Example: 4.7**

Let  $X = \{a, b, c, d\}$ ,  $E = \{e_1, e_2\}$ . Let  $F_1, F_2, F_3, F_4, F_5, F_6$  are functions from  $E$  to  $P(X)$  and are defined as follows:

$F_1(e_1) = \{c\}$	$F_1(e_2) = \{a\}$
$F_2(e_1) = \{d\}$	$F_2(e_2) = \{b\}$
$F_3(e_1) = \{c, d\}$	$F_3(e_2) = \{a, b\}$
$F_4(e_1) = \{a, b\}$	$F_4(e_2) = \{b, d\}$
$F_5(e_1) = \{b, c, d\}$	$F_5(e_2) = \{a, b, c\}$
$F_6(e_1) = \{a, c, d\}$	$F_6(e_2) = \{a, b, d\}$

Then  $\tau = \{\emptyset, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$  is a soft topological space over  $X$ .

Here  $(H, E) = \{\{b, c, d\}, \{a, b, c\}\}$  and  $(G, E) = \{\{c, d\}, \{a, b\}\}$  are soft  $g$ -open sets. Then  $(H, E) \cup (G, E) = \{\{b, c, d\}, \{a, b, c\}\} \neq X$ . Hence  $(X, \tau, E)$  is soft  $g$ -connected but not  $\tilde{\text{S}}\pi g$ -connected.

**Theorem: 4.8**

For a soft space  $(X, \tau, E)$  the following properties are equivalent:

- (1)  $X$  is  $\tilde{\text{S}}\pi g$  -connected
- (2) The only subsets of  $X$  which are both  $\tilde{\text{S}}\pi g$  -open and  $\tilde{\text{S}}\pi g$ -closed are the empty set  $\emptyset$  and  $X$
- (3) Each soft contra  $\pi g$ -continuous function of  $X$  into a soft discrete space  $Y$  with at least two points is a constant function.

**Theorem: 4.9**

If  $f$  is a soft contra  $\pi g$ -continuous function from a  $\tilde{\text{S}}\pi g$  -connected space  $X$  onto any space  $Y$ , then  $Y$  is not a soft discrete space.

**Proof:**

Suppose that  $Y$  is soft discrete. Let  $(A, E)$  be a proper nonempty soft open and soft closed subset of  $Y$ . Then  $f^{-1}(A, E)$  is a proper nonempty  $\tilde{S}\pi g$ -clopen subset of  $X$ , which is a contradiction to the fact that  $X$  is  $\tilde{S}\pi g$ -connected. Hence  $Y$  is not a soft discrete space.

**Theorem: 4.10**

A space  $X$  is  $\tilde{S}\pi g$ -connected if every soft contra  $\pi g$ -continuous function from a space  $X$  into any soft  $T_0$ -space  $Y$  is constant.

**Proof:**

Suppose that  $X$  is not  $\tilde{S}\pi g$ -connected and that every soft contra  $\pi g$ -continuous function from  $X$  into  $Y$  is constant. Since  $X$  is not  $\tilde{S}\pi g$ -connected, there exists a proper nonempty  $\tilde{S}\pi g$ -clopen subset  $(A, E)$  of  $X$ . Then  $f$  is non-constant and soft contra  $\pi g$ -continuous such that  $Y$  is soft  $T_0$ , which is a contradiction. Hence,  $X$  must be  $\tilde{S}\pi g$ -connected.

**Theorem: 4.11**

If  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is a soft contra  $\pi g$ -continuous surjection and  $X$  is  $\tilde{S}\pi g$ -connected, then  $Y$  is soft connected.

**Proof:**

Suppose that  $Y$  is not a soft connected space. Then there exist nonempty disjoint open sets  $(F, E)$  and  $(G, E)$  such that  $Y = (F, E) \cup (G, E)$ . Therefore  $(F, E)$  and  $(G, E)$  are soft clopen in  $Y$ . Since  $f$  is soft contra  $\pi g$ -continuous,  $f^{-1}(F, E)$  and  $f^{-1}(G, E)$  are  $\tilde{S}\pi g$ -open in  $X$ . Moreover,  $f^{-1}(F, E)$  and  $f^{-1}(G, E)$  are nonempty disjoint and  $X = f^{-1}(F, E) \cup f^{-1}(G, E)$ . This shows that  $X$  is not  $\tilde{S}\pi g$ -connected. This contradicts that  $Y$  is not soft connected. Hence  $Y$  is soft connected.

**Theorem: 4.12**

If  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is a  $\tilde{S}\pi g$ -irresolute surjection and  $X$  is  $\tilde{S}\pi g$ -connected, then  $Y$  is  $\tilde{S}\pi g$ -connected.

**Definition: 4.13**

A space  $(X, \tau, E)$  is said to be

- (1)  $\tilde{S}\pi g$ -compact, if every  $\tilde{S}\pi g$ -open cover of  $X$  has a finite sub cover
- (2) countably  $\tilde{S}\pi g$ -compact, if every countable soft cover of  $X$  by  $\tilde{S}\pi g$ -open sets has a finite sub cover
- (3)  $\tilde{S}\pi g$ -Lindelof, if every  $\tilde{S}\pi g$ -open cover of  $X$  has a countable sub cover
- (4) strongly soft  $S$ -closed space, if every soft closed cover of  $X$  has a finite sub cover

**Theorem: 4.14**

If  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft contra  $\pi g$ -continuous and  $A$  is  $\tilde{S}\pi g$ -compact relative to  $X$ , then  $f(A)$  is strongly  $S$ -closed in  $Y$ .

**Proof:**

Let  $\{(V_i, E) : i \in I\}$  be any soft cover of  $f(A, E)$  by soft closed sets of the subspace  $f(A, E)$ . For each  $i \in I$  there exists a soft closed set  $(A_i, E)$  of  $Y$  such that  $(V_i, E) = (A_i, E) \cap f(A, E)$ . For each  $x \in (A, E)$ , there exists  $i(x) \in I$  such that  $f(x) \in (A_{i(x)}, E)$  and there exists  $(F_x, E) \in \tilde{S}\pi GO(X, x)$  such that  $f(F_x, E) \tilde{C} (A_{i(x)}, E)$ . Since the family  $\{(F_x, E) : x \in (A, E)\}$  is a soft cover of  $(A, E)$  by  $\tilde{S}\pi g$ -open sets of  $X$ , there exists a finite subset  $(A_0, E)$  of  $(A, E)$  such that  $(A, E) \tilde{C} \cup \{(F_x, E) : x \in (A_0, E)\}$ . Hence, we obtain  $f(A, E) \tilde{C} \cup \{f(F_x, E) : x \in (A_0, E)\}$  which is a subset of  $\cup \{(A_{i(x)}, E) : x \in (A_0, E)\}$ . Thus,  $f(A, E) = \cup \{(V_{i(x)}, E) : x \in (A_0, E)\}$ . Hence  $f(A, E)$  is strongly soft  $S$ -closed.

**Theorem :4.15**

If the product space of two nonempty soft spaces is  $\tilde{S}\pi g$ -compact, then each factor space is  $\tilde{S}\pi g$ -compact.



**Proof:**

Let  $X \times Y$  be the product space of the nonempty spaces  $X$  and  $Y$  and  $X \times Y$  be  $\tilde{S}\pi g$ -compact. The projection  $p : (X \times Y, \tau, E) \rightarrow (X, \tau, E)$  is  $\tilde{S}\pi g$ -irresolute and then  $p(X \times Y) = X$  is  $\tilde{S}\pi g$ -compact. The proof for the space  $Y$  is similar to the case of  $X$ .

**Theorem: 4.16**

The soft contra  $\pi g$ -continuous images of  $\tilde{S}\pi g$ -Lindelof (resp. countably  $\tilde{S}\pi g$ -compact) spaces are strongly soft S-Lindelof (respectively strongly countably soft S-closed).

**Proof:**

Let  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  be a soft contra  $\pi g$ -continuous surjection. Let  $\{(V_i, E) : i \in I\}$  be any soft closed cover of  $Y$ . Since  $f$  is soft contra  $\pi g$ -continuous, then  $\{f^{-1}(V_i, E) : i \in I\}$  is a  $\tilde{S}\pi g$ -open cover of  $X$ . Hence there exists a countable subset  $I_0$  of  $I$  such that  $X = \cup\{f^{-1}(V_i, E) : i \in I_0\}$ . Therefore, we have  $Y = \cup\{(V_i, E) : i \in I_0\}$  and  $Y$  is strongly soft S-Lindelof.

**Definition: 4.17**

The graph  $G(f)$  of a function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be soft contra  $\pi g$ -graph, if for each  $(x, y) \in (X \times Y) \setminus G(f)$ , there exist a  $\tilde{S}\pi g$ -open set  $(A, E)$  in  $X$  containing  $x$  and a soft closed set  $(B, E)$  in  $Y$  containing  $y$  such that  $(A \times B, E) \cap G(f) = \emptyset$ .

**Proposition: 4.18**

The following properties are equivalent for the graph  $G(f)$  of a function  $f$ :

- (1)  $G(f)$  is soft contra  $\pi g$ -graph
- (2) for each  $(x, y) \in (X \times Y) \setminus G(f)$ , there exist a  $\tilde{S}\pi g$ -open set  $(A, E)$  in  $X$  containing  $x$  and a soft closed set  $(B, E)$  in  $Y$  containing  $y$  such that  $f(A, E) \cap (B, E) = \emptyset$ .

**Theorem: 4.19**

If  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft contra  $\pi g$ -continuous and  $Y$  is soft Urysohn,  $G(f)$  is soft contra  $\pi g$ -graph in  $X \times Y$ .

**Proof:**

Let  $(x, y) \in (X \times Y) \setminus G(f)$ . It follows that  $f(x) \neq y$ . Since  $Y$  is soft Urysohn, there exist soft open sets  $(A, E)$  and  $(B, E)$  such that  $f(x) \in (A, E)$ ,  $y \in (B, E)$  and  $\tilde{S}cl(A, E) \cap \tilde{S}cl(B, E) = \emptyset$ . Since  $f$  is soft contra  $\pi g$ -continuous, there exists a  $\tilde{S}\pi g$ -open set  $(F, E)$  in  $X$  containing  $x$  such that  $f(F, E) \subseteq \tilde{S}cl(A, E)$ . Therefore,  $f(F, E) \cap \tilde{S}cl(B, E) = \emptyset$  and  $G(f)$  is soft contra  $\pi g$ -graph in  $X \times Y$ .

**Theorem :4.20**

Let  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  be a function and  $g : (X, \tau, E) \rightarrow (X \times Y, \tau, E)$  the graph function of  $f$ , defined by  $g(x) = (x, f(x))$  for every  $x \in X$ . If  $g$  is soft contra  $\pi g$ -continuous, then  $f$  is soft contra  $\pi g$ -continuous.

**Proof:**

Let  $(F, E)$  be an soft open set in  $Y$ , then  $X \times (F, E)$  is an soft open set in  $X \times Y$ . It follows that  $f^{-1}(F, E) = g^{-1}(X \times (F, E)) \in \tilde{S}\pi GC(X)$ . Thus,  $f$  is soft contra  $\pi g$ -continuous.

**Theorem : 4.21**

If  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  and  $g : (X, \tau, E) \rightarrow (Y, \tau', E)$  are soft contra  $\pi g$ -continuous and  $Y$  is soft Urysohn, then  $(F, E) = \{x \in X : f(x) = g(x)\}$  is  $\tilde{S}\pi g$ -closed in  $X$ .

**Proof:**

Let  $x \in X \setminus (F, E)$ . Then  $f(x) \neq g(x)$ . Since  $Y$  is soft Urysohn, there exist soft open sets  $(A, E)$  and  $(B, E)$  such that  $f(x) \in (A, E)$ ,  $g(x) \in (B, E)$  and  $\tilde{S}cl(A, E) \cap \tilde{S}cl(B, E) = \emptyset$ . Since  $f$  and  $g$  are soft contra  $\pi g$ -continuous,  $f^{-1}(\tilde{S}cl(A, E)) \in \tilde{S}\pi GO(X)$  and

$g^{-1}(\tilde{\text{Scl}}(B,E)) \in \tilde{\text{S}}\pi\text{GO}(X)$ . Let  $(G,E) = f^{-1}(\tilde{\text{Scl}}(A,E))$  and  $(K,E) = g^{-1}(\tilde{\text{Scl}}(B,E))$ . Then  $(G,E)$  and  $(A,E)$  contains  $x$ . Set  $(V,E) = (G,E) \cap (K,E)$ . Then  $(V,E)$  is  $\tilde{\text{S}}\pi\text{g}$ -open in  $X$ . Hence  $f(V,E) \cap g(V,E) = \emptyset$  and  $x \notin \tilde{\text{S}}\pi\text{g-cl}(F,E)$ . Thus  $(F,E)$  is  $\tilde{\text{S}}\pi\text{g}$ -closed in  $X$ .

**Definition: 4.22**

A subset  $(A, E)$  of a soft space  $(X, \tau, E)$  is said to be  $\tilde{\text{S}}\pi\text{g}$ -dense in  $X$ , if  $\tilde{\text{S}}\pi\text{g-cl}(A,E) = X$ .

**Theorem: 4.23**

Let  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  and  $g : (X, \tau, E) \rightarrow (Y, \tau', E)$  be functions. If

- (1)  $Y$  is soft Urysohn
- (2)  $f$  and  $g$  are soft contra  $\pi\text{g}$ -continuous
- (3)  $f = g$  on  $\tilde{\text{S}}\pi\text{g}$ -dense set  $(A,E) \tilde{\subset} X$ , then  $f = g$  on  $X$ .

**Proof:**

Since  $f$  and  $g$  are soft contra  $\pi\text{g}$ -continuous and  $Y$  is soft Urysohn, by the previous theorem  $(F,E) = \{x \in X : f(x) = g(x)\}$  is  $\tilde{\text{S}}\pi\text{g}$ -closed in  $X$ . Then we have  $f = g$  on  $\tilde{\text{S}}\pi\text{g}$ -dense set  $(A,E) \tilde{\subset} X$ . Since  $(A,E) \tilde{\subset} (F,E)$  and  $(A,E)$  is  $\tilde{\text{S}}\pi\text{g}$ -dense set in  $X$ , then  $X = \tilde{\text{S}}\pi\text{g-cl}(A,E) \tilde{\subset} \tilde{\text{S}}\pi\text{g-cl}(F,E) = (F,E)$ . Hence  $f = g$  on  $X$ .

**Definition :4.24**

A space  $(X, \tau, E)$  is said to be weakly soft Hausdorff, if each element of  $X$  is an intersection of soft regular closed sets.

**Theorem: 4.25**

If  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is a soft contra  $\pi\text{g}$ -continuous injection and  $Y$  is weakly soft Hausdorff, then  $X$  is  $\tilde{\text{S}}\pi\text{g-T}_1$ .

**Proof:**

Suppose that  $Y$  is weakly Hausdorff. For any distinct points  $x$  and  $y$  in  $X$ , there exist soft regular closed sets  $(A,E)$ ,  $(B,E)$  in  $Y$  such that  $f(x) \in (A,E)$ ,  $f(y) \notin (A,E)$ ,  $f(x) \notin (B,E)$  and  $f(y) \in (B,E)$ . Since  $f$  is soft contra  $\pi\text{g}$ -continuous,  $f^{-1}(A,E)$  and  $f^{-1}(B,E)$  are  $\tilde{\text{S}}\pi\text{g}$ -open subsets of  $X$  such that  $x \in f^{-1}(A,E)$ ,  $y \notin f^{-1}(A,E)$ ,  $x \notin f^{-1}(B,E)$  and  $y \in f^{-1}(B,E)$ . This shows that  $X$  is  $\tilde{\text{S}}\pi\text{g-T}_1$ .

**Theorem: 4.26**

Let  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  have a soft contra  $\pi\text{g}$ -graph. If  $f$  is injective, then  $X$  is  $\tilde{\text{S}}\pi\text{g-T}_1$ .

**Proof.**

Let  $x$  and  $y$  be any two distinct soft points of  $X$ . Then we have  $(x, f(y)) \in (X \times Y) \setminus G(f)$ . Then there exist a  $\tilde{\text{S}}\pi\text{g}$ -open set  $(A,E)$  in  $X$  containing  $x$  and  $(F,E) \in \tilde{\text{S}}\text{C}(Y, f(y))$  such that  $f(A,E) \cap (F,E) = \emptyset$ . Hence  $(A, E) \cap f^{-1}(F,E) = \emptyset$ . Therefore we have  $y \notin (A, E)$ . This implies that  $X$  is  $\tilde{\text{S}}\pi\text{g-T}_1$ .

**Definition: 4.27**

A space  $(X, \tau, E)$  is said to be soft ultra Hausdorff, if for each pair of distinct points  $x$  and  $y$  in  $X$ , there exists soft clopen set  $(A,E)$  and  $(B,E)$  containing  $x$  and  $y$  such that  $(A,E) \cap (B,E) = \emptyset$ .

**Theorem: 4.28**

Let  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  be a soft contra  $\pi\text{g}$ -continuous injection. If  $Y$  is a soft ultra Hausdorff space, then  $X$  is  $\tilde{\text{S}}\pi\text{g-T}_2$ .

**Proof:**

Let  $x$  and  $y$  be any two distinct soft points of  $X$ . Then  $f(x) \neq f(y)$  and there exist soft clopen sets  $(A,E)$  and  $(B,E)$  containing  $f(x)$  and  $f(y)$  respectively such that  $(A,E) \cap (B,E) = \emptyset$ . Since  $f$  is soft contra  $\pi\text{g}$ -continuous, then  $f^{-1}(A,E) \in \tilde{\text{S}}\pi\text{GO}(X)$  and  $f^{-1}(B,E) \in \tilde{\text{S}}\pi\text{GO}(X)$  such that  $f^{-1}(A,E) \cap f^{-1}(B,E) = \emptyset$ . Hence  $X$  is  $\tilde{\text{S}}\pi\text{g-T}_2$ .

**Definition : 4.29**

A space  $(X, \tau, E)$  is said to be  $\tilde{S}\pi g$ -normal if each pair of nonempty disjoint soft closed sets can be separated by disjoint  $\tilde{S}\pi g$ -open sets.

**Definition : 4.30**

A space  $(X, \tau, E)$  is said to be soft ultra normal, if each pair of nonempty disjoint soft closed sets can be separated by disjoint soft clopen sets.

**Theorem :4.31**

If  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is a soft contra  $\pi g$ -continuous closed injection and  $Y$  is soft ultra normal, then  $X$  is  $\tilde{S}\pi g$ -normal.

**Proof:**

Let  $(A, E)$  and  $(B, E)$  be disjoint soft closed subsets of  $X$ . Since  $f$  is a soft closed injection,  $f(A, E)$  and  $f(B, E)$  are disjoint and soft closed in  $Y$ . Since  $Y$  is soft ultra normal,  $f(A, E)$  and  $f(B, E)$  are separated by disjoint soft clopen sets  $(F, E)$  and  $(G, E)$  respectively. Thus  $(A, E) \subseteq f^{-1}(F, E)$ ,  $(B, E) \subseteq f^{-1}(G, E) \in \tilde{S}\pi GO(X)$  and  $f^{-1}(F, E) \cap f^{-1}(G, E) = \emptyset$ . Hence  $X$  is  $\tilde{S}\pi g$ -normal.

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