Some Properties of θ-sgp-Neighbourhoods

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***Abstract*— A new generalized closed set called θ-sgp-closed set is introduced in[8]. In this paper, we continue the study of θ-sgp-closed set. Using this set we define the concepts of θ-sgp-neighbourhoods, θ-sgp-limit points, θ-sgp-derived sets, θ-sgp-Ro and θ-sgp-R_1 spaces in topological spaces. We also introduce and study the concept of θ-sgp-closure, θ-sgp-interior and θ-sgp-kernel in topological spaces by using the notion of θ-sgp-closed sets and investigate some of their properties.**

***Keywords*— θ-sgp-closed set, θ-sgp-open set, θ-sgp-nbd, θ-sgpd(A), θ-sgpCl(A), θ-sgpInt(A), θ-sgp-Ro, θ-sgp-R_1.**

1. Introduction

The concept of generalized closed sets introduced by Levine [4] plays a significant role in general topology. This notion has been studied extensively in recent years by many topologists. In 2003, Caldas and Jafari defined θ-semigeneralized closed set [1] using semi-θ-closure operator. Recently in [7] the notion of θgs-closed was introduced. In [8] the notion of θ-semigeneralized pre closed set is introduced utilizing pre-θ-closure operator. In this paper we define θ-sgp-neighbouhoods, θ-sgp-interior, θ-sgp-closure and characterise their properties. We also introduce θ-sgp-R_0 and θ-sgp-R_1 spaces. In addition we define θ-sgp-kernel of a subset A of X to study the behaviours of θ-sgp-R_0 and θ-sgp-R_1 spaces.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of space X, then Cl(A) and Int(A) denote the closure of A and the interior of A in X respectively.

The following definitions are useful in the sequel.

***Definition 2.1*:** A subset A of space X is called
(i) a semi-open set [5] if A ⊆ Cl(Int(A)).
(ii) a semi-closed set [2] if Int(Cl(A)) ⊆ A.
(iii) a pre-open set[6] if A ⊆ Int(Cl(A)).
(iv) a pre-closed set[6] if Cl(Int(A)) ⊆ A.

***Definition 2.2*:**[3] The pre-closure of a subset A of X is the intersection of all pre-closed sets that contain A and is denoted by pCl(A).

***Definition 2.3*:** A point x ϵ X is called a pre-θ-cluster point of A[9] if pCl(U) ∩ A ≠ Ø, for each pre-open set U containing x.

***Definition 2.4*:**[9] The pre-θ-closure denoted by $pCl_θ(A)$, is the set of all pre-θ-cluster points of A. A subset A is called pre-θ-closed set if A = $pCl_θ(A)$. The complement of pre-θ-closed set is pre-θ-open set.

Definition 2.5: [8] A subset A of a topological space X is called θ-Semigeneralized pre closed set (briefly, θ-sgp-closed) if $pCl_\theta(A) \subset U$ whenever $A \subset U$ and U is semi-open in X.
The complement of θ-Semigeneralized pre closed set is called θ-Semigeneralized pre open (briefly, θ-sgp-open).

3. θ-SGP-NEIGHBOURHOODS

In this section we define the notions of θ-sgp-neighbourhoods, θ-sgp-limit points and θ-sgp-derived sets in topological spaces by using the notion of θ-sgp-open sets and obtained some of their properties.

Definition 3.1: A subset A of a topological space X is called θ-semigeneralized pre-neighbourhood (briefly θ-sgp-nbd) of a point x of X if there exists a θ-sgp-open set U such that $x \in U \subseteq A$.

Definition 3.2: Let X be a topological space and A be a subset of X. A subset N of X is said to be θ-sgp-neighbourhood of A if there exists a θ-sgp-open set G such that $A \subseteq G \subseteq N$.

Theorem 3.3: Let A be a subset of a topological space X. Then A is θ-sgp-open if and only if A contains a θ-sgp-nbd of each of its points.
Proof : Let A be a θ-sgp-open set in X. Let $x \in A$, which implies $x \in A \subseteq A$. Thus A is θ-sgp-nbd of x. Hence A contains a θ-sgp-nbd of each of its points.
Conversely, A contains a θ-sgp-nbd of each of its points. For every $x \in A$ there exists a neighbourhood N_x of x such that $x \in N_x \subseteq A$. By the definition of θ-sgp-nbd of x, there exists a θ-sgp-open set G_x such that $x \in G_x \subseteq N_x \subseteq A$. Now we shall prove that $A = \cup \{G_x: x \in A\}$. Let $x \in A$. Then there exist θ-sgp-open set G_x such that $x \in G_x$. Therefore, $x \in \cup\{G_x: x \in A\}$ which implies $A \subseteq \cup\{G_x: x \in A\}$. Now let $y \in \{G_x: x \in A\}$ so that $y \in$ some G_x for some $x \in A$ and hence $y \in A$. Therefore, $\cup\{G_x: x \in A\} \subseteq A$. Hence $A = \cup\{G_x: x \in A\}$. Also each G_x is a θ-sgp-open set. And hence A is a θ-sgp-open set.

Theorem 3.4: If A is a θ-sgp-closed subset of X and $x \in X - A$, then there exists a θ-sgp-nbd N of x such that $N \cap A = \emptyset$.
Proof : If A is a θ-sgp-closed set in X, then X – A is a θ-sgp-open set. By the Theorem 3.3, X – A contains a θ-sgp-nbd of each of its points. Which implies that, there exist a θ-sgp-nbd N of x such that $N \subseteq X-A$. That is, no point of N belongs to A and hence $N \cap A = \emptyset$.

Theorem 3.5: Let X be a topological space. If A is a θ-sgp-closed subset of X and $x \in X - A$ then there exists a θ-sgp-neighbourhood N of x such that $A \cap N \neq \emptyset$.
Proof : Since A is θ-sgp-closed, X – A is θ-sgp-open set in X. By the Theorem 3.3, X – A contains a θ-sgp-neighbourhood of each of its points. Hence there exists a θ-sgp-neighbourhood of N of x such that $N \subseteq X - A$. Thus $N \cap A \neq \emptyset$.

Definition 3.6: Let X be a topological space and A be a subset of X. Then a point $x \in X$ is called a θ-semi generalized pre-limit point of A if and only if every θ-sgp-nbd of x contains a point of A distinct from x. That is $[N - \{x\}] \cap A \neq \emptyset$, for every θ-sgp-nbd N of x. Also equivalently if and only if every θ-sgp-open set G containing x contains a point of A other than x.

The set of all θ-sgp-limit points of A is called θ-sgp-derived set of A and is denoted by θ-sgpd(A).

Theorem 3.7: Let A and B be subsets of X and $A \subseteq B$ implies θ-sgpd(A) $\subseteq$ θ-sgpd(B).

Proof **:** Let x ∈ θ-sgpd(A) implies x is a θ-sgp-limit point of A that is every θ-sgp-nbd of x contains a point of A other than x. Since A ⊆ B, every θ-sgp-nbd of x contains a point of B other than x. Consequently x is a θ-sgp-limit point of B. That is x ∈ θ-sgpd(B). Therefore θ-sgpd(A) ⊆ θ-sgpd(B).

Theorem 3.8**:** Let X be a topological space and A be a subset of X. Then A is θ-sgp-closed if and only if θ-sgpd(A) ⊆A.

Proof **:** If A is θ-sgp-closed set. That is X - A is θ-sgp-open set. Now we prove that θ-sgpd(A) ⊆ A. Let x ∈ θ-sgpd(A) implies x is a θ-sgp-limit point of A, that is every θ-sgp-nbd of x contains a point of A different from x. Now suppose x ∉ A so that x ∈ X - A, which is θ-sgp-open and by definition of θ-sgp-open sets, there exists a θ-sgp-nbd N of x such that N ⊆ X - A. From this we conclude that N contains no point of A, which is a contradiction. Therefore x ∈ A and hence θ-sgpd(A) ⊆ A.

Conversely assume that θ-sgpd(A) ⊆ A and we will prove that A is a θ-sgp-closed set in X or X - A is θ-sgp-open set. Let x ∈ X - A. Let x be an arbitrary point of X - A, so that x ∉ A which implies that x ∉ θ-sgpd(A). That is there exists a θ-sgp-nbd N of x which consists of only points of X – A. This means that X - A is θ-sgp-open. And hence A is θ-sgp-closed set in X.

Theorem 3.9**:** Let X topological space, every θ-sgp-derived set in X is θ-sgp-closed set.

Proof **:** Let A be a subset of X and θ-sgpd(A) is θ-sgp-derived set of A. By Theorem 3.8, A is θ-sgp-closed if and only of θ-sgpd(A) ⊂ A. Hence θ-sgpd(A) is θ-sgp-closed if and only if θ-sgpd(θ-sgpd(A)) ⊆ θ-sgpd(A) . That is every θ-sgp-limit point of θ-sgpd(A) belongs to θ-sgpd(A).

Now let x be a θ-sgp-limit point of θ-sgpd(A). That is x ∈ θ-sgpd(θ-sgpd(A)). So that there exist a θ-sgp-open set G containing x such that {G – {x}} ∩ θ-sgpd(A) ≠ Ø which implies {G – {x}} ∩ A ≠ Ø, because every θ-sgp-nbd of an element of θ-sgpd(A) has at least one point of A. Hence x is a θ-sgp-limit point of A. That is x belongs to θ-sgpd(A). Thus x ∈ θ-sgpd(θ-sgpd(A)) implies x ∈ θ-sgpd(A). Therefore θ-sgpd(A) is θ-sgp-closed set in (X,τ).

4. θ-SGP-CLOSURE AND θ-SGP-INTERIOR

Let us introduce the notion of θ-sgp-closure, θ-sgp-interior in topological spaces by using the notion of θ-sgp-closed sets and investigate some of their basic properties.

Definition 4.1**:** For a subset A of X, θ-semigeneralized pre-closure of A, denoted by θ-sgpCl(A) and is defined as θ-sgpCl(A) = ∩{G : A ⊆ G, G is θ-sgp-closed in X}.

Theorem 4.2**:** For any x ∈ X, x ∈ θ-sgpCl(A) if and only if A ∩ V ≠ Ø for every θ-sgp-open set V containing x.

Proof **:** Let x ∈ θ-sgpCl(A). Suppose there exists a θ-sgp-open set V containing x such that V ∩ A = Ø. Then A ⊆ X–V. Since X–V is θ-sgp-closed, αCl(A) ⊆ X–V. This implies x ∉ θ-sgpCl(A) which is a contradiction. Hence V ∩ A ≠ Ø for every θ-sgp-open set V containing x.

Conversely, let A ∩ V ≠ Ø for every θ-sgp-open set V containing x. To prove that x ∈ θ-sgpCl(A). Suppose x ∉ θ-sgpCl(A). Then there exists a θ-sgp-closed set G containing A such that x ∉ G. Then x ∈ X – G and X – G is θ-sgp-open. Also (X–G) ∩ A = Ø which is a contradiction to the hypothesis. Hence x ∈ θ-sgpCl(A).

Theorem 4.3**:** Let E and F be subsets of X.

a) θ-sgpCl(Ø) = Ø.
b) θ-sgpCl(X) = X.
c) θ-sgpCl(E) is θ-sgp-closed set in X.
d) If E ⊆ F, then θ-sgpCl(E) ⊆ θ-sgpCl(F).
e) θ-sgpCl(E ∪ F) = θ-sgpCl(E) ∪ θ-sgpCl(F).

f) θ-sgpCl [θ-sgpCl(E)] = θ-sgpCl(E).

***Proof*:** The proof of a), b), c) and d) follow from the Definition 4.1.

e) To prove that θ-sgpCl(E) $\cup$ θ-sgpCl(F) $\subseteq$ θ-sgpCl(E $\cup$ F)

We have θ-sgpCl(E) $\subseteq$ θ-sgpCl(E $\cup$ F) and θ-sgpCl(F) $\subseteq$ θ-sgpCl(E $\cup$ F). Therefore θ-sgpCl(E) $\cup$ θ-sgpCl(F) $\subseteq$ θ-sgpCl(E $\cup$ F) -------- (1).

Let x be any point such that x $\notin$ θ-sgpCl(E) $\cup$ θ-sgpCl(F), then there exists θ-sgp-closed sets A and B such that E $\subseteq$ A and F $\subseteq$ B, x $\notin$ A and x $\notin$ B. Then x $\notin$ A $\cup$ B, E $\cup$ F $\subseteq$ A $\cup$ B and A $\cup$ B is θ-sgp-closed set. Thus x $\notin$ θ-sgpCl(E$\cup$F). Therefore we have θ-sgpCl(E $\cup$ F) $\subseteq$ θ-sgpCl(E) $\cup$ θ-sgpCl(F) ---------- (2). Hence from (1) and (2), θ-sgpCl(E $\cup$ F) = θ-sgpCl(E) $\cup$ θ-sgpCl(F).

f). Let A be θ-sgp-closed set containing E. Then by Definition, θ-sgpCl(E) $\subseteq$ A. Since A is θ-sgp-closed set and contains θ-sgpCl(E) and is contained in every θ-sgp-closed set containing E, it follows that θ-sgpCl[θ-sgpCl(E)] $\subseteq$ θ-sgpCl(E). Therefore θ-sgpCl[θ-sgpCl(E)] = θ-sgpCl(E).

***Theorem 4.4*:** A set A $\subset$ X is θ-sgp-closed if and only if θ-sgpCl(A) = A.

***Proof* :** Let A be θ-sgp-closed set in X. Since A $\subseteq$ A and A is θ-sgp-closed set, A $\in$ {G: A $\subseteq$ G, G is θ-sgp-closed set} which implies that $\cap$ {G: A $\subseteq$ G, G is θ-sgp-closed set} $\subset$ A. That is θ-sgpCl(A) $\subset$ A. Note that A $\subset$ θ-sgpCl(A) is always true. Hence A = θ-sgpCl(A).

Conversely, suppose θ-sgpCl(A) = A. Since A $\subseteq$ A and A is θ-sgp-closed set. Therefore A must be a closed set. Hence A is θ-sgp-closed.

Now we introduce the following.

***Definition 4.5*:** For a subset A of X θ-semigeneralized pre-interior of A, denoted by θ-sgpInt(A) and is defined as θ-sgpInt(A) = $\cup$ {G : G $\subseteq$ A and G is θ-sgp-open in X}. That is θ-sgpInt(A) is the union of all θ-sgp-open sets contained in A.

***Theorem 4.6*:** Let A be a subset of X, then θ-sgpInt(A) is the largest θ-sgp-open subset of X contained in A if A is θ-sgp-open.

***Proof* :** Let A $\subseteq$ X be θ-sgp-open. Then θ-sgpInt(A) = $\cup$ {G : G $\subseteq$ A and G is θ-sgp-open in X}. Since A $\subseteq$ A and A is θ-sgp-open, A = θ-sgpInt(A) is the largest θ-sgp-open subset of X contained in A.

The converse of the above theorem need not be true as seen from the following example.

***Example 4.7*:** Let X= {a, b, c} and τ = {X, ϕ, {a}, {c}, {a, c}, {b, c}}. Let A = {a, b}, θ-sgpInt(A) ={a} is θ-sgp-open in X. But A is not θ-sgp-open set in X.

***Theorem 4.8*:** If A $\subseteq$ B, then θ-sgpInt(A) $\subseteq$ θ-sgpInt(B).

***Proof* :** Suppose A $\subseteq$ B, we know that θ-sgpInt(A) $\subseteq$ A. Also we have A $\subseteq$ B. which implies θ-sgpInt(A) $\subseteq$ B, θ-sgpInt(A) is an open set which is contained in B. But θ-sgpInt(B) is the largest open set contained in B. Therefore θ-sgpInt(B) is larger that θ-sgpInt(A). That is θ-sgpInt(A) $\subseteq$ θ-sgpInt(B).

***Remark 4.9*:** θ-sgpInt(A) = θ-sgpInt(B) does not imply that A = B. This shown by the following example.

***Example 4.10*:** Let X = {a, b, c} and τ = {X, ϕ, {a}, {c}, {a, c}, {b, c}}. Then X be a topological space. Here θ-sgp-open sets are X, ϕ, {a}, {c}, {a, c}, {b, c}. Let A = {a} and B = {a, b}, then θ-sgpInt(A) = θ-sgpInt(B) but A $\neq$ B.

***Theorem 4.11*:** For any subset A of X, following holds:

1) θ-sgpInt(Ø) = Ø
2) θ-sgpInt(X) = X
3) If $A \subseteq B$ then θ-sgpInt(A) $\subseteq$ θ-sgpInt(B)
4) θ-sgpInt(A) is the largest θ-sgp-open set contained in A
5) θ-sgpInt(A ∩ B) = θ-sgpInt(A) ∩ θ-sgpInt(B)
6) θ-sgpInt(A ∪ B) $\supseteq$ θ-sgpInt(A) ∪ θ-sgpInt(B)
7) θ-sgpInt[θ-sgpInt(A)] = θ-sgpInt(A)

***Proof* :** Proof follows from the Definition 4.5.

***Theorem 4.12*:** A set $A \subset X$ is θ-sgp-open if and only if θ-sgpInt(A) = A.

***Theorem 4.13*:** For any $A \subseteq X$, [X – θ-sgpInt(A)] = θ-sgpCl(X – A).

***Proof* :** Let $x \in$ X – θ-sgpInt(A). Then $x \notin$ θ-sgpInt(A). That is every θ-sgp-open set G containing x is such that $G \not\subset A$. This implies, every θ-sgp-open set G containing x intersects X–A. That is, $G \cap (X–A) \neq \phi$. Then by Theorem 4.2, $x \in$ θ-sgpCl(X–A) and therefore [X– θ-sgpInt(A)] $\subseteq$ θ-sgpCl(X – A).

Conversely, let $x \in$ θ-sgpCl(X–A). Then every θ-sgp-open set G containing x intersects X–A. That is, $G \cap (X-A) \neq \phi$. That is, every θ-sgp-open set G containing x is such that $G \not\subset A$. Then by Definition 4.5, $x \notin$ θ-sgpInt(A). That is, $x \in$ [X – θ-sgpInt(A)], and so [θ-sgpCl(X-A)] $\subseteq$ [X - θ-sgpInt(A)]. Thus [X - θ-sgpInt(A)] = [θ-sgpCl(X - A)].

***Remark 4.14*:** For any $A \subseteq X$. We have

(i) (X – θ-sgpCl(X – A)) = (θ-sgpInt(A)).

(ii) (X – θ-sgpInt(X – A)) = (θ-sgpCl(A)).

Taking complement in the above Theorem 4.13 and by replacing A by X – A in Theorem 4.13, the above results follows.

5. θ-SGP-R_0 and θ-SGP-R_1 SPACES

***Definition 5.1*:** Let A be a subset of a topological space X. The θ-sgp-kernel of A, denoted by θ-sgp-ker(A) is defined to be the set θ-sgp-ker(A) = ∩{U : A ⊆ U and U is θ-sgp-open in X}.

***Definition 5.2*:** Let x be a point of a topological space X. The θ-sgp-kernel of x, denoted by θ-sgp-ker({x}) is defined to be the set θ-sgp-ker({x}) = ∩ {U : $x \in U$ and U is θ-sgp-open in X}.

***Lemma 5.3*:** Let X be a topological space and $x \in X$. Then θ-sgp-ker(A) = {$x \in X$: θ-sgpCl ({x}) ∩ A ≠ Ø}.

***Proof* :** Let $x \in$ θ-sgp-ker(A) and suppose θ-sgpCl({x}) ∩ A = Ø. Hence $x \notin$ X – θ-sgpCl({x}) which is a θ-sgp-open set containing A. This is absurd, since $x \in$ θ-sgp-ker(A). Hence θ-sgpCl ({x}) ∩ A ≠ Ø.

Conversely, let θ-sgpCl({x}) ∩ A ≠ Ø and assume that $x \notin$ θ-sgp-ker(A). Then there exists a θ-sgp-open set U containing A and $x \notin U$. Let $y \in$ θ-sgpCl({x}) ∩ A. Hence, U is a θ-sgp-nbd of y in which $x \notin U$. By this contradiction $x \in$ θ-sgp-ker(A) and the claim.

***Definition 5.4*:** A topological space X is said to be θ-semigeneralized pre-Ro (briefly θ-sgp-Ro) space if and only if for each θ-sgp-open set G and $x \in G$ implies θ-sgpCl({x}) ⊆ G.

***Lemma 5.5*:** Let X be a topological space and $x \in X$. Then $y \in$ θ-sgp-ker({x}) if and only if $x \in$ θ-sgpCl({y}).

***Proof* :** Suppose that y $\notin$ θ-sgp-ker({x}). Then there exists a θ-sgp-open set V containing x such that y $\notin$ V. Therefore we have x $\notin$ θ-sgpCl({y}). The proof of converse can be done similarly.

***Lemma 5.6*:** The following statements are equivalent for any points x and y in a topological space X:
(i). θ-sgp-ker({x}) ≠ θ-sgp-ker({y}).
(ii). θ-sgpCl({x}) ≠ θ-sgpCl({y}).
***Proof* :** (i) → (ii): Suppose that θ-sgp-ker({x}) ≠ θ-sgp-ker({y}), then there exists a point z in X such that z ∈ θ-sgp-ker({x}) and z $\notin$ θ-sgp-ker({y}). From z ∈ θ-sgp-ker({x}) it follows that {x} ∩ θ-sgpCl({z}) ≠ Ø which implies x ∈ θ-sgpCl({z}). By z $\notin$ θ-sgp-ker({y}), we have {y} ∩ θ-sgpCl({z}) = Ø. Since x ∈ θ-sgpCl({z}), θ-sgpCl({x}) ⊂ θ-sgpCl({z}) and {y} ∩ θ-sgpCl({x}) = Ø. Therefore it follows that θ-sgpCl({x}) ≠ θ-sgpCl({y}).

(ii) → (i): Suppose that θ-sgpCl({x}) ≠ θ-sgpCl({y}). There exists a point z in X such that z ∈ θ-sgpCl({x}) and z $\notin$ θ-sgpCl({y}). Then there exists a θ-sgp-open set containing z and therefore x but not y, namely, y $\notin$ θ-sgp-ker({x}). Hence θ-sgp-ker({x}) ≠ θ-sgp-ker({y}).

***Theorem 5.7*:** A topological space X is θ-sgp-Ro space if and only if for any x, y in X, θ-sgpCl({x}) ≠ θ-sgpCl({y}) implies θ-sgpCl({x}) ∩ θ-sgpCl({y}) = Ø.
***Proof*:** Suppose X is θ-sgp-Ro space and x, y ∈ X such that θ-sgpCl({x}) ≠ θ-sgpCl ({y}). Then there exists a point z ∈ θ-sgp-ker({x}) such that z $\notin$ θ-sgpCl({y}) (or z ∈ θ-sgp-ker({y}) such that z $\notin$ θ-sgpCl({x}). There exist a θ-sgp-open set V such that y $\notin$ V and z ∈ V ; hence x ∈ V. Therefore, we have x $\notin$ θ-sgpCl({y}). Thus x ∈ X- θ-sgpCl({y}) a θ-sgp-open set, which implies θ-sgpCl({x}) ⊆ θ-sgpCl({y}) and θ-sgpCl({x}) ∩ θ-sgpCl({y}) = Ø.

Conversely, let V be a θ-sgp-open set in X and let x ∈ V. Now we have to prove that θ-sgpCl ({x}) ⊂ V. Let y $\notin$ V i.e. y ∈ X – V. Then x ≠ y and x $\notin$ θ-sgpCl({y}). This implies, θ-sgpCl({x}) ≠ θ-sgpCl({y}). By assumption, θ-sgpCl({x}) ∩ θ-sgpCl({y}) = Ø. Hence y $\notin$ θ-sgpCl({x}) and therefore θ-sgpCl({x}) ⊆ V.

***Theorem 5.8*:** A topological space X is θ-sgp-Ro space if and only if for any x, y in X θ-sgp-ker({x}) ≠ θ-sgp-ker({y}) implies θ-sgp-ker({x}) ∩ θ-sgp-ker({y}) = Ø.
***Proof* :** Suppose X is θ-sgp-Ro space. Thus by Lemma 5.6 for any points x, y ∈ X if θ-sgp-ker({x}) ≠ θ-sgp-ker({y}) then θ-sgpCl({x}) ≠ θ-sgpCl({y}).
Now we prove that θ-sgp-ker({x}) ∩ θ-sgp-ker({y}) = Ø.
Suppose that z ∈ θ-sgp-ker({x}) ∩ θ-sgp-ker({y}). By Lemma 5.5 and z ∈ θ-sgp-ker({x}) implies x ∈ θ-sgp-ker({z}). Since x ∈ θ-sgpCl({x}), by Theorem 5.7, θ-sgpCl({x}) = θ-sgpCl({z}). Similarly, we have θ-sgpCl({y}) = θ-sgpCl({x}) a contradiction. Hence θ-sgp-ker({x}) ∩ θ-sgp-ker({y}) = Ø.

Conversely, let X be a topological space such that for any points x and y in X, θ-sgp-ker({x}) ≠ θ-sgp-ker({y}) implies θ-sgp-ker({x}) ∩ θ-sgp-ker({y}) = Ø. If θ-sgpCl({x}) ≠ θ-sgpCl({y}), then by Lemma 5.6, θ-sgp-ker({x}) ≠ θ-sgp-ker({y}). Hence θ-sgp-ker({x}) ∩ θ-sgp-ker({y}) = Ø implies θ-sgpCl({x}) ∩ θ-sgpCl({y}) = Ø. Since z ∈ θ-sgpCl({x}) implies that x ∈ θ-sgp-ker({z}). Therefore θ-sgp-ker({x}) = θ-sgp-ker({z}). Then z ∈ θ-sgpCl({x}) ∩ θ-sgpCl({y}) implies that θ-sgp-ker({x}) = θ-sgp-ker({z}) = θ-sgp-ker({y}), a contradiction. Hence θ-sgpCl({x}) ∩ θ-sgpCl({y}) = Ø. Therefore by Theorem 5.7, X is a θ-sgp-Ro space.

***Theorem 5.9*:** For a topological space X, the following properties are equivalent:
(i). X is a θ-sgp-Ro space.
(ii). For any A ≠ Ø and G is θ-sgp-open in X such that A ∩ G ≠ Ø, there exists θ-sgp-closed F in X such that A ∩ F ≠ Ø and F ⊂ G.
(iii). Any θ-sgp-open set G in X, G = ∪ {F : F ⊆ G, F is θ-sgp-closed in X}.

(iv). θ-sgp-closed F in X, F = ∩ {F : F ⊆ G, F is θ-sgp-open set in X}.
(v). For any x ∈ X, θ-sgpCl({x}) ⊆ θ-sgp-ker({x}).

Proof **:** (i) → (ii): Let A be a non-empty set of X and G be a θ-sgp-open set G in X , such that A ∩ G ≠ Ø. There exists x ∈ A ∩ G. Since x ∈ G where G is θ-sgp-open set in X, θ-sgpCl({x}) ⊆ G. Set F = θ-sgpCl({x}), then F is θ-sgp-closed ⊆ G and A ∩ F ≠ Ø.
(ii) → (iii): Let G be a θ-sgp-open set G in X, then G ⊇ ∪{ F : F ⊆ G , F is θ-sgp-closed in X }. Let x be any point of G then there exists a θ-sgp-closed F in X such that x ∈ F and F ⊆ G. Therefore we have x ∈ F ⊆ ∪{F: F ⊆ G, F is θ-sgp-closed in X} and hence G = ∪ {F: F ⊆ G, F is θ-sgp-closed in X}.
(iii) → (iv): Obvious.
(iv) → (v): Let x be any point of x and y ∉ θ-sgp-ker({x}). Then there exists θ-sgp-open set U such that x ∈ U and y ∉ U; hence θ-sgpCl({y}) ∩ U = Ø . By (iv), (∩ {G: θ-sgpCl({y})}) ⊆ G, G is θ-sgp-open set in X}) ∩ U = Ø. There exists θ-sgp-open set G such that x ∉ G and θ-sgpCl({y}) ⊆ G. Therefore θ-sgpCl({x}) ∩ G = Ø and y ∉ θ-sgpCl({x}). Consequently, we obtain θ-sgpCl({x}) ⊂ θ-sgp-ker({x}).
(v) → (i): Let G be a θ-sgp-open set G in X and x ∈ G. Suppose y ∈ θ-sgp-ker({x}), then x ∈ θ-sgpCl({y}) and y ∈ G . This implies θ-sgpCl({y}) ⊂ θ-sgp-ker({x}). Therefore X is a θ-sgp-Ro space.

Corollary 5.10**:** For a topological space X the following properties are equivalent
(i) X is a θ-sgp-Ro space.
(ii) θ-sgpCl({x}) = θ-sgp-ker({x}) for all x ∈ X.

Proof **:** (i) → (ii): Suppose X is a θ-sgp-Ro space. By the Theorem 5.9, θ-sgpCl({x}) ⊆ θ-sgp-ker({x}) for each x ∈ X. Let y ∈ θ-sgp-ker({x}), then x ∈ θ-sgpCl({y}) and θ-sgpCl({x}) = θ-sgpCl({y}). Therefore y ∈ θ-sgpCl({x}) and hence θ-sgp-ker({x}) ⊆ θ-sgpCl({x}). This shows that θ-sgpCl({x}) = θ-sgp-ker({x}).
(ii) → (i): This is obvious by Theorem 5.9.

Theorem 5.11**:** For a topological space X the following properties are equivalent:
(i) X is a θ-sgp-Ro space.
(ii) x ∈ θ-sgpCl({x}) if and only if y ∈ θ-sgpCl({x}) for any points x and y in X.

Proof **:** (i) → (ii): Assume that X is a θ-sgp-Ro space. Let x ∈ θ-sgpCl({y}) and U be any θ-sgp-open set such that y ∈ U. Now by hypothesis x ∈ U. Therefore, every θ-sgp-open set containing y contains x. Hence y ∈ θ-sgpCl({x}).
(ii) → (i): Let V be a θ-sgp-open set and x ∈ V. If y ∉ V then x ∉ θ-sgpCl({x}) and hence y ∉ θ-sgpCl({x}). This implies that θ-sgpCl({x}) ⊆ V. Hence X is a θ-sgp-Ro space.

Theorem 5.12**:** For a topological space X the following properties are equivalent:
(i) X is a θ-sgp-Ro space.
(ii) If A is a θ-sgp-closed, then F = θ-sgp-ker(A).
(iii) If A is a θ-sgp-closed and x ∈ A, then θ-sgp-ker({x}) ⊆ A.
(iv) If x ∈ X, then θ-sgp-ker({x}) ⊆ θ-sgpCl({x}).

Proof **:** (i) → (ii) : Let A be θ-sgp-closed and x ∉ A. Thus X - A is a θ-sgp-open and x ∈ X –A. Since X is a θ-sgp-Ro space, θ-sgpCl({x}) ⊆ X – A. Thus θ-sgpCl({x}) ∩ A = Ø and by the Lemma 5.3, x ∉ θ-sgp-ker(A). Therefore θ-sgp-ker(A) = A.
(ii) → (iii): In general U ⊆ V implies θ-sgp-ker(U) ⊆ θ-sgp-ker(V). Therefore θ-sgp-ker({x}) ⊆ θ-sgp-ker(A) = A by (ii).
(iii) → (iv): Since x ∈ θ-sgpCl({x}) and θ-sgpCl({x}) is θ-sgp-closed by (iii) θ-sgp-ker({x}) ⊆ θ-sgpCl({x}) .
(iv) → (i): Let x ∈ θ-sgpCl({x}) then by the Lemma 5.5 y ∈ θ-sgp-ker({x}). Since x ∈ θ-sgpCl({x}) and θ-sgpCl({x}) is θ-sgp-closed, by (iv) we obtain y ∈ θ-sgp-ker({x}) ⊆ θ-sgpCl({x}). Therefore x ∈ θ-sgpCl({y}) implies y ∈ θ-sgpCl({x}). The converse is obvious and X is a θ-sgp-Ro space.

***Definition 5.13*:** A topological space X is said to be θ-semigeneralized pre-R_1 (briefly θ-sgp-R_1) if for x, y in X with θ-sgpCl({x}) ≠ θ-sgpCl({y}), there exist disjoint preopen sets U and V such that θ-sgpCl({x}) is a subset of U and θ-sgpCl({y}) is a subset of V.

***Proposition 5.14*:** If (X, τ) is θ-sgp-R_1, then (X, τ) is θ-sgp-R_0.
***Proof* :** Let U be preopen set and x ∈ U. If y ∉ U, then since x ∉ θ-sgpCl({y}), θ-sgpCl({x}) ≠ θ-sgpCl({y}). Hence, there exists a preopen set V_y such that θ-sgpCl({y}) ⊂ V_y and x ∉ V_y which implies y ∉ θ-sgpCl({x}). Thus θ-sgpCl({x}) ⊂ U. Therefore (X, τ) is θ-sgp-R_0.

***Theorem 5.15*:** A topological space X is said to be θ-sgp-R_1 if and only if for x, y ∈ X, θ-sgp-Ker({x}) ≠ θ-sgp-Ker({y}), there exist disjoint preopen sets U and V such that θ-sgp-Ker({x}) ⊂ U and θ-sgp-Ker({y}) ⊂ V.
***Proof* :** It follows from Lemma 5.6.

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