Inverse Edge Domination in Fuzzy Graphs

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Abstract. **In this paper we discuss the concepts of inverse edge domination and total edge domination in fuzzy graph. We determine the inverse edge domination number $\gamma_I'(G)$ and the total edge domination number $\gamma'_t(G)$ for several classes of fuzzy graph and obtain bounds for the same. We also obtain Nordhaus – Gaddum type resuts for these parameters.**

Keywords: **Fuzzy graph; Edge domination; Inverse edge domination; Total edge Domination; Total inverse edge domination.**

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1. INTRODUCTION

The study of domination set in graphs was begun by Ore and Berge. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. A.Somasundram and S.Somasundram discussed domination in Fuzzy graphs. V.R. Kulli and D.K. Patwari discussed the total edge domination number of graph. They defined domination using effective edges in fuzzy graph. In this paper we discuss the inverse edge domination number of fuzzy graph using fuzzy edge cardinality and establish the relationship with other parameter which is also investigated.

2. PRELIMINARIES

Definition 2.1

A fuzzy graph $G = (\sigma,\mu)$ is a set with two functions, $\sigma : V \rightarrow [0,1]$ and $\mu : E \rightarrow [0,1]$ such that $\mu(x\,y) \leq \sigma(x) \wedge \sigma(y)\ \forall\ x,y \in V$.

Definition 2.2

Let $\mu = (\sigma,\mu)$ be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(x) = \sigma(x)$ for all $x \in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(xy) = \mu(xy)$ for all $x,y \in V_1$. Then (σ_1, μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $<V_1>$.

Definition 2.3

The order p and size q of a fuzzy graph $G = (\sigma,\mu)$ are defined to be $p = \sum_{x\in V} \sigma(x)$ and $q = \sum_{xy\in E} \mu(xy)$.

Definition 2.4

Let $\sigma : V \rightarrow [0,1]$ be a fuzzy subset of V then the complete fuzzy graph on σ is defined on $G = (\sigma,\mu)$ where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$ and is denoted by K_σ.

Definition 2.5

The complement of a fuzzy graph G denoted by $\overline{G}$ is defined to be
$\overline{G} = (\sigma, \overline{\mu})$ where $\overline{\mu}(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$.

Definition 2.6

Let $G = (\sigma,\mu)$ be a fuzzy graph on V and $S \subseteq V$. Then the fuzzy cardinality of S is defined to be $\sum_{v \in S} \sigma(v)$.

Definition 2.7

Let $G = (\sigma,\mu)$ be a fuzzy graph on E and $D \subseteq E$ then the fuzzy edge cardinality of D is defined to be $\sum_{e \in D} \mu(e)$.

Definition 2.8

An edge e = xy of a fuzzy graph is called an effective edge if $\mu(xy) = \sigma(x) \wedge \sigma(y)$. $N(x) = \{y \in V \mid \mu(xy) = \sigma(x) \wedge \sigma(y)\}$ is called the neighbourhood of x and $N[x] = N(x) \cup \{x\}$ is the closed neighbourhood of x.

Definition 2.9

The effective degree of a vertex u is defined to be sum of the weights of the effective edges incident of 'u' and is denoted by dE(u). $\sum_{v \in N(v)} \sigma(v)$ is called the neighbourhood degree of u and is denoted by dN(u).

Definition 2.10

The minimum effective degree $\delta_E(G) = \min\{dE(u) \mid u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u) \mid u \in V(G)\}$.

Definition 2.11

The effective edge degree of an edge e = uv, is defined to be $d_E(e) = dE(u)+dE(v)$.

The minimum edge effective degree and the maximum edge effective degree are $\delta'_E(G) = \min\{d_E(e) / e \in X\}$ and $\Delta'_E(G) = \max\{d_E(e) / e \in X\}$ respectively.

N(e) is the set of all effective edges incident with the vertices of e.

In a similar way minimum neighbourhood degree and the maximum neighbourhood degree denoted by δ'_N and Δ'_N respectively can also be defined.

Definition 2.12

A fuzzy graph $G = (\sigma,\mu)$ is said to be bipartite if the vertex set V can be partitioned into two non-empty sets V_1 and V_2 such that $\mu(v_1v_2) = 0$ if $v_1, v_2 \in V_1$ (or) $v_1, v_2 \in V_2$.

Further if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u\in V_1$ and $v\in V_2$ than G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are, respectively, the restrictions of σ to V_1 and V_2.

3. INVERSE EDGE DOMINATION IN FUZZY GRAPHS

Definition 3.1

Let $G = (\sigma,\mu)$ be a fuzzy graph on (V, X). A subset S of X is said to be an edge domination set in G if for every edge in X-S is adjacent to atleast one effective edge in S.

The minimum fuzzy cardinality of an edge dominating set is G is called the edge domination number of G and it is denoted by $\gamma'(G)$ or γ'.

Definition 3.2

Let $G = (\sigma,\mu)$ be a fuzzy graph on (V, X). A subset D of X is a minimal edge dominating set of a fuzzy graph G.If V-D contains an edge dominating set D' is called an inverse edge dominating set of G with respect to D. The minimum fuzzy edge cardinality taken over all inverse edge dominating set of G is called the inverse edge domination number of the fuzzy graph G and it is denoted by $\gamma_I'(G)$ or γ_I'

Remark 3.3

It is clear that if G has atleast one edge, then $0 \leq \gamma_I'(G) \leq q$. However if a graph G has no effective edges, then $\gamma_I'(G) = 0$

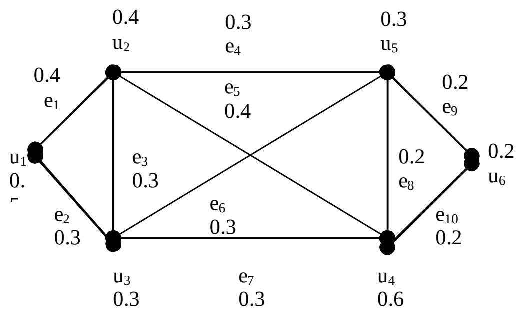
Theorem 3.4

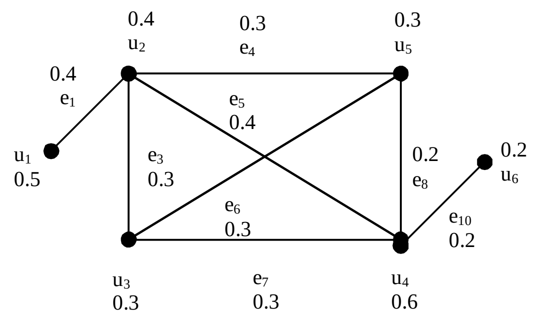
For any fuzzy graph G, $\gamma'(G) \leq \gamma_I'(G)$.

Theorem 3.5

For any fuzzy graph G, $\gamma_I'(G) \leq \gamma_I'(H)$ where H is the spanning sub graph of G.

Example:





$\gamma^{\square} = 0.5 \;\; \gamma_I^{\square}(G) = 0.6$ $\gamma^{\square} = 0.6 \;\; \gamma_I^{\square}(H) = 0.6$

Therefore, $\gamma_I^{\square}(G) \leq \gamma_I^{\square}(H)$

For the inverse edge domination number $\gamma_I^{\square}(G)$, the following theorem gives a Nordhaus – Gaddum type result.

Theorem 3.6

For any fuzzy graph G, $\gamma_I' + \bar{\gamma}_I' \leq 2q$. where γ_I' is the edge domination number of $\bar{G}$ and equality holds if and only if $0 < \mu(xy) < \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Proof.

The inequality is trivial. Further $\gamma_I' = q$ if and only if $\mu(xy) < \sigma(x) \wedge \sigma(y)$ for all $xy \in E$ and $\bar{\gamma}_I' = q$ if and only if $\sigma(x) \wedge \sigma(y) - \mu(xy) < \sigma(x) \wedge \sigma(y)$ for all $xy \in E$ which is equivalent to $\mu(xy) > 0$. Hence , $\gamma_I' + \bar{\gamma}_I' = 2q$ if and only $0 < \mu(xy) < \sigma(x) \wedge \sigma(y)$.

Definition 3.7

An inverse edge dominating set D of a fuzzy graph G is said to be minimal inverse edge dominating set if no proper subset D is an inverse edge dominating set of G.

Theorem 3.8

An inverse edge dominating set D□ is minimal if and only if for each edge $e \in D□$, one of the following two conditions holds.

(a) $N(e) \cap D□ = \varphi$
(b) There exists an edge $f \in X – D□$ such that $N(f) \cap D□ = \{e\}$ and f is an effective edge.

Proof.

Let $D\Box$ be a minimal edge dominating set and $e \in D\Box$. Then $D\Box_e = D\Box - \{e\}$ is not an inverse edge dominating set and hence there exists $f \in X - D\Box_e$ such that f is not dominated by any element of $D\Box_e$.

If f =e we get (a) and if $f \neq e$ we get (b). The converse is obvious.

Definition 3.9

An edge e of a fuzzy graph G is said to be an isolated edge if no effective edges incident with the vertices of e.

Thus an isolated edge does not dominate any other edge in G.

Theorem 3.10

If G is a fuzzy graph without isolated edges then for every minimal inverse edge dominating set D, X-D is also an inverse edge dominating set.

Proof.

Let f be any edge in D. Since G has no isolated edges, there is an edge $c \in N(f)$. It follows from theorem 3.7 that $c \in X - D$. thus every element of S is dominated by some element of X-D.

Corollary 3.11

For any fuzzy graph G without isolated edges $\gamma_I' \leq \frac{q}{2}$.

Proof.

Any graph without isolated edges has two disjoint inverse edge dominating sets and hence the result follows.

Corollary 3.12

Let G be a fuzzy graph such that both G and $\overline{G}$ have no isolated edges. Then $\gamma_I' + \bar{\gamma}_I' \leq q$, where $\bar{\gamma}_I'$ is the edge domination number of $\overline{G}$. Further equality holds if and only if $\gamma_I' = \bar{\gamma}_I' = \frac{q}{2}$.

Theorem 3.13

If G is a fuzzy graph without isolated edges then $\frac{q}{\Delta^{\Box}(G)+1} \geq \gamma_I'(G)$.

Proof.

Let D be an inverse edge dominating set of G

Since, $|D|\,\Delta'(G) \leq \sum_{e\in D} d_E(e) = \sum_{e\in D} |N(e)|$

$$\le |\bigcup_{e \in D} N(e)|$$
$$\le |E - D|$$
$$\le q - |D|$$

$\therefore |D|\Delta'(G) + |D| \le q$

Thus $\gamma_I'(G) \le \frac{q}{\Delta'(G)+1}$.

Theorem 3.14

For any fuzzy graph $\gamma_I^{\square}(G) \ge q - \Delta'(G)$.

Definition 3.15

Let G be a fuzzy graph without isolated edges. A total inverse edge dominating set D of a fuzzy graph G is set of fuzzy edges of X for which each $e \in X$ is incident with edges of D, i.e., N[D] = X.

The minimum fuzzy cardinality of a total edge dominating set is called the total edge domination number of G and is denoted by $\gamma'_t(G)$.

Theorem 3.16

If a fuzzy graph G has no isolated edges then $\gamma_I^{\square}(G) \le \gamma_t'(G)$.

Theorem 3.17

For any fuzzy graph $\frac{q}{\Delta'(G)} \ge \gamma_t'(G)$.

Proof.

Let D be a total edge dominating set with minimum number of fuzzy edge cardinality. Then every edge in D is adjacent to atleast $\Delta'(G)$ fuzzy edge cardinality, therefore,

$|D| \Delta'(G) \le q$. Hence $\frac{q}{\Delta'(G)} \ge \gamma_t'(G)$.

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REFERENCES

1. Harary, E., 1969. *Graph Theory*. Addison Wesley, Reading, MA. McAlester, M.L.N., 1988. Fuzzy intersection graphs. Comp. Math. Appl. 15(10), 871-886.
2. Haynes, T.W., Hedetniemi S.T. and Slater P.J. (1998). *Domination in Graphs: Advanced Topics*, Marcel Dekker Inc. New York, U.S.A.
3. Haynes, T.W., Hedetniemi S.T. and Slater P.J. (1998). *Fundamentals of domination in graphs,* Marcel Dekker Inc. New York, U.S.A.
4. Kulli, V.R. and Janakiram B. (1997). *The split domination number of graph. Graph Theory notes of New York*. New York Academy of Sciences, XXXII, pp. 16-19.
5. Ore, O. (1962). *Theory of Graphs*. American Mathematical Society Colloq. Publi., Providence, RI, 38.
6. Ponnappan.C.Y.,.Basheer Ahamed.S and Surulinathan.P.,*Edge domination in fuzzy graph – new approach,* International journal of IT,Engg&Applied sciences Research.Vol 4,No.1,Jan (2015)
7. Ponnappan.C.Y.,.Basheer Ahamed.S and Surulinathan.P.*The Split,Edge domination in fuzzy graphs*,Int.Jour.of Mathematics Trends and Tech.Vol 18,No 1 Feb(2015)
8. Rosenfeld, A., 1975. Fuzzy graphs. In : Zadeh, L.A., Fu, K.S., Shimura, M. (Eds.), *Fuzzy Sets and Their Applications*. Academic Press, New York.
9. Somasundaram, A., and Somasundaram, S., *Domination in fuzzy graphs-I*, Pattern Recognit. Lett. 19(9) 1998), 787-791.