Study the Effect of Variable Viscosity and Thermal Conductivity of Micropolar Fluid past a Continuously Moving Plate with Suction or Injection in Presence of Magnetic Field

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Abstract

This paper investigates the influence of thermal conductivity and variable viscosity on the problem of micropolar fluid with suction or injection in presence of magnetic field. The fluid viscosity is assumed to vary as an exponential function of temperature and the thermal conductivity is assumed to vary as an exponential function of temperature. The governing equations are approximated by a system of non linear ordinary differential equations and are solved numerically by using shooting method. Numerical results are presented for the distribution of velocity, microrotation and temperature profiles within the boundary layer.

Key words: *injection, suction, micropolar fluid, variable viscosity, variable thermal conductivity, shooting method.*

1. Introduction

The theory of micropolar fluid, first introduced and formulated by Eringen (1966, 1972) and derived the constitutive laws of fluid with micro-structure has been a field of very active research for the last decades. This theory is capable to explain the complex fluids behaviour such as liquid crystals, polymeric suspensions, animal blood etc. by taking into account the effect arising from local structure and micro-motions of the fluid elements. An extensive review of micropolar fluids and their applications has been done by Ariman et al. (1973). The study of the dynamics of micropolar fluids has received considerable interest, because of its wide applicability in energy, such as geothermal energy technology, petroleum recovery, glass fiber production, metal extrusion, hot rolling, the cooling and/or drying of paper and textiles and wire drawing.

Many researchers have been studied the existing analytical studies for this problem are based on the constant physical properties of the ambient fluid [2,6,7]. However, it is known that these properties may change with temperature [3]. To accurately predict the flow and heat transfer rates it is necessary to take into account this variation of viscosity and thermal conductivity. The study of heat transfer and the flow field is necessary for determining the quality of the final products of these processes as explained by Karwe and Jaluria [4,5].

Hydrodynamics flows of a viscous and incompressible fluid have been studied under different physical conditions with variable fluid properties by Hassanien [3] and Seddek [8]. In many particle engineering system, both the plane surface and the ambient fluid are moving in parallel. Influence of thermal conductivity and variable viscosity on the flow of a micropolar fluid past a continuously moving plate with suction or injection was studied by Salem and Odda [1].

Motivated by the above investigations, the present paper considers a steady incompressible, micropolar fluid to study the effects of variable viscosity and variable thermal conductivity on heat transfer from moving plate with suction or injection in presence of steady magnetic field.

2. Mathematical formulation

A steady, two dimensional flow of an incompressible electrically conducting micropolar fluid past a continuously moving plate with suction or injection in presence of a magnetic field is considered. The origin is located at the spot through which the plate is drawn in the fluid medium, the x-axis is chosen along the plate and y-axis is taken normal to it. We assume that the fluid properties are isotropic and constant, except for the fluid viscosity μ , which is assumed to vary as an exponential function of temperature T, in the form

$$\mu = \mu_0 e^{-\beta_1 \theta} \qquad (1)$$

Also assume that, the fluid thermal conductivity k is assumed to vary as a linear function of temperature in the form [1].

$$k = k_0(1+\beta_2\theta) \qquad (2)$$

where β_1 and β_2 are parameters depending on the nature of the fluid and μ_0, k_0 are the thermal diffusivity and viscosity at temperature T_w respectively.

Under the usual boundary layer approximation, the governing equation for this problem can be written as follows.

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3)$$

The equation of momentum is

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y}) + k_1\frac{\partial \sigma}{\partial y} - \frac{\sigma}{\rho}B_0{}^2 u \qquad (4)$$

The equation of angular momentum is

$$G_1\frac{\partial^2\sigma}{\partial y^2} - 2\sigma - \frac{\partial u}{\partial y} = 0 \qquad (5)$$

The equation of energy is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(k\frac{\partial u}{\partial y}\right) + \frac{v}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (6)$$

Subject to the boundary conditions

$$u = U_0, v = V_w, T = T_w, \sigma = 0 \quad \text{at } y=0$$

$$u \to 0, T - T_\alpha, \sigma \to 0 \quad \text{as } y \to \alpha \qquad (7)$$

where x and y are the coordinate direction, u, v, σ and T are the fluid velocity components in the x and y directions, the microrotation constant and density of the fluid, respectively, c_p, U_0, T_w, and T_α are the specific heat of the fluid at constant pressure, the uniform velocity of the plate, a non-zero velocity component at the wall, the temperature of the plate and the temperature of the fluid far away from the plate and B_0 is an external magnetic field.

The governing equations subject to the boundary conditions can be expressed in a simpler form by introducing the following transformations:

$$u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}$$

$$\eta = y\sqrt{\frac{U_0}{2vx}}, \psi = \sqrt{2vU_0 x f(\eta)}, T = (T_w - T_\alpha)\theta(\eta) + T_\alpha,$$

$$\sigma = \sqrt{\frac{U_0{}^3}{2vx}} g(\eta) \qquad (8)$$

to obtain the ordinary differential equations for the function $f(\eta), g(\eta)$ and $\theta(\eta)$

The transformed ordinary differential equations are:

$$f''' + e^{\beta_1\theta}(ff'' + Kg' - Mgf') + \beta_1 f''\theta' = 0 \qquad (9)$$

$$Gg'' - 2(2g + f'') = 0 \qquad (10)$$

$$\theta''(1+\beta_2\theta) + p_r(E_c f''^2 + f\theta') + \beta_2\theta'^2 = 0 \qquad (11)$$

subject to the boundary conditions (7) which become

at $\eta = 0$: $f = F_w, f' = 1, \theta = 1, g = 0$

as $\eta \to \alpha$: $f' = \theta = g = 0$ (12)

In the above equations, a p[rime denotes differentiation with respect to η, and

$$K = \frac{k_1}{v}, G = \frac{G_1 U_0}{vx}, p_r = \frac{\rho v C_p}{k_0}$$

$$E_c = \frac{U_0{}^2}{c_p(T_w - T_\alpha)}, F_w = -V_m\sqrt{\frac{2x}{vU_0}}, M = \sqrt{\frac{2}{v}}\frac{1}{\rho}B_0{}^2 \qquad (13)$$

as the coupling constant parameter, Microrotation parameter, prandtl number, Eckert number and mass transfer parameter, respectively. Here F_w is positive for suction and negative for injection. For micropolar boundary layer flow, the wall skin τ_w is given by

$$\tau_w = \left[(\mu + k)\frac{\partial u}{\partial y} + k\sigma\right]_{y=0} \qquad (14)$$

The skin friction coefficient c_f can be defined as

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U_0{}^2} = -2\,\mathrm{Re}_x^{-\frac{1}{2}} f''(0) \qquad (15)$$

where $\mathrm{Re}_x = \frac{U_0 x}{v}$ the local Reynolds number.

The local heat flux coefficient (or local Nusselt number) may be written as

$$Nu_x = \frac{q_w x}{k(T_w - T_\alpha)} = -\frac{1}{2}\mathrm{Re}_x^{\frac{1}{2}}\theta'(0), q_w = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (16)$$

RESULTS AND DISCUSSION

Equations (9), (10) and (11) with boundary conditions (12) can be integrated numerically by the shooting method with a systematic guessing of $f''(0), g'(0)$ and $\theta'(0)$ by the shooting technique. To study the behaviour of the velocity, the angular velocity and the temperature profiles, curves are drawn for various values of the parameters that describe the flow in the case of air (pr=0-.733) and water (pr=3) at $\eta_\alpha = 7, k = 0.1, G = 2$ and *Ec=0.02*.

Fig. 1-3 shows the effect of variable viscosity parameter β_1 on the velocity, the angular velocity and the temperature distribution, respectively. As shown, the velocity and the angular velocity are decreasing with increasing β_1, but the temperature increases as β_1 increases. So, we conclude that for both air and water, the consequence of having a significant temperature dependent viscosity is to produce a marked effect on the temperature filed in these convection flows. The temperature variation for both air and water is shown in Fig. 4 for varius values of the thermal conductivity β_2 parameter. Clearly the temperature profiles increases as the thermal conductivity parameter β_2 increases in two cases. The velocity distribution for both air and water is shown in Fig. 5 for various values of the magnetic parameter M_1. Clearly the velocity distribution decreases as the magnetic parameter M_1 increases in two cases.

Fig. 6-8 illustrates the velocity, angular velocity and temperature fields, respectively, for different values of porosity parameter *Fw*. These figures indicate that, all of these quantities decreases as *Fw* decreases. It can be seen that the velocity increases monotonically with injection (*Fw*<0) and decreases with increase in suction (*Fw>0*). Also, increasing values of the injection parameter move the location of the maximum value of the microrotation away from the surface.

CONCLUSION

In the present work we consider a steady incompressible, micropolar fluid to study the influence of variable viscosity and variable thermal conductivity heat transfer from moving plate with suction or injection in presence of steady magnetic field. The fluid viscosity is assumed to vary as an exponential function of temperature and the thermal conductivity is assumed to vary as an exponential function of temperature. The governing equations are approximated by a system of non linear ordinary differential equations and are solved numerically by using shooting method. Numerical results are presented for the distribution of velocity, microrotation and temperature profiles within the boundary layer.

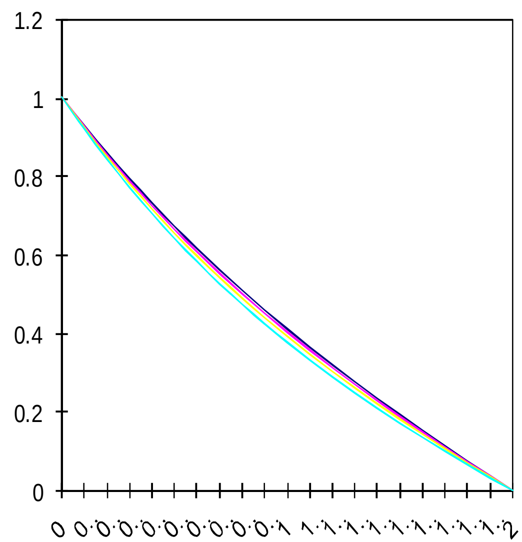

Fig. 1. Variation velocity distribution f' against η for various values of temperature corresponding to the viscosity parameter β_1 taking *Pr=0.733*, β_2=0.2, *k=0.1, Ec=0.2, G=2, Fw=0, M1=0.1*

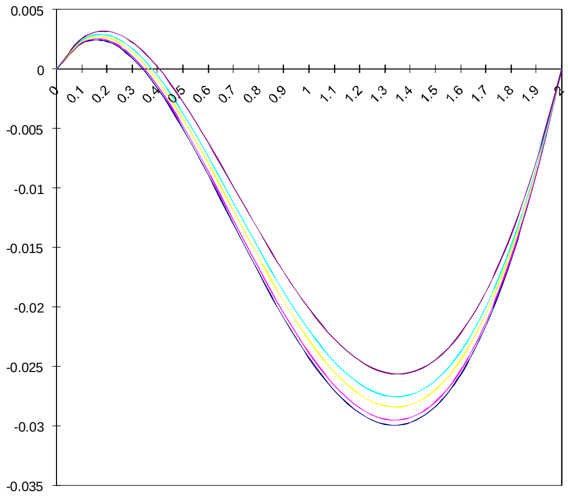

Fig. 2. Variation angular velocity g against η for various values of temperature corresponding to the viscosity parameter β_1 taking $Pr=0.733$, $\beta_2=0.2$, $k=0.1$, $Ec=0.2$, $G=2$, $Fw=0$, $M1=0.1$

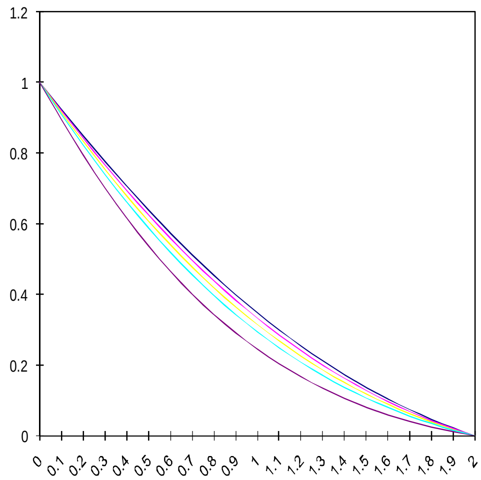

Fig. 3. Variation temperature distribution θ against η for various values of temperature corresponding to the viscosity parameter β_1 taking $Pr=0.733$, $\beta_2=0.2$, $k=0.1$, $Ec=0.2$, $G=2$, $Fw=0$, $M1=0.1$

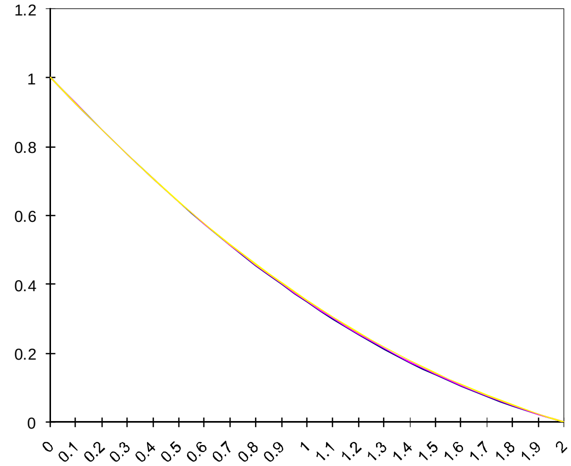

Fig. 4. Variation temperature distribution θ against η for various values of temperature corresponding to the conductivity parameter β_2 taking $Pr=0.733$, $\beta_1=0.2$, $k=0.1$, $Ec=0.2$, $G=2$, $Fw=0$, $M1=0.1$

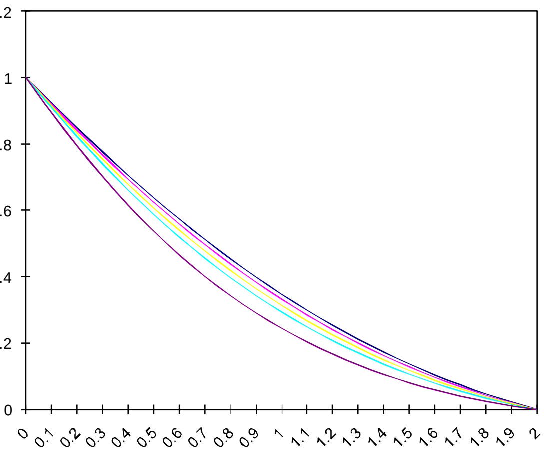

Fig. 5. Variation velocity distribution f' against η for various values of temperature corresponding to the magnetic parameter M_1 taking $Pr=0.733$, $\beta_1 = 0.3$, $\beta_2 = 0.2$, $k=0.1$, $Ec=0.2$, $G=2$, $Fw=0$, $M1=0.1$

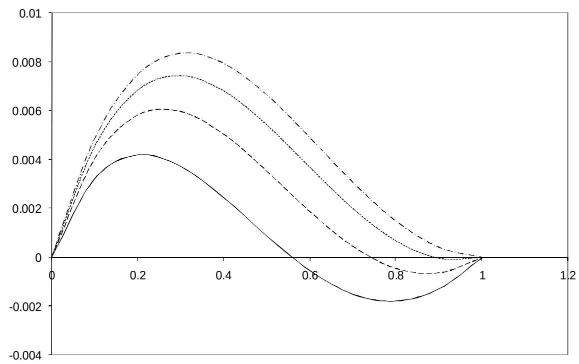

Fig. 6. Variation velocity distribution f' against η for various values of temperature corresponding to the porosity parameter F_w taking *Pr=0.733,* $\beta_1 = 0.3$, $\beta_2 = 0.2$, *k=0.1, Ec=0.2, G=2, Fw=0, M1=0.1*

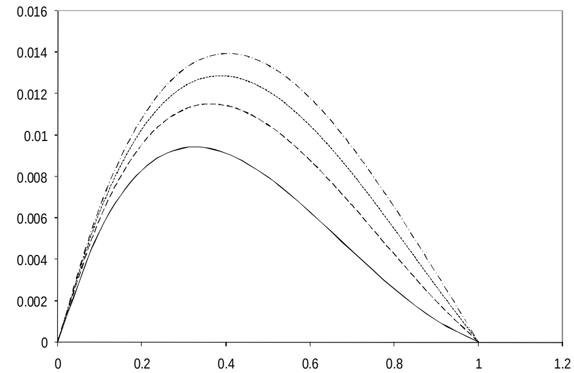

Fig. 8. Variation temperature distribution θ against η for various values of temperature corresponding to the porosity parameter F_w taking *Pr=0.733,* $\beta_1 = 0.3$, $\beta_2 = 0.2$, *k=0.1, Ec=0.2, G=2, Fw=0, M1=0.1*

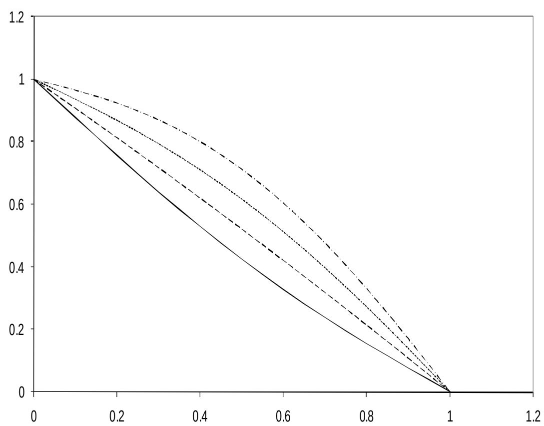

Fig. 7. Variation angular velocity *g* against η for various values of temperature corresponding to the porosity parameter F_w taking *Pr=0.733,* $\beta_1 = 0.3$, $\beta_2 = 0.2$, *k=0.1, Ec=0.2, G=2, Fw=0, M1=0.1*

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