

Volume mensuration relation of two Cuboids (Relation All Mathematics)

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Abstract:

In this research paper, Cube, Cuboid and two cuboids volume relation is explained with the help of formula. We can understand difference between their volumes, also with this formula. This volume relation is considered in two parts as i) when height is same and ii) when height is un-equal.

We are trying to give a new concept "Relation All Mathematics" to the world. I am sure that this concept will be helpful in Agricultural, Engineering, Mathematical world etc.

Keywords:

Volume, Sidemeasurement, Relation, Cube, Cuboid

I. Introduction

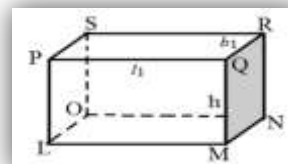
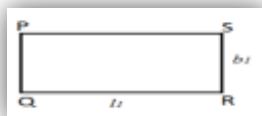
Relation All Mathematics is a new field and the various relations shown in this research, "Volume mensuration relation of two cuboids" is a 3rd research paper of Relation All Mathematics. and in future, the research related to this concept, that must be part of "Relation Mathematics" subject. Here, we have studied and shown new variables, letters, concepts, relations, and theorems. Inside the research paper explained relation between two cuboids explained in two parts. i.e. i) When height is same and ii) when height is un-equal. Sidemeasurement is a explained new concept which is very important related to Relation Mathematics subject.

In this "Relation All Mathematics" we have proved the relation between cube – cuboid and two cuboids with the help of formula. This "Relation All Mathematics" research work is near by 300 pages. This research is done considering the Agricultural sector mainly, but I am sure that it will also be helpful in other sector also.

II. Basic concept of Cube and Cuboids

2.1. Sidemeasurement(B) :- If sides of any geometrical figure are in right angle with each other, then those sides or considering one of the parallel and equal sides after adding them, the addition is the sidemeasurement. Sidemeasurement indicated with letter 'B'

Sidemeasurement is a one of the most important concept and maximum base of the Relation All Mathematics depend upon this concept.



Sidemeasurement of right angled triangle - B (ΔPQR) = b+h

In ΔPQR, sides PQ and QR are right angle, performed to each other.

Sidemeasurement of rectangle-B(□PQRS)= l₁+ b₁

In □PQRS, opposite sides PQ and RS are similar to each other and m∠Q = 90°. here side PQ and QR are right angle performed to each other.

Sidemeasurement of cuboid-E_B(□PQRS) = l₁+ b₁+ h₁

In $E(\square PQRS)$, opposite sides are parallel to each other and QM are right angle performed to each other. Sidemeasurement of cuboid written as $= E_B(\square PQRS)$

2.2) Important points of square-rectangle relation :-

I) For explanation of square and rectangle relation following variables are used

- i) Area – A
- ii) Perimeter – P
- iii) Sidemeasurement – B

II) For explanation of square and rectangle relation following letters are used

- i) Area of square ABCD – $A(\square ABCD)$
- ii) Perimeter of square ABCD – $P(\square ABCD)$
- iii) Sidemeasurement of square ABCD – $B(\square ABCD)$
- iv) Area of rectangle PQRS – $A(\square PQRS)$
- v) Perimeter of rectangle PQRS – $P(\square PQRS)$
- vi) Sidemeasurement of rectangle PQRS – $B(\square PQRS)$

2.3) Important points of cube-cuboid relation:-

I) For explanation of Cube-Cuboid relation following variables are used

- i) cuboid -E
- ii) Cube -G
- iii) Volume -V
- vi) Vertical surface area -U
- v) Total surface area -A
- vi) Sidemeasurement -B

II) Concept of explanation of Cube-Cuboid

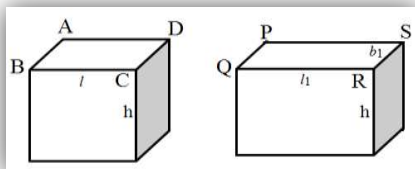


Figure I : Concept of cube and cuboid

Explanation of cube and cuboid is given with the reference of its upper side .

In Fig.I ,cuboid explaine with the reference of rectangle.i.e. $E(\square PQRS)$ and cube explaine with reference of square. i.e. $G(\square ABCD)$.

III) For explanation of Cube-Cuboid relation following letters are used

- i) Volume of cube ABCD – $G_V(\square ABCD)$
- ii) Volume of cuboid PQRS – $E_V(\square PQRS)$
- iii) Vertical surface area of the cube ABCD – $G_U(\square ABCD)$
- iv) Total surface area of the cube ABCD – $G_A(\square ABCD)$
- v) Vertical surface area of the cuboid PQRS – $E_U(\square PQRS)$
- vi) Total surface area of the cuboid PQRS – $E_A(\square PQRS)$
- vii) Sidemeasurement of cube ABCD – $G_B(\square ABCD)$
- viii) Sidemeasurement of cuboid PQRS – $E_B(\square PQRS)$

2.4) Un-equal height Volume Relation formula of cube and cuboid(Z) :

In cube and cuboid when perimeter of square and rectangle is same but height of both are un-equal then difference between volume of both are maintained with the help of ‘Un-equal height Volume Relation formula of cube and cuboid(Z) ’ and both sides volume relation of cube and cuboid become equal.

Un-equal height Volume Relation formula of cube and cuboid indicated with letter ‘Z’

$$Z=[l_1 \cdot b_1 (h - h_1)] \quad \dots \text{ here } h = \frac{(l_1 + b_1)}{2} \text{ and } l_1 \cdot b_1 = L^2$$

2.5) Important Reference theorem of previous paper which used in this paper:-

Theorem :Basic theorem of area relation of square and rectangle

Perimeter of square and rectangle is same then area of square is more than area of rectangle, at that time area of square is equal to sum of the, area of rectangle and Relation area formula of square-rectangle(K) .

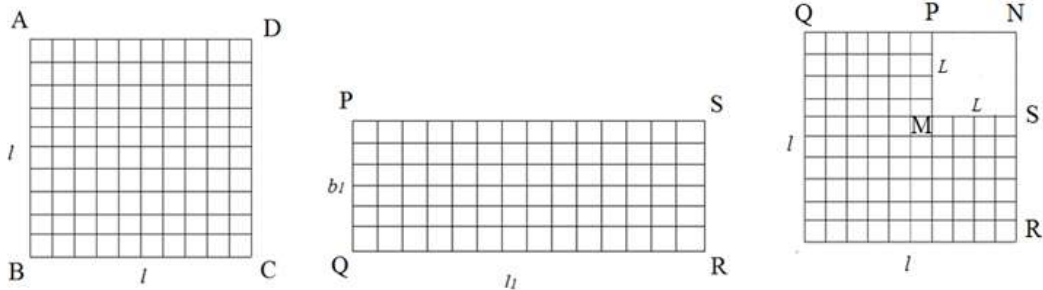


Figure II : Area relation of square and rectangle

Proof formula :- $A(\square ABCD) = A(\square PQRS) + \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2$

[Note :-The proof of this formula given in previous paper and that available in reference]

Theorem :-Basic theorem of perimeter relation of square-rectangle

Area of square and rectangle is same then perimeter of rectangle is more than perimeter of square , at that time perimeter of rectangle is equal to product of the, perimeter of square and Relation perimeter formula of square-rectangle(V).

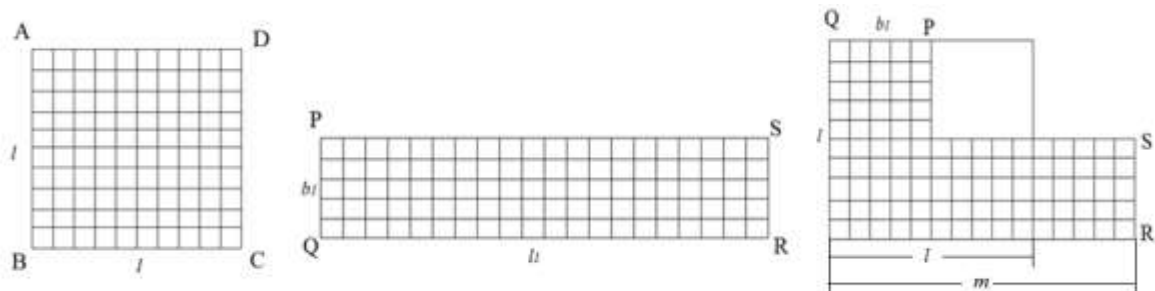


Figure III : Perimeter relation of square-rectangle

Proof formula :- $P(\square PQRS) = P(\square ABCD) \times \frac{1}{2} \left[\frac{(n^2 + 1)}{n} \right]$

[Note :-The proof of this formula given in previous paper and that available in reference]

III. Relation between cube and cuboids.

Relation –I: Volume relation of cube and cuboid when height is same

Known information: Side of cube $G(\square ABCD)$ is 'l'.and length ,width and height of cuboid $E(PQRS)$ is l_1, b_1 and h respectively.

$$G_B(\square ABCD) = E_B(\square PQRS) \dots l_1 > l.$$

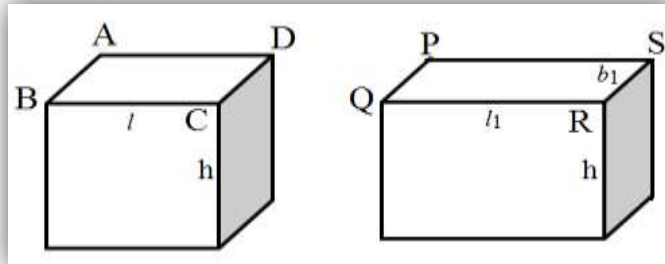


Figure –IV : Equal height Volume relation of cube and cuboid

To prove : $G_V(\square ABCD) = E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2$

Proof: In $G(\square ABCD)$ and $E(\square PQRS)$,

$$A(\square ABCD) = A(\square PQRS) + \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 \dots K = \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2$$

... (Basic theorem of area relation of square and rectangle)

Multiply to both side with height 'h'.

$$A(\square ABCD) \times h = [A(\square PQRS) \times h] + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2$$

Here, $h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2$ is a 'Volume Relation formula of cube and cuboid' and it explain with letter 'T'

$$G_V(\square ABCD) = E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2$$

Hence , we are prove that Volume relation of cube and cuboid when height is same .

This Relation cleared that following points-

- 1)Sidemeasurement of cube and cuboid are equal .
- 2)But volume of cube is more than volume of cuboid and that relation explained with the help of formula.

Example :-

	WATER TANK	□ABCD (CUBE)	□PQRS (CUBOID)	+ T	REMARK
GIVEN	LENGTH	10	14		
	WIDTH	10	6		
	HEIGHT	10	10		EQUAL
EXPLANATION	VERTICAL SURFACE AREA	400	400		EQUAL COST
	TOTAL SURFACE AREA	600	568		DIFFERENCE 32 NEGLIGIBLE
		LHS	RHS	RHS	
	VOLUME	1000	840	160	
RESULT IN WATER CAPACITY	WATER CAPACITY ON TANK	27,300 ltr (27.3 ltr/sqft)	22,932 ltr (27.3 ltr/sqft)		Water store Differenc 4368 ltr
	ANSWER	1000	1000		LHS=RHS

Relation –II:Volume relation of Cube and Cuboid when height is un-equal.

Known information: Side of cube G(□ABCD) is ‘l’.and length ,width and height of cuboid E(PQRS) is l1,b1 and h1respectively.

$$B(\square ABCD)=B(\square PQRS) \dots l_1 > l.$$

But , side of cube (h)≠ height of cuboid (h1)

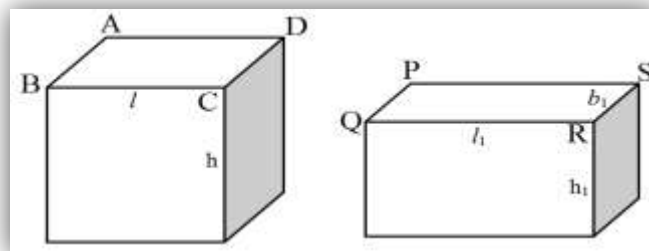


Figure –V :Un-equal height Volume relation of Cube and Cuboid

To prove : $G_V(\square ABCD) = E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 + [A(\square PQRS) \times (h - h_1)]$

Proof:In G(□ABCD) and E(□PQRS) ,

$$G_V(\square ABCD) = E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 \dots(i) \dots(\text{Volume relation of cube and cuboid when height is same})$$

But ,

$$G_V(\square ABCD) \neq E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2$$

here height of cube and cuboid is unequal,so add value of ‘Z’ in RHS .so equation become,

$$G_V(\square ABCD) = E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 + Z$$

$$G_V(\square ABCD) = E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 + [l_1 \cdot b_1 \times (h - h_1)] \dots [Z = l_1 \cdot b_1 \times (h - h_1)]$$

$$G_V(\square ABCD) = E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 + [A(\square PQRS) \times (h - h_1)]$$

Hence , we are prove that Volume relation of Cube and Cuboid when height is un-equal..

This relation cleared that following points,

- 1)In cube and cuboid ,sidemeasurement of square and rectangle are same but height is unequal .
- 2)be remember related to cube, length and width of cube are equal but height is not necessary to equal with its side
- 3)height of cube –cuboid is unequal at that time volume relation between them explained here with the help of formula.

Relation –III: Volume relation of two cuboids when height is same

Known information: The length ,width and height of cuboid E(PQRS) is l_1, b_1 and h_1 and cuboid E(LMNO) is l_2, b_2 and h_2 respectively .

$$G_B(\square ABCD) = E_B(\square PQRS) = E_B(\square LMNO) \dots l_2 > l_1 > h \quad \& \quad h = h_1 = h_2$$

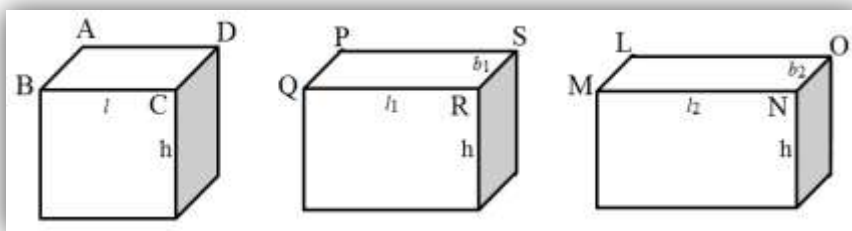


Figure –VI :Equal height Volume relation of two cuboids

To prove : $E_V(\square PQRS) = E_V(\square LMNO) + h \times (b_1 - b_2) \times [(l_1 + b_1) - (b_1 + b_2)]$

Proof: In $G(\square ABCD)$ and $E(\square PQRS)$,

$$G_V(\square ABCD) = E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 \dots (i)$$

... (Volume relation of cube and cuboid when height is same)

In $G(\square ABCD)$ and $E(\square LMNO)$,

$$G_V(\square ABCD) = E_V(\square LMNO) + h \cdot \left[\frac{(l_2 + b_2)}{2} - b_2 \right]^2 \dots (ii)$$

... (Volume relation of cube and cuboid when height is same)

$$E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 = E_V(\square LMNO) + h \cdot \left[\frac{(l_2 + b_2)}{2} - b_2 \right]^2$$

$$E_V(\square PQRS) = E_V(\square LMNO) + h \cdot \left[\frac{(l_2 + b_2)}{2} - b_2 \right]^2 - h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 \dots \text{From equatin no. (i) and (ii)}$$

$$= E_V(\square LMNO) + h \times \left[\left(\frac{(l_2 + b_2)}{2} - b_2 \right)^2 - \left(\frac{(l_1 + b_1)}{2} - b_1 \right)^2 \right]$$

$$= E_V(\square LMNO) + h \times \left[\frac{(l_2 + b_2)}{2} - b_2 - \frac{(l_1 + b_1)}{2} + b_1 \right] \times \left[\frac{(l_2 + b_2)}{2} - b_2 + \frac{(l_1 + b_1)}{2} - b_1 \right]$$

$$\dots (a^2 - b^2) = (a+b)(a-b)$$

$$E_V(\square PQRS) = E_V(\square LMNO) + h \times (b_1 - b_2) \times [(l_1 + b_1) - (b_1 + b_2)]$$

Hence , we are prove that , Volume relation of two cuboids when height is same

This relation cleared that following points,

1) sidemeasurement of two cuboid are equal

2) But among the both cuboid ,minimum length of cuboid campaired with another cuboid that's valume is more than valume of another cuboid ,and that relation here explained with the help of formula.

Relation -IV: Volume relation of two cuboids when height is un-equal.

Known information:The length ,width and height of cuboid E(PQRS) is l_1, b_1 and h_1 and cuboid E(LMNO) is l_2, b_2 and h_2 respectively .

$$B(\square ABCD) = B(\square PQRS) = B(\square LMNO) \dots l_2 > l_1 > l \quad \& \quad h \neq h_1 \neq h_2$$

But , side of cube (h) \neq height of cuboid (h_1)

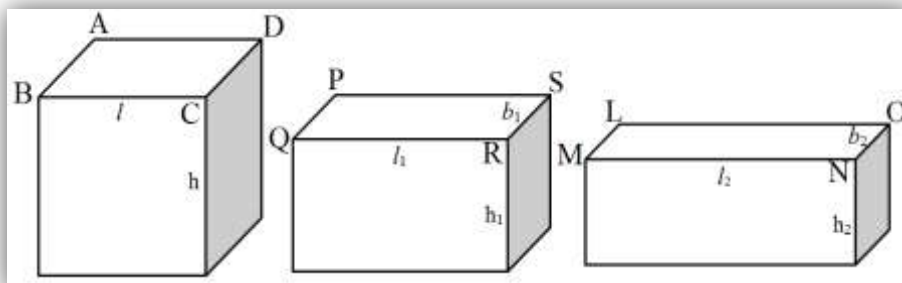


Figure –VII :Un-equal Volume relation of two cuboids

To prove :

$$E_V(\square PQRS) = E_V(\square LMNO) + [l_2 \cdot b_2 \cdot (h-h_2)] - [l_1 \cdot b_1 \cdot (h-h_1)] + h \cdot [(b_1 - b_2)] \times [(l_1 + b_1) - (b_1 + b_2)]$$

Proof: In $G(\square ABCD)$ and $E(\square PQRS)$,

$$G_V(\square ABCD) = E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 + [l_1 \cdot b_1 \cdot (h-h_1)] \quad \dots (i)$$

...(Volume relation of Cube and Cuboid when height is un-equal.)

In $G(\square ABCD)$ and $E(\square LMNO)$,

$$G_V(\square ABCD) = E_V(\square LMNO) + h \cdot \left[\frac{(l_2 + b_2)}{2} - b_2 \right]^2 + [l_2 \cdot b_2 \cdot (h-h_2)] \quad \dots (ii)$$

...(Volume relation of Cube and Cuboid when height is un-equal.)

$$E_V(\square PQRS) + h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 + [l_1 \cdot b_1 \cdot (h-h_1)] = E_V(\square LMNO) + h \cdot \left[\frac{(l_2 + b_2)}{2} - b_2 \right]^2 + [l_2 \cdot b_2 \cdot (h-h_2)]$$

...From equation no.(i) and (ii)

$$E_V(\square PQRS) = E_V(\square LMNO) + h \left[\frac{(l_2 + b_2)}{2} - b_2 \right]^2 + [l_2 \cdot b_2 \cdot (h-h_2)] - h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2 - [l_1 \cdot b_1 \cdot (h-h_1)]$$

$$= E_V(\square LMNO) + [l_2 \cdot b_2 \cdot (h-h_2)] - [l_1 \cdot b_1 \cdot (h-h_1)] + h \cdot \left[\frac{(l_2 + b_2)}{2} - b_2 \right]^2 - h \cdot \left[\frac{(l_1 + b_1)}{2} - b_1 \right]^2$$

$$= E_V(\square LMNO) + [l_2 \cdot b_2 \cdot (h-h_2)] - [l_1 \cdot b_1 \cdot (h-h_1)] + h \cdot \left[\left(\frac{(l_2 + b_2)}{2} - b_2 \right)^2 - \left(\frac{(l_1 + b_1)}{2} - b_1 \right)^2 \right]$$

$$= E_V(\square LMNO) + [l_2 \cdot b_2 \cdot (h-h_2)] - [l_1 \cdot b_1 \cdot (h-h_1)]$$

$$+ h \cdot \left[\frac{(l_2 + b_2)}{2} - b_2 + \frac{(l_1 + b_1)}{2} - b_1 \right] \cdot \left[\frac{(l_2 + b_2)}{2} - b_2 + \frac{(l_1 + b_1)}{2} - b_1 \right]$$

$$\dots (a^2 - b^2) = (a + b)(a - b)$$

$$E_V(\square PQRS) = E_V(\square LMNO) + [l_2 \cdot b_2 \cdot (h-h_2)] - [l_1 \cdot b_1 \cdot (h-h_1)] + h \cdot [(b_1 - b_2)] \times [(l_1 + b_1) - (b_1 + b_2)]$$

Hence, we have proved that Volume relation of two cuboids when height is un-equal.

This Relation cleared that following points-

1) Two cuboid inside side measurement of two rectangle are same but height is un-equal.

2) As like project, inside height depends upon their situation.

3) In this relation two cuboid relation explained with the help of formula when height is un-equal.

Note : in above relation $l_1 + b_1 = l_2 + b_2$ and h is defined as $h = l_1 + b_1 / 2$

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