Some Common Fixed Point Theorems in Fuzzy Mappings

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ABSTRACT:

In this paper we established some fixed point and common fixed point theorem for sequence of fuzzy mappings and also taking rational inequalities which generalized the result of Heilpern [2], Lee, Cho, Lee and Kim [16].

KEYWORDS: Fixed point theory, Fuzzy Mappings, Contraction mappings, Upper semi-continuous, common fixed point.

Introduction:

The concept of fuzzy sets was introduced by Zadeh [1] in 1965 . After that a lot of work has been done regarding fuzzy sets and fuzzy mappings. The concept of fuzzy mapping was first introduced by Heilpern[2], he proved fixed point theorem for fuzzy contraction mappings which is a fuzzy analogue of the fixed point theorem for multy valued mappings of Nadler[3], vijayraju and Marudai[4], generalized the Bose and Mukherjee's[5] fixed point theorems for contractive types fuzzy mappings. Marudai and srinmivasan [6] derived the simple proof of Heilpern's [2] theorem and generalization of Nadler's [3] theorem for fuzzy mappings. Bose and sahani [7], Butnariu [8-10], Chang and Huang , Non-jing [11], Chang [12], chitra [13], som and Mukharjee [14] , studied fixed point theorems for fuzzy mappings.

Bose and Sahani [7], extends Heilpern's result for a pair of generalized fuzzy contraction mappings. Lee and cho [15], described a fixed point theorem for contractive type fuzzy mappings which is generalization of Heilpern's [2], result. Lee, cho, lee and kim [16] obtained a common fixed point theorem for a sequence of fuzzy mappings satisfying certain conditions, which is generalization of the second theorem of Bose and Sahini [7].

Recently Rajendra and Bala Subramanian [21], worked on fuzzy contraction mappings. More recently Vijayraju and Mohanraj [17], obtained some fixed point theorems for contractive type fuzzy mappings which are generalization of Beg and Azam [18], fuzzy extension of kirk and Downning [19], and which obtained simple proof of park and jeong [20]. In this paper we are proving some fixed point theorems in fuzzy mapping containing the rational expressions. Perhaps this is first time when we are including such types of rational expressions. These results are extended form of Heilpern [2], Lee, Cho, Lee and Kim [16]. Common fixed point theorems in fuzzy metric spaces for weakly compatible mappings along with property (E.A.) satisfying implicit relation by Asha Rani[22].

Preliminaries:

Fussy mappings: Let X be any metric linear spaces and d be any metric in X.A fuzzy set in X is a function with domain X and values in [0,1]. If A is a fuzzy set and x∈X, the function value A(x) is called the grade of membership of x in A. The collection of all fuzzy sets in X is denoted by F(x).

Let $A \in F(x)$ and $\propto \in$ [0,1] .The $\propto -$level set of A, denoted by $A_\propto$ is denoted by $A_\propto$={x : A(x) ≥∝ } if $\propto \in$ [0,1].

$A_0 = \{ \overline{x: A(x)} > 0\}$, whenever B is closure of B.

Now we distinguish from the collection F(x) a sub collection of approximate quantities, denoted W(x).

Definition 2.1:

A fuzzy subset A of X is an approximate quantity iff it's $\propto$-level set is a compact subset (non fuzzy) of X for each $\propto \in$ [0,1], and $\sup_{x\in X} A(x) = 1$

When A∈ $W(x)$ and A(x_0) = 1 for some $x_0 \in W(x)$, we will identify A with an approximation of x_0. Then we shall define a distance between two approximate quantities.

Definition 2.2:

Let A,B∈W(x), $\propto \in$[0,1], define $P_\propto(A,B) = inf_{x\in A_\propto, y\in B_\propto} d(x,y)$,

$D_\propto(A,B) = dist(A_\propto, B_\propto), d(A,B) = sup_\propto D_\propto(A,B)$.

Whenever dist. Is Hausdorf distance. The function $P_\propto$ is called $\propto$-spaces, and a distance between A and B . it is easy to see that $P_\propto$is non decreasing function of $\propto$. We shall also define an order 0 the family W(x), which characterizes accuracy of a given quantity.

Definition 2.3:

Let A,B ∈W(x). an approximate quantity A is more accurate then B, denoted by A⊂ *B* iff A(x) ≤ B(x), for each x ∈ X.

Now we introduce a notation of fuzzy mapping , i.e a mapping with value in the family of approximate quantities.

Definition 2.4:

Let X be an arbitrary set and Y be any metric linear space. F is called a fuzzy mapping iff F is mapping from the set X into W(Y), i.e .F(x) ∈ W(Y) for each x ∈

X.

A fuzzy mapping Fis a fuzzy subset on $X \times Y$ with membership function F(x,y). the function value F(x, y) is grade of membership of y in F(x).

Let A $\in$ F(X), B $\in$ F(Y). the fuzzy set F(A) in F(Y) is defined by F(A)(Y)=$sup_{x \in X}(F(X,Y), A(Y))$, $y \in Y$

And the fuzzy set $F^{-1}(B)$ in F(X), is defined as $F^{-1}(B)(x) = sup_{y \in Y}(F(X,Y), B(Y))$ where x $\in$ X.

First of all we shall give here the basic properties of $\propto$-space and $\propto$-distance between some approximate quantities.

Lemma 3.1:

Let x $\in$ X, A $\in$ W(X) and {x} be a fuzzy set with membership function equal a characteristic function of set {x} , if {x} is subset of A then $P_\propto(X, A) = 0$ for each $\propto \in$ [0,1].

Lemma 3.2:

$P_\propto(x, A) \leq d(x, y) + P_\propto(y, A)$ for any x,y $\in$ X

Lemma 3.3:

if $\{x_0\}$ is subset of A then $P_\propto(x_0, B) \leq D_\propto(A, B)$ for each B $\in$ W(X).

Lemma 3.4[15]:

Let (X,d) be a complete metric linear space, T be a fuzzzy mapping from X into W(X) and $x_0 \in X$, then there exist $x_1 \in X$ such that $\{x_1\} \subset T\{x_0\}$

Lemma 3.5[16]:

Let A,B $\in$ W(X) then for each {x} $\subset$ A , there exists {y} $\subset$ B such that

$$D(\{x\}, \{y\}) \leq D(A, B)$$

Theorem 1:

Let X be a complete metric linear space and T be a fuzzy mapping from X to W(X) ,p,q , r $\in$(0,1) such that

D(T(x), T(y)) $\leq$ p max {d(x,y)+d(x, T(x))+ d(x, T(y))} +q d(x,y)
$+r \dfrac{d(x,y)+d(x,T(x))+d(x,T(y))}{1+d(x,y)d(x,T(y))d(x,T(y))}$

where $\forall$ x $\neq$ y then there exists $x^* \in X$ such that $\{x^*\} \subset T(x^*)$

proof:

let $x_0 \in X$ and $\{x_1\} \subset T(x_0)$, then there exist $\{x_2\} \subset T(x_1)$ and

$d(x_{2,}x_1) \leq D_1(T(x_1), T(x_0))$. If $\{x_3\} \subset T(x_2)$, then there exists $\{x_4\} \subset T(x_3)$ such that $d(x_{3,}x_4) \leq D_1(T(x_2), T(x_3))$

on continuing this in this way we produce a sequence (x_n) in X such that $\{x_n\} \subset T(x_{n-1})$ and $d(x_{n,}x_{n+1}) \leq D_1(T(x_{n-1}), T(x_n))$, for each n$\in N$

Now we shall show that (x_n) is a cauchy sequence.

$d(x_{k+1,}x_k) \leq D_1\big(T(x_k), T(x_{k-1})\big)$

$\qquad \leq D\big(T(x_k), T(x_{k-1})\big)$

$\leq$p max $\{d(x_k, x_{k-1}) + d(x_k, T(x_k) + d(x_k, T(x_{k-1}))\}$ +q $d(x_k, x_{k-1})$ +r

$\qquad \dfrac{d(x_k, x_{k-1}) + d\big(x_k, T(x_k)\big) + d\ (x_k, T(x_{k-1}))}{1 + d(x_k, x_{k-1})d\ (x_k, T(x_k))d\ (x_k, T(x_{k-1}))}$where p, q, r $\in (0,1)$

$d(x_{k+m,}x_k) \leq \sum_{j=k}^{k+m+1} d\big(x_{j+1}, x_j\big) \leq \sum_{j=k}^{k+m-1} s^j d(x_1, x_0)$, where s=(p+q+r)

$$\leq \frac{s^k}{1-k} d(x_1, x_0), s^k \text{ converges to 0 as k} \to \infty$$

Then since X is a complete space and (x_n) is a cauchy sequence, there exists a limit of sequence (x_n), such that we assume $\lim_{n \to \infty} x_n = x^*$

$P_0\big(x^*, T(x^*)\big) \leq d(x^*, x_n) + P_0(x_n, T(x^*))$ by lemma 3.2

$\qquad \leq d(x^*, x_n) + D_0(T(x_{n-1}), T(x^*))$ by lemma 3.3

$\leq d(x^*, x_n) + sd(x_{n-1}, x^*)$.

$d(x^*, x_n)$ coverges to o as n$\to \infty$. hence from lemma 3.1, we conclude that $\{x^*\} \subset T\{x^*\}$

Remark:

If we put T=F and p=0, r=0, we get the result of Heilpern, S[54]

Now we are giving a new result which also includes rational inequalities and which is extended from of Lee, Cho[15], Lee, Cho, Lee and Kim[16] for three mappings.

Theorem 2:

Let g be a non expansive mapping from complete metric spaces X, into itself. If $\{T_i\}_{i=1}^{\infty}$ is a sequence of fuzzy mappings from X into W(X) satisfying the following conditions:

For three fuzzy mapping T_i, T_j, T_k and for any $x \in X$, $\{u_x\} \subset T_i(x)$, there exist $\{v_y\} \subset T_j(y), \{w_z\} \subset T_k(z)$ for all y,z $\in X$ such that

$$D\big[(u_x),(v_y)\big] = \propto_1 d\big(g(x),g(u_x)\big) + \propto_2 d\big(g(x),g(v_y)\big) + \propto_3 d\big(g(x),g(w_z)\big)$$

$$+ \propto_4 d(g(y),g(u_x)) + \propto_5 d(g(y),g(v_y)) + \propto_6 d(g(y),g(w_z))$$
$$+ \propto_7 d(g(z),g(u_x)) + \propto_8 d(g(z),g(v_y)) + \propto_9 d(g(z),g(w_z)) +$$
$$\propto_{10} \frac{d(g(x),g(u_x)) + d(g(y),g(u_x))}{1 + d(g(x),g(u_x))d(g(y),g(u_x))}$$
$$+ \propto_{11} \frac{d\big(g(x),g(v_y)\big) + d(g(x),g(w_z)) + d\big(g(z),g(v_y)\big)}{1 + d\big(g(x),g(v_y)\big)d(g(x),g(w_z))d\big(g(z),g(v_y)\big)} + \propto_{12} d\big(g(x),g(y)\big)$$

For all $\propto_1, \propto_2, \propto_3, \propto_4, \propto_5, \propto_6, \propto_7, \propto_8, \propto_9, \propto_{10}, \propto_{11}$ are non negative real and $[\propto_2 + \propto_3 + \propto_5 + \propto_6 + \propto_7 + \propto_9 + 2\propto_{11}] < 1$ then there exists r $\in X$ such that $\{r\} \subset \cap_{i=1}^{\infty} T_i(r)$

Proof :

Let $x_0 \in X$, then we can choose $\{x_1\} \in X$ such that $\{x_1\} \subset T_1(x_0)$, so by assumption there exist $x_2, x_3 \in X$, such that $\{x_2\} \subset T_2(x_1), \{x_3\} \subset T_3(x_2)$, and
$$D\lfloor(x_1),(x_2)\rfloor = \propto_1 d(g(x_0),g(x_1)) + \propto_2 d(g(x_0),g(x_2)) + \propto_3 d(g(x_0),g(x_3))$$

$$+ \propto_4 d(g(x_1),g(x_1)) + \propto_5 d(g(x_1),g(x_2)) + \propto_6 d(g(x_1),g(x_3))$$
$$+ \propto_7 d(g(x_2),g(x_1)) + \propto_8 d(g(x_2),g(x_2)) + \propto_9 d(g(x_2),g(x_3)) +$$
$$\propto_{10} \frac{d(g(x_0),g(x_1)) + d(g(x_1),g(x_1))}{1 + d(g(x_0),g(x_1))d(g(x_1),g(x_1))}$$
$$+ \propto_{11} \frac{d(g(x_0),g(x_2)) + d(g(x_0),g(x_3)) + d(g(x_2),g(x_2))}{1 + d(g(x_0),g(x_2))d(g(x_0),g(x_3))d(g(x_2),g(x_2))} + \propto_{12} d\big(g(x_0),g(x_1)\big)$$

$$D(\{x_1\},\{x_2\}) \leq \propto_1 d(x_0,x_1) + \propto_2 d(x_0,x_2) + \propto_3 d(x_0,x_3)$$

$$+ \propto_4 d(x_1,x_1) + \propto_5 d(x_1,x_2) + \propto_6 d(x_1,x_3)$$
$$+ \propto_7 d(x_2,x_1) + \propto_8 d(x_2,x_2) + \propto_9 d(x_2,x_3) + \propto_{10} \frac{d[(x_0),(x_1)] + d[(x_1),(x_1)]}{1 + d[(x_0),(x_1)]d[(x_1),(x_1)]}$$
$$+ \propto_{11} \frac{d[(x_0),(x_2)] + d[(x_0),(x_3)] + d[(x_2),(x_2)]}{1 + d[(x_0),(x_2)]d[(x_0),(x_3)]d[(x_2),(x_2)]} + \propto_{12} d\big((x_0),(x_1)\big)$$

We can find $x_4 \in X$, such that $\{x_4\} \subset T_4(x_3)$, and

$$D(\{x_2\},\{x_3\}) \leq \propto_1 d(x_1,x_2) + \propto_2 d(x_1,x_3) + \propto_3 d(x_1,x_4)$$

$$+\alpha_4\, d(x_2,x_2)+\alpha_5\, d(x_2,x_3)+\alpha_6\, d(x_2,x_4)$$

$$+\alpha_7\, d(x_3,x_2)+\alpha_8\, d(x_3,x_3)+\alpha_9\, d(x_3,x_4)+\alpha_{10}\, \frac{d[(x_1),(x_2)]+d[(x_2),(x_2)]}{1+d[(x_1),(x_2)]d[(x_2),(x_2)]}$$

$$+\alpha_{11}\, \frac{d[(x_1),(x_3)]+d[(x_1),(x_4)]+d[(x_3),(x_3)]}{1+d[(x_1),(x_3)]d[(x_1),(x_4)]d[(x_3),(x_3)]} +\alpha_{12}\, d\big((x_1),(x_2)\big)$$

On continuing this process we can obtain a sequence $\{x_n\}$ in X such that $\{x_{n+1}\} \subset T_{n+1}(x_n)$ and

$$D[\{x_n\},\{x_{n-1}\}] \leq \alpha_1\, d(x_{n-1},x_n)+\alpha_2\, d(x_{n-1},x_{n+1})+\alpha_3\, d(x_{n-1},x_{n+2})$$

$$+\alpha_4\, d(x_n,x_n)+\alpha_5\, d(x_n,x_{n+1})+\alpha_6\, d(x_n,x_{n+2})$$

$$+\alpha_7\, d(x_{n+1},x_n)+\alpha_8\, d(x_{n+1},x_{n+1})+\alpha_9\, d(x_{n+1},x_{n+2})+$$

$$\alpha_{10}\, \frac{d[(x_{n-1}),(x_n)]+d[(x_n),(x_n)]}{1+d[(x_{n-1}),(x_n)]d[(x_n),(x_n)]}$$

$$+\alpha_{11}\, \frac{d[(x_{n-1}),(x_{n+1})]+d[(x_{n-1},x_{n+2})]+d[(x_{n+1},x_{n+1})]}{1+d[(x_{n-1}),(x_{n+1})]d[(x_{n-1},x_{n+2})]d[(x_{n+1},x_{n+1})]} +\alpha_{12}\, d\big((x_{n-1}),(x_n)\big)$$

Since $D[\{x_n\},\{x_{n+1}\}] = d(x_n,x_{n+1})$, and using triangular inequality in metric spaces

$$d(x_n,x_{n+1}) \leq \alpha_1\, d(x_{n-1},x_n)+\alpha_2\, [d(x_{n-1},x_n) + d(x_n,x_{n+1})]+\alpha_3\, [d(x_{n-1},x_n)$$

$$+d(x_n,x_{n+1}) + d(x_{n+1},x_{n+2})]$$

$$+\alpha_4\, d(x_n,x_n)+\alpha_5\, d(x_{n+1},x_n)+\alpha_6\, [d(x_{n+1},x_n) + d(x_{n+1},x_{n+2})]$$

$$+\alpha_7\, d(x_{n+1},x_n)+\alpha_8\, d(x_{n+1},x_{n+1})+\alpha_9\, d(x_{n+1},x_{n+2})+\alpha_{10}\, d(x_{n-1},x_n)$$

$$+\alpha_{11}\, [d(x_{n-1},x_n) + d(x_{n+1},x_n) + d(x_{n-1},x_n) + d(x_{n+1},x_n) +$$

$$d(x_{n+1},x_{n+2})] +\alpha_{12}\, d\big((x_{n-1}),(x_n)\big)$$

$$d(x_n,x_{n+1}) \leq \frac{\alpha_1+\alpha_2+\alpha_3+\alpha_9+ 2\,\alpha_{11}+\alpha_{12}}{1 - [\alpha_2+\alpha_3+\alpha_5+\alpha_6+\alpha_7+ 2\,\alpha_{11}]}d(x_{n-1},x_n)$$

$$+\frac{\alpha_3+\alpha_6+\alpha_9 +\alpha_{11}}{1 - [\alpha_2+\alpha_3+\alpha_5+\alpha_6+\alpha_7+ 2\,\alpha_{11}]}d(x_{n+1},x_{n+2})$$

On continuing this process we get,

$$d(x_n,x_{n+1}) \leq p^n d(x_0,x_1) + q^n d(x_1,x_2)$$

Hence $\{x_n\}_{n=1}^{\infty} is$ cauchy sequence in X.

Where $p = \left[\frac{\alpha_1+\alpha_2+\alpha_3+\alpha_9+2\alpha_{11}+\alpha_{12}}{1-[\alpha_2+\alpha_3+\alpha_5+\alpha_6+\alpha_7+2\alpha_{11}]}\right]$ and $q = \left[\frac{\alpha_3+\alpha_6+\alpha_9+\alpha_{11}}{1-[\alpha_2+\alpha_3+\alpha_5+\alpha_6+\alpha_7+2\alpha_{11}]}\right]$

Since X is complete, there exists r $\in$X, such that $\lim_{n\to\infty} x_n = r$,

Let T_m be an arbitrary member of $\{T_i\}_{i=1}^{\infty}$. since$\{x_n\} \subset T_n(x_{n-1})$ for all n there exists $v_n \in X$, such that $\{v_n\} \subset T_m(r)$, for each value of n.

Clearly $D(\{r\}, \{x_n\}) \to 0$ as $n \to \infty$

Since, $T_m(r) \in W(X), T_m(r)$ is upper semi-continuos and so,

$$\lim_{n \to \infty} \sup[T_m(r)](v_n) \leq [T_m(r)](r)$$

Since $\{v_n\} \subset T_m(r)$ for all n, so

$$[T_m(r)](r) = 1$$

Hence $\{r\} \subset T_m(r)$ and $T_m(r)$ is arbitrary so $\{r\} \subset \cap_{i=1}^{\infty} T_i(r)$

Remarks:

If we write $\propto_1 as a_1, \propto_5 as a_2, \propto_4 as a_3, \propto_2 as a_4, \propto_{12} as a_5$

And $\propto_3 = \propto_5 = \propto_6 = \propto_7 = \propto_8 = \propto_9 = \propto_{10} = \propto_{11} = 0$. then we get the result of Lee,cho,Lee and kim [16]

Corollary 2.1:

Let (X,d) be a complete linear metric spaces . if $\{T_i\}_{i=1}^{\infty}$ is a sequence of fuzzy mappings from X into W(x) satisfying the following conditions:

For three fuzzy mapping T_i, T_j, T_k and for any $x \in X, \{u_x\} \subset T_i(x)$, there exist

$\{v_y\} \subset T_j(y), \{w_z\} \subset T_k(z)$, for all y,z $\in$ X such that,

$D[(u_x), (v_y)] \leq \propto_1 d((x), (u_x)) + \propto_2 d((x), (v_y)) + \propto_3 d((x), (w_z))$

$$+ \propto_4 d((y), (u_x)) + \propto_5 d((y), (v_y)) + \propto_6 d((y), (w_z))$$
$$+ \propto_7 d((z), (u_x)) + \propto_8 d((z), (v_y)) +$$
$$\propto_9 d((z), (w_z)) + \propto_{10} \frac{d((x),(u_x)) + d((y),(u_x))}{1 + d((x),(u_x)) d((y),(u_x))}$$
$$+ \propto_{11} \frac{d(g(x),g(v_y)) + d(g(x),g(w_z)) + d(g(z),g(v_y))}{1 + d(g(x),g(v_y)) d(g(x),g(w_z)) d(g(z),g(v_y))} + \propto_{12} d((x), (y))$$

For all $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}$ are non negative reals, and

$[\propto_2 + \propto_3 + \propto_5 + \propto_6 + \propto_7 + \propto_9 + 2 \propto_{11}] < 1$ Then there exists r$\in$ X, such that $\{r\} \subset \cap_{i=1}^{\infty} T_i(r)$.

Proof:

On putting g(x)=x in result 2, we can get this corollary.

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