Conceptions Of Mathematics To Mathematics Education Research

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***Abstract*— The survey of the literature shows that conceptions of mathematics fall along an externally-internally developed continuum, [4] comments, along with others [13], indicate that mathematicians behave like construction lists until challenged. Similar findings may hold for mathematics teachers. The retreat to the external model to discuss their conceptions shows a strong predilection for Platonic views of mathematics. Such conceptions are strongly flavored by dualistic or multiplistic beliefs about mathematics, allowing few teachers to reject an authoritarian teaching style. Even so, the leaders and professional organizations in mathematics education are promoting a conception of mathematics that reflects a decidedly relativistic view of mathematics [14]. Steps to address the gaps between the philosophical bases for current mathematics instruction are important ones that must be addressed in the development and study of mathematics education at all levels.**

Keywords-Web Mining; Web Content Mining; Web Structure Mining; Web Usage Mining; Cloud Mining

I. INTRODUCTION

Mathematics is one of the most demanding and difficult subjects for a student to master [1]. Mathematics is taught every year from the beginning of elementary education through post-secondary education and in graduate education. Basic mathematic skills are essential to everyday life. From shopping to traveling, "math problems" exist in every aspect of daily living. However, the emphasis placed on mathematics in education and the pervasive nature of mathematics in everyday life are not enough to motivate some students to learn, master, and retain its concepts.

Perceptions of the nature and role of mathematics held by our society have a major influence on the development of school mathematics curriculum, instruction, and research. The understanding of different conceptions of mathematics is as important to the development and successful implementation of programs in school mathematics as it is to the conduct and interpretation of research studies. The literature of the reform movement in mathematics and science education (American Association for the Advancement of Science, 1989; Mathematical Sciences Education Board, 1989, 1990; National Council of Teachers of Mathematics, 1989) portrays mathematics as a dynamic, growing field of study. Other conceptions of the subject define mathematics as a static discipline, with a known set of concepts, principles, and skills [2].

Mathematics is the body of knowledge focusing on the concepts of quantity, structure, space, and change. Through abstract reasoning and logic, mathematics has evolved from counting, calculation, and measurement to be integrated into many fields such as science, medicine, economics, and everyday life. The theories within mathematics were developed in order to solve problems associated within commerce, to understand the relationships between numbers, to measure land, and to predict astronomical events [1].

Today mathematics is integrated into the educational systems of all developed countries. Most schools begin teaching addition and subtraction to students who are between the ages of five and seven years old. Students progress through mathematics courses focusing on different subdivisions of mathematics as they move through the grades in school. By the time the student graduates from high school or any secondary education institution, he will have been taught the major subdivisions of mathematics including algebra, geometry, and calculus [15].

A. Historical Evolution of Mathematical Conceptions

The rapid growth of mathematics and its applications over the past 50 years has led to a number of scholarly essays that examine its nature and its importance (Consortium for Mathematics and Its Applications, 1988; Committee on Support of Research in the Mathematical Sciences, 1969 [3] [4][5]; [16];[17];[18]; [19];[20] ilder, 1968). This literature has woven a rich mosaic of conceptions of the nature of mathematics, ranging from axiomatic structures to generalized heuristics for solving problems. These diverse views of the nature of mathematics also have a pronounced impact on the ways in which our society conceives of mathematics and reacts to its ever-widening influence on our daily lives. Regarding this, [6] writes:

Discussions of the nature of mathematics date back to the fourth century BC. Among the first major contributors to the dialogue were Plato and his student, Aristotle. Plato took the position that the objects of mathematics had an existence of their own, beyond the mind, in the external world. In doing so, Plato drew clear distinctions between the ideas of the mind and their representations perceived in the world by the senses. This caused Plato to draw distinctions between arithmetic-the theory of numbers-and logistics-the techniques of computation required by businessmen. In the *Republic* (1952a), Plato argued that the study of arithmetic has a positive effect on individuals, compelling them to reason about abstract numbers. Plato consistently held to this view, showing indignation at technicians' use of physical arguments to "prove" results in applied settings. For Plato, mathematics came to "be identical with philosophy for modern thinkers, though they say that it should be studied for the sake of other things"[6]. This elevated position for mathematics as an abstract mental activity on externally existing objects that have only representations in the sensual world is also seen in Plato's discussion of the five regular solids in *Timaeus* (1952b) and his support and encouragement of the mathematical development of Athens [8].

For since the name "Mathematics" means exactly the same as "scientific study," , , . we see that almost anyone who has had the slightest schooling, can easily distinguish what relates to Mathematics in any question from that which belongs to the other sciences. . . . I saw consequently that there must be some general science to explain that element as a whole which gives rise to problems about order and measurement restricted as these are to no special subject matter. This, I perceived, was called "Universal Mathematics," not a far fetched designation, but one of long standing which has passed into current use, because in this science is contained everything on account of which the others are called parts of Mathematics. (1952, p. 7).

B. Late 19th and Early 20th Century Views

In many ways, the ideas put forth by Brouwer were based on a foundation not unlike that professed by Kant. Brouwer did not argue for the "inspection of external objects, but [for]'close introspection'" [9]. This conception portrayed mathematics as the objects resulting from "valid" demonstrations. Mathematical ideas existed only insofar as they were constructible by the human mind. The insistence on construction placed the mathematics of the intuitionists within the Aristotelian tradition. This view took logic to be a subset of mathematics. The intuitionists' labors resulted in a set of theorems and conceptions different from those of classical mathematics. Under their criteria for existence and validity, it is possible to show that every real-valued function defined for all real numbers is continuous. Needless to say, this and other differences from classical mathematics have not attracted a large number of converts to intuitionism.

The third conception of mathematics to emerge near the beginning of the 20th century was that of formalism. This school was molded by the German mathematician David Hilbert. Hilbert's views, like those of Brouwer, were more in line with the Aristotelian tradition than with Platonism. Hilbert did not accept the Kantian notion that the structure of arithmetic and geometry existed as descriptions of a priori knowledge to the same degree that Brouwer did. However, he did see mathematics as arising from intuition based on objects that could at least be considered as having concrete representations in the mind.

The three major schools of thought created in the early 1900s to deal with the paradoxes discovered in the late 19th century advanced the discussion of the nature of mathematics, yet none of them provided a widely adopted foundation for the nature of mathematics. All three of them tended to view the contents of mathematics as products. In logicism, the contents were the elements of the body of classical mathematics, its definitions, its postulates, and its theorems. In intuitionism, the contents were the theorems that had been constructed from first principles via "valid" patterns of reasoning. In formalism, mathematics was made up of the formal axiomatic structures developed to rid classical mathematics of its shortcomings. The influence of the Platonic and Aristotelian notions still ran as a strong undercurrent through these theories. The origin of the "product" -either as a pre-existing external object or as an object created through experience from sense perceptions or experimentation - remained an issue.

C. Modern Views

Mathematics is used every day. Whether it is an engineer designing a machine, a clothes shopper determining how much he/she will save, or a student in the classroom, all have used mathematical concepts. The importance of mathematics has not diminished because of its importance; its presentation to students has become equally important. High-tech calculators can carry out extremely complex mathematical equations in a fraction of a second. This ability has created a debate on whether or not the use of calculators benefits or hurts students in their mathematical understanding.

The use of a product orientation to characterize the nature of mathematics is not a settled issue among mathematicians. They tend to carry strong Platonic views about the existence of mathematical concepts outside the human mind. When pushed to make clear their conceptions of mathematics, most retreat to a formalist, or Aristotelian, position of mathematics as a game played with symbol systems according to a fixed set of socially accepted rules [3]. In reality, however, most professional mathematicians think little about the nature of their subject as they work within it. The formalist tradition retains a strong influence on the development of mathematics [21] [13] [10] argues that the search for the foundations of mathematics is misguided. He suggests that the focus be shifted to the study of the contemporary practice of mathematics, with the notion that current practice is inherently fallible and, at the same time, a very public activity [13]. To do this, [10] begins by describing the plight of the working mathematician. During the creation of new mathematics, the mathematician works as if the discipline describes an externally existing objective reality: But when discussing the nature of mathematics, the mathematician often rejects this notion and describes it as a meaningless game played with symbols. This lack of a commonly accepted view of the nature of mathematics among mathematicians has serious ramifications for the practice of mathematics education, as well as for mathematics itself.

Mathematics must be accepted as a human activity, an activity not strictly governed by anyone school of thought (logicist, formalist, or constructivist). Such an approach would answer the question of what mathematics is by saying that: Mathematics deals with ideas. Not pencil marks or chalk marks, not physical triangles or physical sets, but ideas (which may be represented or suggested by physical objects). What are the main properties of mathematical activity or mathematical knowledge, as known to all of us from daily experience?

i. Mathematical objects are invented or created by humans.
ii. They are created, not arbitrarily, but arise from activity with already existing mathematical objects, and from the needs of science and daily life.
iii. Once created, mathematical objects have properties which are well determined, which we may have great difficulty in discovering, but which are possessed independently of our knowledge of them. [10]

The development and acceptance of a philosophy of mathematics carries with it challenges for mathematics and mathematics education. A philosophy should call for experiences that help mathematician, teacher, and student to experience the invention of mathematics. It should call for experiences that allow for the mathematization, or modeling, of ideas and events. Developing a new philosophy of mathematics requires discussion and communication of alternative views of mathematics to determine a valid and workable characterization of the discipline.

II. CONCEPTIONS OF MATHEMATICS TO MATHEMATICS EDUCATION RESEARCH

The focus on mathematics education and the growth of research in mathematics education in the late 1970s and the 1980s reflects a renewed interest in the philosophy of mathematics and its relation to learning and teaching. At least five conceptions of mathematics can be identified in mathematics education literature [11]. These conceptions include two groups of studies from the external (Platonic) view of mathematics. The remaining three groups of studies take a more internal (Aristotelian) view.

A. *Platonic View*

It [12] provides warnings about the nature of findings from research based on the external conception point of view. First, the findings provide a picture of the existing situation, not a picture of what could be achieved under dramatically changed instruction. Second, the findings reflect the type of performance that was used to separate the teachers into the different categories initially. That is, when teachers were selected as experts on the basis of specific criteria, the results reflect the teaching patterns of instruction related to those criteria. The conduct of the studies and the external conception of the mathematics employed tend to direct the type of research questions asked, and those *not* asked. This research must include teachers with a wide variety of styles if findings generalizable to all teachers or all classrooms are desired.

The second group of researchers adopting the external view espouses a more dynamic view of mathematics, but they focus on adjusting the curriculum to reflect this growth of the discipline and to see how students acquire knowledge of the related content and skills. The underlying focus is, however, still on student mastery of the curriculum or on the application of recent advances in technology or instructional technology to mathematics instruction.

B. *Aristotelian View*

The remaining three conceptions of mathematics found in mathematics education research focus on mathematics as a personally constructed, or internal, set of knowledge. In the first of these, mathematics is viewed as a process. Knowing mathematics is equated with doing mathematics. Research in this tradition focuses on examining the features of a given context that promotes the "doing." Almost everyone involved in the teaching and learning of mathematics holds that the learning of mathematics is a personal matter in which learners develop their own personalized notions of mathematics as a result of the activities in which they participate.

III. CONCLUSION & FUTURE WORK

We provide a survey the research in the area of conceptions of mathematics to mathematics education research. The emergence of a process view of mathematics embedded in the NCTM*Standards* (1989) and in the works of modern mathematical philosophers [13] presents many new and important challenges. Teacher educators and curriculum developers must become aware of the features and ramifications of the internal and external conceptions, and their ramifications for curricular development and teacher actions. Further, all involved in applying mathematics education research must recognize the important influences of each conception of mathematics on both the findings cited and on the interpretation and application of such findings. Mathematics educators need to focus on the nature of mathematics in the development of research, curriculum, teacher training, instruction, and assessment as they strive to understand its impact on the learning and teaching of mathematics.

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