Classification of Matrices on the Basis of Special Characteristics

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1. Abstract:

In this note, we classify square matrices using special characteristics in column, rows, leading and non-leading diagonal entries. All these properties may exist singly or in a group in each class of square matrices. Besides, we have established basic algebraic structure of each class and also algebraic properties between the members of different classes. Some operations change the resultant from a class while some preserves the class. The outstanding property of all such classes is its libra value property that remains in correspondence of the corresponding mathematical operation.

2. Key Words and Notations:

Key-words: Class, Libra Value, Class Shift, Zero Class

Notations: CJ1 (m x n, L (A) = p), CJ11 (m x n, p), Z1 (m, 0), Z1

3. Introduction:

In this note, as a part of the total work, we classify square matrices depending upon certain predefined characteristic in the elements of given column, row, non-leading diagonal and leading diagonal. Each class corresponds to pre-defined characteristic properties. Member matrices of given classes will checked for different standard structural properties like commutatively, associatively on the basis of standard algebraic operations. In addition to this, we have tries to establish inter connectivity between the classes based on certain matrix structures.

4. Properties and corresponding Classes and Libra Values

Our work is encircled in classification of matrices in more than six classes. As planned, these classes are formed as the result on properties that we are going to introduce in successive steps in which each higher class shall involve some more properties than the class on discussion. In this paper, we discuss the first three properties and hence the first three classes.

Properties:

(a)Property 1 (= P_1)

Let A= $(a_{ij})_{m \times n}$ be a matrix on the field of real numbers R , ∀ i = 1 to m and j= 1 to n. where m, n ϵ N

If $\sum_{i=1}^{m} a_{ij}$= Constant for each j = 1,2,….n.

I.e. If the sum of all the entries of a column for each one of the columns of the given matrix A, remains the same real constant than the matrix is said to satisfy the property P_1.

(b) Property 2 (= P_2)

Let A= $(a_{ij})_{m \times n}$ be a matrix on the field of real numbers R , ∀ i = 1 to m and j= 1 to n. Where m, n ϵ N

If $\sum_{j=1}^{n} a_{ij}$= Constant for each i = 1,2,….m.

I.e. If the sum of all the entries of a row for each one of the rows of the given matrix A, remains the same real constant than the matrix is said to satisfy the property P_2.

(c) Property 3 (= P_3)

Let A= $(a_{ij})_{m \times n}$ be a matrix on the field of real numbers R , ∀ i = 1 to m and j= 1 to n. where

m, n ϵ N
If $\sum_{i=1}^{m} a_{ij}$= Constant for each j = 1,2,….n.
and $\sum_{j=1}^{n} a_{ij}$= Constant for each i = 1,2,….m.
I.e. If the sum of all the entries of a column and a row for each one of the columns and rows of the given matrix A, remains the same real constant than the matrix is said to satisfy the property P_3.

(d) Libra Value:

Libra value of a given class of matrices is the real number which is associated with the property of a class. This is the prime property which is the calling value of the property P_i for i=1, 2, 3 etc. Libra value will be denoted by the symbol L(A); Where A is the given matrix.
L(A) ϵ R.

(e) Zero Libra class 1:

As a special case to the above note there is a special case when L(A) = 0 where A is a matrix satisfying property P1. This class is denoted as Z1 (m x n, 0) or simply Z1(m) if there is no ambiguity for the matrix A being a square one of order m x m and
L(A) = 0
e.g. Consider a case of 3 x 3 square matrices.

$A_1 = \begin{pmatrix} 3 & 4 & -1 \\ -4 & 1 & 1 \\ 1 & -5 & 0 \end{pmatrix} \in Z1(3,0)$

The null matrix $\bar{O}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in Z1(3, 0) \subset CJ1(3,0)$

(f) Class Notations and Infinite Sub-Classes:

(1) Class CJ1:

A set of matrices which observe the property P1 constitutes class1; denoted as CJ1.
CJ1 = { A |A = $(a_{ij})_{m \times n}$, A satisfies P_1 and L(A)= p; p ϵ R for a given matrix A} -----(1)
We denote, for the given matrix A, the notation A ϵ CJ1 (m x n, p), the first notation 'm x n' in the parenthesis denotes the order of the given matrix and the second notation shows the libra value of the given matrix.
[In the case of square matrix the order may be shown by a single letter
i.e. instead of writing m x m , one can write ' m ' only.]

E.G. $A = \begin{pmatrix} 2 & 1 & -1 \\ -3 & 2 & 3 \\ 5 & 1 & 2 \end{pmatrix}$

Sum of all entries for each column is the same constant is the prime property of the matrix. The underlying matrix A satisfies property P1. The real value of the constant sum =4 = L(A) is called the libra value of the matrix. We denote this as A ϵ CJ1 (3,L(A) = 4) or A ϵ CJ1 (3,4)
Note: The identity matrix = $I_{m \times m}$, the null matrix = $\bar{O}_{m \times m}$ and the scalar matrix A(α); α ϵ R are, as a virtue of property P1, are members of CJ1

E.G $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in CJ1 (3, L(I_3) = 1)$

$\bar{O}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in CJ1 (3, L(\bar{O}_3) = 0)$

$$A(\alpha) = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{pmatrix} \in CJ1(3, L(A(\alpha) = \alpha), \alpha \in R$$

[What we require for the member matrix of class 1 i.e. CJ1, is that the matrix satisfies property P1.]

According to the general format of class 1 matrix given by defining property (1), for each fixed value of p ϵ R there exists a class of square matrices denoted as CJ1 (m , L(A) = p).

i.e. for each p ϵ R, there corresponds infinite square matrices.

At this junction we introduce an important point that leads to division of CJ1(m x n, L(A) = p) where p ϵ R.

We consider two cases; L(A) = p = 0 and p ≠ 0

This makes a dichotomous classification of the class CJ1(m x n, L(A) = p)

We denote an infinite sub-class when L(A) = p ≠ 0 . We shall denote this class by the notation **CJ11.**

Recalling here the class Z1(m x n, p =0); we have

CJ1(m x n, p) = CJ11(m x n, p ≠ 0) U Z1(m x n, p = 0) (2)

If the matrices under consideration are square matrices then, without any ambiguity and loss of generality we write;

CJ1(m, p) = CJ11(m, p ≠ 0) U Z1(m, p = 0)

It is important to mention at this point that these classes CJ11 and Z1 are mutually disjoint. i.e. **CJ11(m x n, p ≠ 0)** ∩ **Z1(m x n, p = 0)** = φ (3)

These classes **CJ11(m x n, p ≠ 0** and **Z1(m x n, p = 0)** possess different structural properties with respect to different algebraic operations. We will establish these properties in some of the next units to follow in the sequence.

General structure of 3 x 3 matrix of class CJ1 is as follows:

$$A = \begin{pmatrix} a & c & e \\ b & d & f \\ p-(a+b) & \mathrm{p-(c+d)} & p-(e+f) \end{pmatrix} \qquad (4)$$

Where all the letters used are real numbers.

We write A ϵ CJ1 (3, L(A) = p)

There are many ways of expressing the same matrix but without loss of generality, we will follow the above style.

Also we note that the columns of A ϵ CJ1 are Linearly Independent.

(2) Class 2:

In the same way as we have defined property P_1 and its corresponding class CJ1, we define class 2(CJ2) and class3 (CJ3) in tune with properties P_2 and P_3. It is important at this stage to mention that by taking the transpose of the member matrices of class 1, we get the matrices of class 2 and hence the class **CJ2.**

We have

CJ2 = { A |A = $(a_{ij})_{m \times n}$, A satisfies P_2 and L(A)= p; p ϵ R for a given matrix A} **(5)**

General structure of 3 x 3 matrix of class CJ2 is as follows:

$$A = \begin{pmatrix} a & b & p-(a+b) \\ c & d & p-(c+d) \\ e & f & p-(e+f) \end{pmatrix} \qquad (6)$$

We write A ϵ CJ2(3, L(A) = p) where p ϵ R

We note that (1) the identity matrix, (2) the null matrix, and (3) the scalar matrices, by virtue of the definition of the class 2 are also the members of class2.
As we discussed for the class1 same applies to class 2 also.
We consider two cases; L(A) = p = 0 and p ≠ 0
This makes a dichotomous classification of the class CJ2(m x n, L(A) = p)
For libra value L(A) = p = 0, we have Zero libra class as an infinite sub-class of class CJ2(m x n, p = 0); we denote this class by the notation **Z2(m x n, p = 0)** (7)

We denote an infinite sub-class when L(A) = p ≠ 0 by the notation **CJ22.**
Recalling here the class Z2(m x n, p =0); we have
CJ2(m x n, p) = CJ22(m x n, p ≠ 0) U Z2(m x n, p = 0) (8)

If the matrices under consideration are square matrices then, without any ambiguity and loss of generality we write;
CJ2(m, p) = CJ22(m, p ≠ 0) U Z2(m, p = 0)
It is important to mention at this point that these classes CJ22 and Z2 are mutually disjoint. i.e. **CJ22(m x n, p ≠ 0) ∩ Z2(m x n, p = 0)** = φ (9)
These classes **CJ22(m x n, p ≠ 0** and **Z2(m x n, p = 0)** possess different structural properties with respect to different algebraic operations. We will establish these properties in some of the next units to follow in the sequence.
We cite some examples of member matrices of class 2.

$$A = \begin{pmatrix} 2 & 3 & -1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix} \in CJ2(3, L(A) = 4) \text{ in fact } A \in CJ22(3, L(A) = 4) \subset CJ2(3, L(A) = 4)$$

$$B = \begin{pmatrix} 2 & 3 & -5 \\ 1 & -2 & 1 \\ 2 & -2 & 0 \end{pmatrix} \in Z2(3, L(B) = 0)$$

(3) Class 3:
Combining the property 1(= P1) and the property 2 (=P2) and allowing them to exist in a given matrix, we constitute a class 3 (=**CJ3**). It is a class in which the member matrix will observe both the properties 1 and 2 at a time. For the given matrix A , we define class 3 as follows.

CJ3 = { A |A = $(a_{ij})_{m \times n}$, A satisfies P_1 and P_2 both at a time. and L(A)= p; p ϵ R } **(10)**
As a virtue of the definition of class 3, the identity matrix, the null matrix, and the scalar matrix are the members of the class 3.

The general format of square matrices of class 3 is as follows.

$$A = \begin{pmatrix} a & b & p-(a+b) \\ c & d & p-(c+d) \\ p-(a+c) & p-(b+d) & -p+(a+b+c+d) \end{pmatrix} \quad (11)$$

We write A ϵ CJ3(3, L(A) = p) with all entries being real numbers.
[In fact there are many ways of expressing the general format of class 3 but for simplicity we have accepted the above form.]
[Note: We have identified many properties of matrices of class 3; some of them are as

mentioned below.

(1) For b = c, we have a symmetric matrix of class 3.

(2) The Identity matrix, the null matrix, and the scalar matrix are common to all the three classes.

(3) As we have two infinite sub-classes in the case of class1 and class 2, the class 3 does possess the same characteristics.

In this paper, we concentrate on algebraic properties and their relevance with abstract algebra.

5. Some Structural Properties:

In this section, we mention some structural properties of class 1 and provide details wherever necessary. We accept, without loss of generality, that

$A = A(a_{ij}) \in CJ1(m, L(A) = p)$; $p \in R$ for $\forall$ i, and j from 1 to m , $m \in N$.

In a similar way, where ever necessary, we will follow the same type of notations.

5.1 Closure Property:

Under regular operation addition of two matrices of the same order, for

$A \in CJ1$ (n, $L(A)= \alpha_1$) and $B \in CJ1$ (n, $L(B)= \alpha_2$) , $\alpha_1, \alpha_2 \in R$

The result of Addition of matrices A and B denoted as A + B is also a matrix,

Let C = A + B ;

C = A + B with $c_{ij} = a_{ij} + b_{ij}$ $\forall\, i, and\ j$ from 1 to n, $n \in N$

$C(c_{ij}) = C = A + B \in CJ1$ (n, $L(C) = L(A) + L(B) = \alpha_1 + \alpha_2$); which can be easily verified.

We conclude that Closure (Binary operation) property preserves the class.

[This also holds true for the member matrices of class2.]

5.2 Multiplication by Scalar:

Let $A \in CJ1$ (n, $L(A)= \alpha$) then for some $k \in R$

$kA \in CJ1$ (n, $k\alpha$)

We note that multiplication by a scalar is a class preserving property.

If k = -1 then the matrix -1A will be denoted as –A; which is also a member of class CJ1; $-A \in CJ1(n)$

[This also holds true for the member matrices of class2.]

5.3 Equality of Two Matrices:

Two matrices of the same order and same class are equal if and only if their corresponding entries are equal. This can hold true if and only if both the matrices under consideration have the same libra value. $L(A) = L(B) = \alpha$

Let $A \in CJ1$ (n, $L(A)= \alpha$) and $B \in CJ1$ (n, $L(B)= \alpha$); $\alpha \in R$

$A = B \leftrightarrow a_{ij} = b_{ij}\ \forall\, i\ and\ j = 1\ to\ n$

[This also holds true for the member matrices of class2.]

5.4 Associative Property:

Let $A_1 \in CJ1$ (m xn, $p(A_1)= \alpha_1$) , $A_2 \in CJ1$ (m x n, $p(A_2)= \alpha_2$) and $A_3 \in CJ1$ (m x n, $p(A_3)= \alpha_3$)

$\alpha, \alpha_2, \alpha_3 \in R$ then, it can be verified that $(A_1 + A_2) + A_3 = A_1 + (A_2 + A_3)$

We note that $L(A_1 + A_2) + L(A_3) = L(A_1) + L(A_2 + A_3) = L(A_1) + L(A_2) + L(A_3) = \alpha_1 + \alpha_2 + \alpha_3$

As Z1(m x n, 0) is an infinite sub-class of CJ11(m x n, 0); we claim that associative property for addition operation on the members of CJ1(m x n, p) and hence on Z1(m x n, 0) also holds true.

[This also holds true for the member matrices of class2.]

5.5 Existence of Null Matrix of class Z1 (m x m, 0):

Let A_1 be a given matrix $A_1 \in$ CJ1 (m, p)

then there exists a matrix $\bar{O}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in$ CJ1 (3 x 3, L($\bar{O}_3$) = 0)

so that $A_1 + \bar{O}_3 = \bar{O}_3 + A_1 = A_1$; The matrix $\bar{O}_3$ is a null matrix.

To be more specific, the null matrix belongs to the Zero Libra class

$\bar{O}_3 \in$ Z1(m x n, 0) $\subset$ CJ1 (m x n, p).

We conclude that $\bar{O} \in$ CJ1(m x m, 0) is an additive identity.

[This also holds true for the member matrices of class2.]

5.6 Existence of Additive Inverse:

As a virtue of the result of the property 'multiplication by a scalar' mentioned in 6,2 above, we conclude that for a given matrix A $\in$ CJ1(m xn, p) there exists its additive inverse denoted as –A such that A + (-A) = -A + A = $\bar{O} \in Z_1$(m x n, 0)

For every A $\in$ CJ1(m x n, L(A) =p), there exists exactly one matrix B $\in$ CJ1(m x n, L(B) = -p) such that A + B = B + A = $\bar{O}$ and $\bar{O} \in$ CJ1(m x n, L($\bar{O}$) =0)

[This also holds true for the member matrices of class2.]

Some Deductions:

At this junction, it is important to note the following points.

(A) It is note-worthy that the set of matrices of class1 under regular binary operation '+' forms a group. i.e. **CJ1(m x n, p) is a group under the binary operation addition of matrices.**]

(B) For some A $\in$ Z1(m, 0) there exists B $\in$ Z1(m,0) such that A + B = $\bar{O} \in Z_1$(n, 0)

(C) For the matrices A $\in$ CJ1(n, L(A) = $\alpha \neq 0$), we have for **any** matrix B $\in$ Z1(n, 0) $\subset$ CJ1(n, p) such that A + B $\in$ CJ1(n, L(A) = α) and $\alpha \neq 0$

(D) For a non–null matrix of class Z1(m, 0) there exists another non-null matrix, say B, of the same class Z1(m,0)

A + B = B + A = $\bar{O} \in$ Z1(m, 0)

This is a very important property and we shall discuss more on some proper time.

5.7 Commutative Property:

Let A = A(a_{ij}) $\in$ CJ1 (m xn, L(A)= α_1) and B = B(b_{ij}) $\in$ CJ1 (m xn, L(B)= α_2) , $\alpha_1, \alpha_2 \in$ R

Then under regular operation addition '+' as defined by binary operation on matrices of the same class, we have A + B = B + A

As $a_{ij} + b_{ij} = b_{ij} + a_{ij}$ for all i , and j $\in$ N where a_{ij} and b_{ij} are real values.

To add to this, we observe that L(A) + L(B) = L(B) + L(A)

Deduction:

To add to the deduction (3) of 6.6 above, we conclude that **CJ1(n, p) is an Abelian group** with respect to regular addition operation on class CJ1(n, p).

5.8 Matrix Multiplication:

Let $A_1 = A_1(a_{ij}) \in$ CJ1 (n, L(A_1)= α_1) and $A_2 = A_2(b_{jk}) \in$ CJ1 (n, L(A_2)= α_2) , $\alpha_1, \alpha_2 \in$ R

As defined by regular multiplicative operation of two matrices, here A_1 and A_2, is denoted as $A_1 A_2$; which is also a matrix , say $A_3 = A_1 A_2$

Also $A_3 = A_1 A_2(c_{ik} = \sum_{j=1}^{j=n} a_{ij} b_{jk}) \in CJ1\ (m \times k, L(A_1 A_2) = L(A_1) L(A_2))$

The important point is about the libra value of the product matrix A_3.

We have $L(A_3) = L(A_1) L(A_2)$

We have some important deductions as follows.

(1) If both the products $A_1 A_2$, and $A_2 A_1$ are well defined yet, In general, $A_1 A_2 \neq A_2 A_1$ but $L(A_1 A_2) = L(A_2 A1) = L(A_1) L(A_2)$

(2) The product of A, a square matrix, with itself i.e. A x A, is denoted by the symbol A^2. For $A \in CJ1\ (n, L(A)= \alpha)$, $A^2 \in CJ1\ (n, L(A^2)= \alpha^2)$

(3) Associative property in case of well defined products is also satisfied. i.e. $A_1(A_2 A_3) = (A_1A_2) A_3$; with libra value of the resultant matrix being $L(A_1)L(A_2)L(A_3)$

5.9 Distributive Property:

Let $A_1 \in CJ1\ (n, L(A_1)= \alpha_1)$, $A_2 \in CJ1\ (n, L(A_2)= \alpha_2)$, and $A_3 \in CJ1\ (n, L(A_3)= \alpha_3)$ with $\alpha_1, \alpha_2, \alpha_3 \in R$ then in case of square matrices we have

$A_1 (A_2 + A_3) = A_1 A_2 + A_1 A_3$ (Left Distributive Law)

$(A_1 + A_2) A_3 = A_1A_3 + A_2A_3$ (Right Distributive Law)

The resultant matrix at the end is of the class CJ1.

We note that $L(A_1) L(A_2 + A_3) = L(A_1 A_2) + L(A_1 A_3)$

Note:

1. We know that in general $A_1 A_2 \neq A_2A_1$ but $L(A_1 A_2) = L(A_2 A_1) = L(A_1) L(A_2)$
2. If either A_1 or A_2 or both belong to class Z1 (m) then the resultant $A_1 A_2$ also belongs to the same class Z1 (m)

5.10 Determinant of a Matrix:

Let $A \in CJ1\ (m, p(A_1) = \alpha_1)$ then its determinant denoted as $|A|$ is a real value.

$|A|$ in the case of the matrix $A \in CJ1\ (3, L(A) = p)$

According to the general format of class 1 matrix,

let $A = \begin{pmatrix} a & c & e \\ b & d & f \\ p-(a+b) & \mathrm{p}-(\mathrm{c}+\mathrm{d}) & p-(e+f) \end{pmatrix} \in CJ1\ (3, L(A) = p)$

and $|A|$ = adp – afp – bcp + bep + cfp – dep

= p(ad - bc) + p(cf – de) + p(be – af)

= p [(ad - bc) + (cf – de) + (be – af)]

If p = 0 then the matrix $A \in Z1(m,0)$ and hence $|A| = 0$.

If $p \neq 0$ then $|A| \neq 0$ (Non-Singularity) only if at least one of the expression in parenthesis in the above result is other than zero or final result on evaluation of the bracketed expression is not zero. In our usual term, we call matrix ‘A’ a non-singular matrix.

5.11 Inverse of a Non-Singular Matrix:

We consider a matrix $A \in CJ1\ (m,p)$

If p = 0 then the matrix $A \in Z1(m,0)$ and hence $|A| = 0$, and in this case Inverse of the matrix A does not exist.

If $p \neq 0$ then $|A| \neq 0$ (Non-Singularity) then Inverse of the matrix A, denoted as A^{-1}, exists.

$\therefore$ A^{-1} exists and as per the known result $A^{-1} = (adj.\ A) /|A|$

There are two important points in this connection;

(1) $A^{-1} \in$ CJ11 (m. p ≠ 0) ⊂ CJ1(m, p)

(2) $L(A^{-1}) = 1/p$

Thus we conclude that process of finding inverse of a non-singular matrix is a binary operation.

6. Abstract Algebraic Properties of CJ11, Z1, and CJ1:

We recall that

CJ1(m x n, p) = CJ11(m x n, p ≠ 0) U Z1(m x n, p = 0)

and now using the references of all the points that we have derived we state some important characteristic properties of Algebraic Structure.

(A) The infinite sub-class CJ11 is a non-commutative group under the operation matrix multiplication of members of CJ11.

*Associative property for multiplication has been established. (Ref.5.8 (3))

** The identity matrix $I_m \in$ CJ11(m, p ≠ 0)

*** Multiplicative Inverse A^{-1} exists and $A^{-1} \in$ CJ11 (m. p ≠ 0) (Ref. 5.11)

The points above coordinate the fact that the class CJ11(m, p ≠ 0) is a **group** under matrix multiplication.

As matrix multiplication is non-commutative, the group is a non-commutative group.

(B) The infinite sub-class Z1(m,0) is a commutative group under the operation matrix addition of members of Z1.

*Associative property for addition has been established. (Ref.5.4)

** The identity matrix Ō ∊ Z1(m, 0) (Ref. 5.5)

*** Additive Inverse --A exists and --A ∊ Z1 (m. p = 0) (Ref. 5.6)

The points above coordinate the fact that the class Z1(m, p = 0) is a **group** under matrix addition.

As matrix addition is commutative, (Ref. 5.7) the group Z1(m, 0) is a commutative group.

(C) The Class C1(m x n, p) under the two binary operation (1) matrix addition and (2) matrix multiplication is Ring.

We have already established in units 5.1 through 5.7 that viz. (1) associative property (2) Existence of identity (3) Existence of additive inverse and (4) Commutative property hold good, become a valid claim for a commutative group.

With reference to multiplication operation, 5.8 and 5.9 units discussed above establish (5) associative property and (6) doubly distributive laws.

All these together claim for the class1, CJ1(m x n, p) to become a Ring.

CJ1((m x n, p), + , .) is a Ring.

(D) As the members of class2 i.e. CJ2 (m x n, p) , by definition of class 2, are the corresponding transposed members of those of class1, it is sufficient to claim for all the properties mentioned above in section (c) above.

Hence CJ2((m x n, p), + , .) is a Ring.

7. Some special matrices of class-1

In this section, we discuss properties of some special matrices of class-1.These properties play important role in connecting matrices of other class also which we discuss in the further papers to follow on the same unit. We quote some matrices of class CJ1 and show some applications in the context of algebraic operations.

We introduce a Square matrix of order 3 denoted as PJ1, defined as follows.

$$PJ1 = \begin{bmatrix} 1 & 3 & 6 \\ -3 & -8 & -15 \\ 3 & 6 & 10 \end{bmatrix} \quad (12)$$

We note that PJ1 € CJ1 (3, L(PJ1) = 1)

For this matrix, $(PJ1)^{-1} = \begin{bmatrix} 10 & 6 & 3 \\ -15 & -8 & -3 \\ 6 & 3 & 1 \end{bmatrix}$ (13)

and $(PJ1)^{-1}$ € CJ1 (3,L$(PJ1)^{-1}$= 1); PJ1 being non-singular

Looking at both PJ1 and $(PJ1)^{-1}$ some special characteristic of the matrix can be identified. Some properties can be identified by performing special operation on them.

Special characteristics:

(1) PJ1 and $(PJ1)^{-1}$ both belong to CJ1 and with the same Libra value

L (PJ1) = L [$(PJ1)^{-1}$] = 1 (14)

(2) det.(PJ1) = det.(PJ^{-1}) = 1 (15)

(3) The order of element of the first row and that of third row in PJ1 and $(PJ1)^{-1}$ is interchanged, while the second row in both the matrices have order of the elements interchanged.

(4) Characteristic equation of both PJ1 and that of $(PJ1)^{-1}$ one same.

Characteristic equation is $\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$ (16)

This equation also reflects that IPJ1I = I $(PJ1)^{-1}$I = 1

(5) We define matrix multiplication on both (PJ1) and $(PJ1)^{-1}$and obtain the following results without loss of generality.

$$(PJ1)^n = \begin{bmatrix} 3^2(\sum(n-1))+1 & \frac{3n(3n-1)}{2} & \frac{3n(3n+1)}{2} \\ -(3n-1)^2+1 & -(3n)^2+1 & -(3n+1)^2+1 \\ \frac{3n(3n-1)}{2} & \frac{3n(3n+1)}{2} & 3^2(\sum(n))+1 \end{bmatrix} \quad \text{for } n \in N \quad (17)$$

We note that $(PJ1)^n \epsilon$ CJ1 (3,L$(PJ1)^n$ = 1)

Also we can, in the same way generalize $(PJ1)^{-1}$ to n[th] order (n ϵ N)

$$(PJ1)^{-n} = \begin{bmatrix} 3^2(\sum(n))+1 & \frac{3n(3n+1)}{2} & \frac{3n(3n-1)}{2} \\ -(3n+1)^2+1 & -(3n)^2+1 & -(3n-1)^2+1 \\ \frac{3n(3n+1)}{2} & \frac{3n(3n-1)}{2} & 3^2(\sum(n-1))+1 \end{bmatrix} \quad (18)$$

Also $(PJ1)^{-n} \in$ CJ1 (3 , $L((PJ1)^{-n})= 1$)

Application:

(1) We introduce a matrix SP1 = $\frac{1}{4}\begin{pmatrix} 3 & 0 & -1 \\ 6 & 8 & 6 \\ -1 & 0 & 3 \end{pmatrix} \in$ CJ1(3, L(SP1)=2) (19)

with |SP1| = 16

Consider a Pythagorean matrix whose column vectors are Pythagorean triplets

of consecutive odd integers.

The matrix OD1 = $\begin{pmatrix} 3 & 5 & 7 \\ 4 & 12 & 24 \\ 5 & 13 & 25 \end{pmatrix}$; It is a matrix of Plato family with triplets of the form

(**a, b, b+1**) where **b** = ($\mathbf{a}^2$- 1)/2 and **a** is an odd integer greater than 1. The first two

elements of the next two rows being the form (**a** + 2) and (**a** + 4**)**

We carry out multiplication operation of the two matrices defined above.

(OD1)(SP1) = $\begin{pmatrix} 3 & 5 & 7 \\ 4 & 12 & 24 \\ 5 & 13 & 25 \end{pmatrix} \frac{1}{4}\begin{pmatrix} 3 & 0 & -1 \\ 6 & 8 & 6 \\ -1 & 0 & 3 \end{pmatrix}$ which results into a Pythagorean matrix

of consecutive even triplets of Pythagorean family.

In the connection of the given odd matrix, the resultant matrix has leading element of the each column being an even integer. Each column vector is a Pythagorean triplet and the first one, of the first column, beginning with the immediate even integer off (**a** + 4**) i.e.** (**a** + 5**) .**

[Note that **a** is an odd integer.]

(OD1)(SP1) = $\begin{pmatrix} 3 & 5 & 7 \\ 4 & 12 & 24 \\ 5 & 13 & 25 \end{pmatrix} \frac{1}{4}\begin{pmatrix} 3 & 0 & -1 \\ 6 & 8 & 6 \\ -1 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 8 & 10 & 12 \\ 15 & 24 & 35 \\ 17 & 26 & 37 \end{pmatrix}$ = EV1, an even

Pythagorean matrix.

[In the member triplets of the form (**a, b, b+1**) where **b** = ($\mathbf{a}^2$- 1)/2 and **a** is an odd integer greater than 1. In the triplets of Plato family primitivity of the triplets is assured but not to all the triplets of the resultant matrix; except at least one triplet.]

(2) We apply associative property to extend the above property.

[(OD1)(SP1)] (SP1) = (OD1) [(SP1) (SP1)] = (OD1) $(SP1)^2$

We find $(SP1)^2 = \frac{1}{16}\begin{pmatrix} 10 & 0 & -6 \\ 60 & 64 & 60 \\ -6 & 0 & 10 \end{pmatrix}$

It may be noted that the matrix $(SP1)^2 \in$ CJ1 (3, $L((SP1)^2) = 4$) ;

∵ $L((SP1)^2 = (L(SP1))^2$

As a result of application of above property,

$$(OD1)\ (SP1)^2 = \begin{pmatrix} 3 & 5 & 7 \\ 4 & 12 & 24 \\ 5 & 13 & 25 \end{pmatrix} \frac{1}{16} \begin{pmatrix} 10 & 0 & -6 \\ 60 & 64 & 60 \\ -6 & 0 & 10 \end{pmatrix} = \begin{pmatrix} 18 & 20 & 22 \\ 80 & 99 & 120 \\ 82 & 101 & 122 \end{pmatrix}$$

[As the result shows the next set of even Pythagorean triplets, we deduce that the matrix $(SP1)^2$ shifts a given set to the next set of triplet to the higher triplet.

We call, in general, that the matrix $(SP1)^n$ for n ϵ N, is a shift operator of class **n** for n ϵ N.

The matrix changes the status of matrix bringing it to a Pythagorean family of even triplets from any matrix of Plato family of the format (**a, b, b+1**) where **b** = (**a**^{2}- 1)/2 and **a** is an odd integer greater than 1, where **a** ϵ N

Even (**a** is an even integer greater than 4) triplets of Pythagorean family are of the form

(a, b, b +2) where **b** is an odd positive integer only in case of Primitive triplets.

There are many such matrices of class1 which demonstrate mathematical properties.

Conclusion:

Classification of matrices based upon special characteristic has played a very important roll. Matrices with certain properties in columns, rows, leading and non-leading diagonal either individually or jointly exhibit connectivity to different branches like, abstract algebra, graph theory, application of integration, and space geometry. Some of the applications connecting class 1 matrices with Pythagorean matrices of primitive and non-primitive even and odd triplets established here are useful in developing algebraic relations on Pythagorean triplets.

Projection:

As a part of on-going work, further classes have been developed and linear transformation using integration by treating elements of a given matrix as coefficients of algebraic polynomial has contributed interesting results

References:

(1) Hazra,,A.K., matrix: Algebra,Calculus and Generalized Inverse, Viva Books Pvt. Ltd. First Indian Edition 2009.