Peristaltic Flow of Blood through Artery with a Wave of Small Amplitude Travelling Down its Wall

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Abstract—
In this paper, we study the flow of blood through artery when a progressive wave of area contraction and expansion travels along the arterial wall. For the sake of convince, we have used moving coordinate system (r, z) travelling with the wave. To get an expression for transverse and horizontal velocity component (U, V) we have reverted moving system to the stationary coordinate system (R, Z). For every fixed z, we draw the velocity profile for U/c and V/c under zero pressure drop.

Keywords- Peristaltic flow, wave velocity, wave length, Sinusoidal wave

Glossary of terms

b = the radius of the original undisturbed artery

λ = wave length

δ = amplitude of the wave

c =wave velocity

u, v = velocity components along moving co-ordinate system (r, z)

U, V = velocity components along stationary co-ordinate system (R, Z)

I. Introduction:

Peristaltic motion is a significant mechanism for transporting blood, where the cross-section of artery is contracted and expanded periodically by the progression of progressive wave. Peristaltic motion happened generally when stenosis is created in the functioning of ureter, chyme movement in intestine, movement of egg in fallopian tube, the transport of spermatozoa in the cervical canal, transport of bile in bile duct, transport of cilia etc. **[1]** assumed blood to be incompressible viscous Newtonian fluid to discuss two dimension peristaltic motions through a circular tube. **[2]** Studied the peristaltic transport of a viscous incompressible fluid through the gap between coaxial tubes, where the outer tube is non-unifrom and a sinusoidal wave travelling down its wall and the inner tube is rigid, uniform and moving with a constant velocity. **[3]** Investigated the effect of the effects of amplitude ratio, mean pressure gradient, yield stress and the power law index on the velocity distribution, wall shear stress, streamline pattern and trapping. The effects of viscoelastic wall properties and micropolar fluid parameters on the flow are investigated by **[4]** using the equation of fluid as well as of the deformable boundaries. A perturbation technique is used to determine floe characteristic by them. The use of catheters is of immense importance in many areas of scientific significance and has become a criterion for diagnosis and treatment of cardiovascular diseases. **[5]** Investigated that the pressure drop increases with the flow rate for any given particle concentration, catheter size and amplitude ratio. **[6]** Studied the peristaltic motion of electrically conducting Jeffrey fluid in a tube of a uniform magnetic field with sinusoidal wave travelling down its wall.The blood flow induced by peristaltic waves in a uniform small diameter tube studied by **[7].** Blood has been represented by a two-fluid model consisting of a core region of suspension of all erythrocytes, assumed to be cassion fluid and a peripheral layer of plasma as a Newtonian fluid. **[8]** Investigated the problem of peristaltic transport of blood in a uniform and non-uniform tube under zero Reynolds number and long wave length approximation.

II. Mathematical Formulation:

Let the equation of arterial wall be given by

$$k = b\left[1 + \delta\cos\frac{2\pi}{\lambda}(Z - ct)\right] \qquad (1)$$

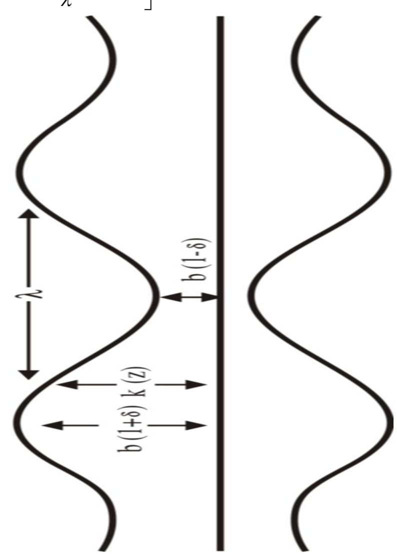

Fig.1: Geometry of two dimensional peristaltic artery

Where b is the radius of the orginal undisturbed artery, δ is the amplitude of the wave, λ is the wave length and c is the wave velocity.

We will use the moving coordinate system (r, z) traveling with the wave so that

$$r = R\,,\; z = Z - ct \qquad (2)$$

In this system, p is a function of z only. The equation of continuity and motion reduce respectively to

$$\frac{\partial}{\partial r}(ru)+\frac{\partial}{\partial z}(rv)=0 \tag{3}$$

$$\frac{\partial p}{\partial z}=\mu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right)=\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) \tag{4}$$

Where u and v are the components of velocity for the motion of the blood in relation to the moving coordinate system.

The boundary conditions for solving (3) and (4) are

$$u=\frac{\partial k}{\partial t}, v=-c, r=k \tag{5}$$

Integrating (4), we obtain

$$v=-c-\frac{1}{4\mu}\frac{dp}{dz}(k^2-r^2) \tag{6}$$

Now $q=2\pi\int_0^k rvdr$ (7)

Using (6), we have

$$q=-\pi k^2 c-\frac{\pi k^4}{8\mu}\frac{dp}{dz} \tag{8}$$

Or $\frac{dp}{dz}=-\frac{8\mu q}{\pi k^4}-\frac{8\mu c}{k^2}$ (9)

Substituting in (6), we get

$$v=-c+2\left(\frac{q}{\pi k^4}+\frac{c}{k^2}\right)(k^2-r^2) \tag{10}$$

To find the transverse velocity component u we integrate the continuity equation (3) at the constant z. Remembering that $u=0$ at $r=0$, we obtain

$$ru=-\int_0^r r\frac{\partial v}{\partial z}dr \tag{11}$$

Using (10) and remembering that $u(0,z)=0$, we get

$$u=-\frac{dk}{dz}\left(\frac{cr^3}{k^3}-\frac{2qr}{\pi k^3}+\frac{2qr^3}{\pi k^5}\right) \tag{12}$$

We now revert to the stationary coordinate system with the coordinates R, Z. The velocity components U, V and the flow rate Q so that

$$V=v+c, U=u \tag{13}$$

$Q=2\pi\int_0^k VRdR$ Or $Q=q+\pi ck^2$ (14)

Let Q_1 be the time average of Q over a complete time period T for k so that

$$Q_1=\frac{1}{T}\int_0^T Qdt=q+\pi cb^2\left(1+\frac{1}{2}\delta^2\right) \tag{15}$$

From equation (1) and (2),

$$k(z)=b\left[1+\delta\cos\left\{\frac{2\pi}{\lambda}\right\}(Z-ct)\right]$$

$$=b\left[1+\delta\cos\left(\frac{2\pi}{\lambda}z\right)\right] \tag{16}$$

$$\frac{dk}{dz}=-\frac{2\pi\delta b}{\lambda}\sin\left(\frac{2\pi}{\lambda}z\right)$$

$$=-\frac{2\pi\delta b}{\lambda}\sin\left\{\frac{2\pi}{\lambda}(Z-ct)\right\} \tag{17}$$

From (2), (10),(12) and (13), we obtain

$$U=\frac{2\pi\delta b}{\lambda}\sin\left\{\frac{2\pi}{\lambda}(Z-ct)\right\}\left(\frac{cR^3}{k^3}-\frac{2qR}{\pi k^3}+\frac{2qR^3}{\pi k^5}\right) \tag{18}$$

$$V=2\left(\frac{q}{\pi k^4}+\frac{c}{k^2}\right)(k^2-R^2) \tag{19}$$

Here, k is determine as a function of z and t from (17) and q is known from (16) after Q_1 is determined experimentally.

To determine the pressure drop across a length equal to the wavelength λ, we integrate (9)

$$(\Delta p)_\lambda=-\frac{8\mu q}{\pi b^4}\int_0^\lambda\frac{1}{\left[1+\delta\cos\left(\frac{2\pi}{\lambda}z\right)\right]^4}dz-\frac{8\mu c}{b^2}\int_0^\lambda\frac{1}{\left[1+\delta\cos\left(\frac{2\pi}{\lambda}z\right)\right]^2}$$

$$=-\frac{4\mu\lambda q}{\pi b^4}\int_0^{2\pi}\frac{1}{(1+\delta\cos\phi)^4}d\phi-\frac{4\mu c\lambda}{\pi b^2}\int_0^{2\pi}\frac{1}{(1+\delta\cos\phi)^2}d\phi$$

$$=-\frac{8\mu\lambda}{\pi b^4}\left[\frac{q(2+3\delta^2)}{2(1-\delta^2)^{\frac{7}{2}}}+\frac{\pi cb^2}{(1-\delta^2)^{\frac{3}{2}}}\right] \tag{20}$$

The pressure drop across $\lambda=1$ would be zero if

$$q=-2\pi c\frac{b^2(1-\delta^2)^2}{2+3\delta^2} \tag{21}$$

Then from (16)

$$Q_1=\frac{\pi b^2 c(16\delta^2-\delta^4)}{2(2+3\delta^2)} \tag{22}$$

Substituting (22) in (19) and (20) to obtain

$$U=\frac{2\pi\delta bRc}{\pi k^3}\sin\left\{\frac{2\pi}{\lambda}(Z-ct)\right\}\left[R^2+\frac{4b^2(1-\delta^2)^2}{2+3\delta^2}\left(1-\frac{R^2}{k^2}\right)\right] \tag{23}$$

$$V=2c\left[1-\frac{2b^2(1-\delta^2)^2}{k^2(2+3\delta^2)}\right]\left(1-\frac{R^2}{k^2}\right) \tag{24}$$

III. NUMERICAL RESULTS:

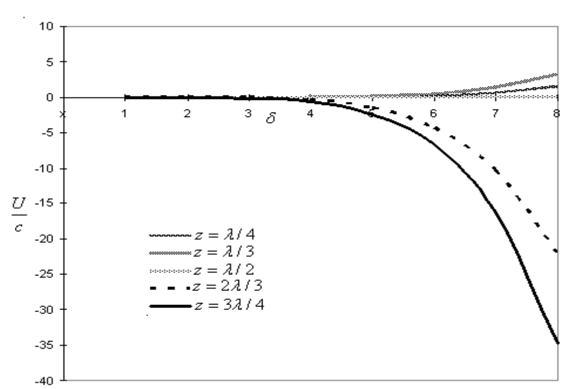

Fig.2: Velocity Profile for U/c

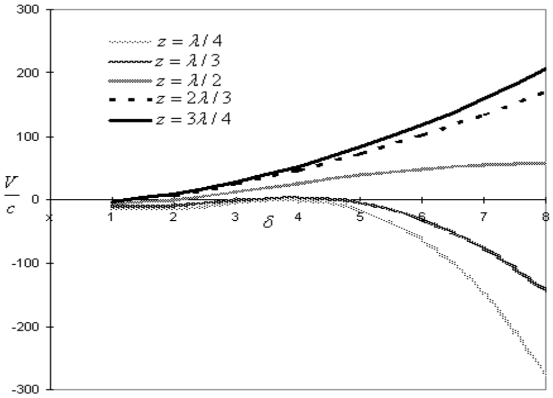

Fig.3: Velocity Profile for V/c

IV. CONCLUSION

In the present study, the analysis of long-wave length approximation to peristaltic flow of blood in an artery has been presented. We have drawn the graphs of velocity profiles for U/c and V/c for different values of z which may be useful to improve the function of ureter, chyme, movement in intestine and fallopian tube for many clinical purposes.

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