Generalized Union and Intersection of Fuzzy Soft Sets

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***Abstract*— The generalization of the notion of union and intersection laid down by Zadeh in his pioneer work in 1965 was initiated by Chakrabarty. The necessity of such an attempt was also stated with practical examples. Maji initiated the notion of fuzzy soft set and introduced some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, DeMorgan Laws etc. These results were further revised and improved by Ahmad and Kharal. They defined arbitrary fuzzy soft union and intersection and proved DeMorgan Inclusions and DeMorgan Laws in Fuzzy Soft Set Theory. In 2011, Neog and Sut studied further and put forward some more propositions on fuzzy soft set theory. In this paper, we have studied the notion of union and intersection of two fuzzy soft sets in two fuzzy soft classes and propose some related results. Our work is an attempt to generalize the notion of union and intersection of fuzzy soft sets.**

***Keywords*— Soft Set, Fuzzy Soft Set, Union of Fuzzy Soft Sets, Intersection of Fuzzy Soft Sets.**

I. INTRODUCTION

In many complicated problems arising in the fields of engineering, social science, economics, medical science etc involving uncertainties, classical methods are found to be inadequate in recent times. Molodstov [3] pointed out that the important existing theories viz. Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. He further pointed out that the reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory. In 1999 he initiated the novel concept of Soft Set as a new mathematical tool for dealing with uncertainties. Soft Set Theory, initiated by Molodstov [3], is free of the difficulties present in these theories.

In recent times, researches have contributed a lot towards fuzzification of Soft Set Theory. Maji et al. [2] introduced the concept of Fuzzy Soft Set and some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, DeMorgan Law etc. These results were further revised and improved by Ahmad and Kharal [1]. In 2011, Neog and Sut [8] put forward some more propositions regarding fuzzy soft set theory. They studied the notions of fuzzy soft union, fuzzy soft intersection, complement of a fuzzy soft set and several other properties of fuzzy soft sets along with examples and proofs of certain results. In this paper, we have studied the notion of union and intersection of two fuzzy soft sets in two fuzzy soft classes and propose some related properties. Our work is an attempt to generalize the notion of union and intersection of fuzzy soft sets. This paper is, in fact, a modification of our earlier work [9] regarding union and intersection of fuzzy soft sets in two universes.

II. PRELIMINARIES

Chakrabarty et al. [7] generalized the notion of union and intersection of two fuzzy sets laid down by Zadeh [10]. In fact the concept of union of two fuzzy sets A and B in the universes X could be treated as a particular case of the following two fuzzy sets:

(i) Fuzzy set A in the universe X and
(ii) Fuzzy set B in the universe Y,

where X and Y are two different universes, in general. Thus the definitions of union and intersection of two fuzzy sets as proposed in [7] are as follows:

Definition 1. [7]

Let A be a fuzzy set of X with membership function μ_A and B be a fuzzy set of Y with membership function μ_B. Then the union of two fuzzy sets A and B denoted by $A \tilde{\cup} B$ is a fuzzy set of $X \cup Y$ with the membership function defined by

$$\mu_{A\tilde{\cup}B}(z) = \begin{cases} \mu_A(z) & \text{if } z \in X - Y \\ \mu_B(z) & \text{if } z \in Y - X \\ \max\{\mu_A(z), \mu_B(z)\} & \text{if } z \in X \cap Y \end{cases}$$

It may be observed that the fuzzy union defined by Zadeh [10] is a particular case of the above defined union when $X = Y$.

Definition 2. [7]

Let A be a fuzzy set of X with membership function μ_A and B be a fuzzy set of Y with membership function μ_B. Then the intersection of two fuzzy sets A and B denoted by $A \tilde{\cap} B$ is a fuzzy set of $X \cup Y$ with the membership function defined by

$$\mu_{A\tilde{\cap}B}(z) = \min\{\mu_A(z), \mu_B(z)\}, \qquad \forall z \in X \cup Y$$

It may be observed that the fuzzy intersection defined by Zadeh [10] is a particular case of the above defined union when $X = Y$.

It may also be observed that $\forall z \in (X \Delta Y), \mu_{A \tilde{\cap} B}(z) = 0$ and in this sense $A \tilde{\cap} B$ is a fuzzy set in $X \cap Y$.

Definition 3. [5, 6]

Let *U* be a set and *X* be a subset of *U*. Then for any fuzzy set *A* of *X*, the fuzzy set μ_A^U of *U* given by

$$\mu_A^U(x) = \begin{cases} \mu_A(x), & \forall x \in X \\ 0, & \text{otherwise} \end{cases}$$

is called the 'fuzzy set of U generated by μ_A'.

Example 1.

Let $U = \{a,b,c,d,e\}$ and $X = \{a,b,e\} \subseteq U$

Let $A = \{a/01, b/0.3, e/0.7\}$ be a fuzzy set of *X*. Then the fuzzy set μ_A^U of U generated by μ_A (i.e. *A*) is given by

$$\mu_A^U = \{a/01, b/0.3, c/0, d/0, e/0.7\}$$

Chakrabarty et al [7] gave the following proposition:

Proposition 1. [7]

For any two fuzzy sets *A* and *B* of the sets *X* and *Y*, respectively, the following holds:

(i) If $A = \varphi$, $B = \varphi$, then $A \tilde{\cap} B = \varphi$ but not conversely;

(ii) $\varphi \tilde{\cup} \varphi = \varphi$, $A \tilde{\cup} \varphi = A$, $A \tilde{\cap} \varphi = \varphi$, where φ is the null fuzzy set.

(iii) $A \tilde{\cup} B = B \tilde{\cup} A$, $A \tilde{\cap} B = B \tilde{\cap} A$

Chakrabarty et al [4] put forward the following proposition for fuzzy sets *A*,*B*,*C* obtained from three different universes *X*,*Y* and *Z* respectively.

Proposition 2. [4]

Let *A*,*B*,*C* be three fuzzy sets obtained from three different universes *X*,*Y* and *Z* respectively. Then the following holds .

(i) $A \tilde{\cup} (B \tilde{\cup} C) = (A \tilde{\cup} B) \tilde{\cup} C$

(ii) $A \tilde{\cap} (B \tilde{\cap} C) = (A \tilde{\cap} B) \tilde{\cap} C$

(iii) $A \tilde{\cap} (B \tilde{\cup} C) = (A \tilde{\cap} B) \tilde{\cup} (A \tilde{\cap} C)$

(iv) $A \tilde{\cup} (B \tilde{\cap} C) = (A \tilde{\cup} B) \tilde{\cap} (A \tilde{\cup} C)$

Definition 4. [3]

A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*.

In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε - approximate elements of the soft set.

Definition 5. [2]

A pair (*F*, *A*) is called a fuzzy soft set over *U* where $F : A \to \tilde{P}(U)$ is a mapping from *A* into $\tilde{P}(U)$.

Definition 6. [1]

Let *U* be a universe and *E* a set of attributes. Then the pair (*U*, *E*) denotes the collection of all fuzzy soft sets on *U* with attributes from *E* and is called a fuzzy soft class.

Definition 7. [2]

For two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a fuzzy soft class (*U*, *E*), we say that (*F*, *A*) is a fuzzy soft subset of (*G*, *B*), if

(*i*) $A \subseteq B$,

(*ii*) For all $\varepsilon \in A$, $F(\varepsilon) \le G(\varepsilon)$ and is written as $(F, A) \tilde{\subseteq} (G, B)$.

Definition 8. [2]

Union of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } x \in A - B \\ G(\varepsilon), & \text{if } x \in B - A \\ F(\varepsilon) \tilde{\cup} G(\varepsilon), & \text{if } x \in A \cap B \end{cases}$$

and is written as $(F, A) \tilde{\vee} (G, B) = (H, C)$.

Definition 9. [2]

Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same fuzzy set) and is written as $(F, A) \tilde{\wedge} (G, B) = (H, C)$.

Ahmad and Kharal [1] pointed out that generally $F(\varepsilon)$ or $G(\varepsilon)$ may not be identical. Moreover in order to avoid the degenerate case, he proposed that $A \cap B$ must be non-empty and thus revised the above definition as follows.

Definition 10. [1]

Let (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets in a soft class (*U*, *E*) with $A \cap B \neq \phi$ .Then Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. We write $(F, A) \tilde{\wedge} (G, B) = (H, C)$.

III. UNION AND INTERSECTION OF FUZZY SOFT SETS IN TWO FUZZY SOFT CLASSES

In view of the definition of union and intersection of two fuzzy sets over two universes proposed by Chakrabarty et al [7], we propose the union and intersection of two fuzzy soft sets in two fuzzy soft classes as follows:

Definition 11.

Union of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in the fuzzy soft classes (*X*, *E*) and (*Y*, E') respectively, is a fuzzy soft set

(*H*, *C*) in the fuzzy soft class $\left(X \cup Y, E \cup E^{/}\right)$ where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} [F(\varepsilon)]^{X \cup Y}, & \text{if } x \in A - B \\ [G(\varepsilon)]^{X \cup Y}, & \text{if } x \in B - A \\ F(\varepsilon)\tilde{\cup}G(\varepsilon), & \text{if } x \in A \cap B \end{cases}$$

and is written as $(F,A)\tilde{\vee}(G,B) = (H,C)$, where $[F(\varepsilon)]^{X \cup Y}$ is the fuzzy set of $X \cup Y$ generated by $F(\varepsilon)$ and $[G(\varepsilon)]^{X \cup Y}$ is the fuzzy set of $X \cup Y$ generated by $G(\varepsilon)$. It is to be noted that whenever *X* = *Y*, $[F(\varepsilon)]^{X \cup Y} = F(\varepsilon)$, which is a fuzzy set of X and $[G(\varepsilon)]^{X \cup Y} = G(\varepsilon)$, which is a fuzzy set of *Y* and then the above definition reduces to the form

$$H(\varepsilon) = \begin{cases} [F(\varepsilon)]^{X \cup X}, & \text{if } x \in A - B \\ [G(\varepsilon)]^{X \cup X}, & \text{if } x \in B - A \\ F(\varepsilon)\tilde{\cup}G(\varepsilon), & \text{if } x \in A \cap B \end{cases}$$

$$= \begin{cases} [F(\varepsilon)]^{X}, & \text{if } x \in A - B \\ [G(\varepsilon)]^{X}, & \text{if } x \in B - A \\ F(\varepsilon)\tilde{\cup}G(\varepsilon), & \text{if } x \in A \cap B \end{cases}$$

$$= \begin{cases} F(\varepsilon), & \text{if } x \in A - B \\ G(\varepsilon), & \text{if } x \in B - A \\ F(\varepsilon)\tilde{\cup}G(\varepsilon), & \text{if } x \in A \cap B \end{cases}$$

which is nothing but the definition of union of two fuzzy soft sets in the same fuzzy soft class as laid down by Maji et al [2].

Example 2.

Let $X = \{c_1, c_2, c_3, c_4\}$ be the set of four cars manufactured by a certain car manufacturer of India and $Y = \{c_3, c_4, c_5, c_6, c_7\}$ be the set of five sedan cars manufactured by the car manufacturers of India. Let

$E = \{e_1(\text{costly}), e_2(\text{Beautiful}), e_3(\text{Fuel Efficient}),$
$e_4(\text{ModernTechnology}), e_5(\text{Luxurious})\}$

and

$E^{/} = \{e_1(\text{costly}), e_2(\text{Beautiful}), e_6(\text{Sporty look})\}$ be the set of parameters

$A = \{e_1, e_2, e_3\} \subseteq E$ and $B = \{e_2, e_6\} \subseteq E^{/}$.

We consider the fuzzy soft sets

$(F, A) = \{ F(e_1) = \{ c_1/0.9, c_2/0.1, c_3/0.4, c_4/0.6\},$
$F(e_2) = \{ c_1/1, c_2/0, c_3/0.9, c_4/0.5\},$
$F(e_3) = \{ c_1/0.8, c_2/0.2, c_3/0.7, c_4/0.6\}\}$ and
$(G, B) = \{ G(e_2) = \{ c_3/0.7, c_4/0.2, c_5/0.2, c_6/0.7, c_7/0.4\},$
$G(e_6) = \{ c_3/0.9, c_4/0.6, c_5/0.5, c_6/1, c_7/0.6\}\}$ in the fuzzy soft classes (*X*, *E*) and (*Y*, $E^{/}$) respectively.

Then $(F,A)\tilde{\vee}(G,B) = (H,C)$,
where $C = A \cup B = \{e_1, e_2, e_3, e_6\}$ and

$(H, C) = \{ H(e_1) = \{ c_1/0.9, c_2/0.1, c_3/0.4, c_4/0.6, c_5/0, c_6/0, c_7/0\},$
$H(e_2) = \{ c_1/1, c_2/0, c_3/0.9, c_4/0.5, c_5/0.2, c_6/0.7, c_7/0.4\},$
$H(e_3) = \{ c_1/0.8, c_2/0.2, c_3/0.7, c_4/0.6, c_5/0, c_6/0, c_7/0\},$
$H(e_6) = \{ c_1/0, c_2/0, c_3/0.9, c_4/0.6, c_5/0.5, c_6/1, c_7/0.6\}\}$

It is obvious from the above example that $(F,A)\tilde{\vee}(G,B)$ is a fuzzy soft set in the fuzzy soft class $\left(X \cup Y, E \cup E^{/}\right)$.

Definition 12.

Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in the fuzzy soft classes (*X*, *E*) and (*Y*, $E^{/}$) respectively, is a fuzzy soft set (*H*, *C*) in the fuzzy soft class $\left(X \cup Y, E \cup E^{/}\right)$ where $C = A \cap B$ with $A \cap B \neq \varphi$ and

$\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)\tilde{\cap}G(\varepsilon)$

and is written as $(F,A)\tilde{\wedge}(G,B) = (H,C)$, where $F(\varepsilon)$ is a fuzzy set in *X* and $G(\varepsilon)$ is a fuzzy set in *Y*. It is to be noted that whenever *X* = *Y*, the above definition gives the intersection of two fuzzy soft sets in the same fuzzy soft class as laid down by Maji et al [2].

Example 3.

We take **Example 2**. Here
$(F,A)\tilde{\wedge}(G,B) = (H,C)$,
where $C = A \cap B = \{ e_2 \}$ and
$(H, C) = \{ H(e_2) = \{ c_1/0, c_2/0, c_3/0.7, c_4/0.2, c_5/0, c_6/0, c_7/0\} \}$

It is obvious from the above example that $(F,A)\tilde{\wedge}(G,B)$ is a fuzzy soft set in the fuzzy soft class $\left(X \cup Y, E \cup E^{/}\right)$.

We now propose some propositions related to the union and intersection of two fuzzy soft sets in two fuzzy soft classes.

Proposition 3.

For two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in the fuzzy soft classes (*X*, *E*) and (*Y*, $E^{/}$) respectively, the following holds:

(i) If $(F,A) = \tilde{\varphi}$, $(G,B) = \tilde{\varphi}$, then $(F,A)\tilde{\wedge}(G,B) = \tilde{\varphi}$ but not conversely.

(ii) $\tilde{\varphi}\tilde{\vee}\tilde{\varphi} = \tilde{\varphi}, (F,A)\tilde{\vee}\tilde{\varphi} = (F,A), (F,A)\tilde{\wedge}\tilde{\varphi} = \tilde{\varphi}$

(iii) $(F,A)\tilde{\vee}(G,B) = (G,B)\tilde{\vee}(F,A)$
$(F,A)\tilde{\wedge}(G,B) = (G,B)\tilde{\wedge}(F,A)$

Proof:

We only give the proof of (i). The results (ii) and (iii) are straight forward.

Let $(F,A) = \tilde{\varphi}$, $(G,B) = \tilde{\varphi}$

Then $\forall \varepsilon \in A, F(\varepsilon) = \varphi$ and $\forall \alpha \in B, G(\alpha) = \varphi$

Let $(F, A) \tilde{\wedge} (G, B) = (H, C)$

Where $C = A \cap B$ with $A \cap B \neq \varphi$ and

$$\forall \gamma \in C, H(\gamma) = F(\gamma) \tilde{\cap} G(\gamma)$$
$$= F(\gamma) \tilde{\cap} G(\gamma)$$
$$= \varphi \tilde{\cap} \varphi$$
$$= \varphi$$

It follows that $(F, A) \tilde{\wedge} (G, B) = \tilde{\varphi}$

For the converse part, we take the following example:

Let $X = \{c_1, c_2, c_3, c_4\}$ be the set of four cars manufactured by a certain car manufacturer of India and $Y = \{c_3, c_4, c_5, c_6, c_7\}$ be the set of five sedan cars manufactured by the car manufacturers of India. Let $E = \{e_1(\text{costly}), e_2(\text{Beautiful}), e_3(\text{Fuel Efficient}), e_4(\text{ModernTechnology}), e_5(\text{Luxurious})\}$

and

$E^{/} = \{e_1(\text{costly}), e_2(\text{Beautiful}), e_6(\text{Sporty look})\}$ be the set of parameters

$A = \{ e_1, e_2, e_3\} \subseteq E$ and $B = \{ e_2, e_6\} \subseteq E^{/}$.

We consider the fuzzy soft sets

$(F, A) = \{ F(e_1) = \{ c_1/0.9, c_2/0.1, c_3/0.4, c_4/0.6\},$

$F(e_2) = \{ c_1/1, c_2/0, c_3/0.9, c_4/0\},$

$F(e_3) = \{ c_1/0.8, c_2/0.2, c_3/0.7, c_4/0.6\}\}$ and

$(G, B) = \{ G(e_2) = \{ c_3/0, c_4/0.2, c_5/0.2, c_6/0.7, c_7/0.4\},$

$G(e_6) = \{ c_3/0.9, c_4/0.6, c_5/0.5, c_6/1, c_7/0.6\}\}$ in the fuzzy soft classes (X, E) and $(Y, E^{/})$ respectively.

Here

$(F, A) \tilde{\wedge} (G, B) = (H, C)$,

where $C = A \cap B = \{ e_2 \}$ and

$(H, C) = \{ H(e_2) = \{c_1/0, c_2/0, c_3/0, c_4/0, c_5/0, c_6/0, c_7/0\}\}$

$= \tilde{\varphi}$

In what follows, it is obvious that $(F, A) \tilde{\wedge} (G, B) = \tilde{\varphi}$ but $(F, A) \neq \tilde{\varphi}$, $(G, B) \neq \tilde{\varphi}$.

Proposition 4.

For three fuzzy soft sets (F, A), (G, B) and (H, C) in the fuzzy soft classes (X_1, E_1), (X_2, E_2) and (X_3, E_3) respectively with $A \cap B \cap C \neq \varphi$, the following holds:

(i) $(F, A) \tilde{\vee} ((G, B) \tilde{\vee} (H, C)) = ((F, A) \tilde{\vee} (G, B)) \tilde{\vee} (H, C)$

(ii) $(F, A) \tilde{\wedge} ((G, B) \tilde{\wedge} (H, C)) = ((F, A) \tilde{\wedge} (G, B)) \tilde{\wedge} (H, C)$

(iii) $(F, A) \tilde{\wedge} ((G, B) \tilde{\vee} (H, C)) = ((F, A) \tilde{\wedge} (G, B)) \tilde{\vee} ((F, A) \tilde{\wedge} (H, C))$

(iv) $(F, A) \tilde{\vee} ((G, B) \tilde{\wedge} (H, C)) = ((F, A) \tilde{\vee} (G, B)) \tilde{\wedge} ((F, A) \tilde{\vee} (H, C))$

Proof:

The proof immediately follows from definition and corresponding properties of fuzzy sets.

IV. CONCLUSION

The occurrence of union and intersection of two fuzzy soft sets in two fuzzy soft classes is very natural in many real-life situations. We have generalized the notion of union and intersection of fuzzy soft sets in two fuzzy soft classes. A comparative study with the earlier notion laid down by Maji et al. [2] is made. We have put forward some propositions based on this new notion. It is hoped that our findings would lead to a fruitful result.

ACKNOWLEDGMENT

Our special thanks to the anonymous referee and the editor of this journal for their valuable comments and suggestions which have improved this paper.

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