A SPECIAL DIOPHANTINE QUADRUPLE WITH PROPERTY D (17)

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***Abstract*— In this paper, we exhibit a quadruple $(a, b, c_n, c_n, c_{n+1})$ such that the product of any two members minus the some of the same members and increased by 17**

***Keywords*- Diophantine Quadruples, Integer Sequences.**

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NOTATIONS USED:

$$t_{m,n} = n\left(1+\frac{(n-1)(m-2)}{2}\right)$$

$$ct_{2,n} = n(n+1)+1$$

$$P_n^5 = \frac{n^2(n+1)}{2}$$

$$G_n = 2n-1$$

I. INTRODUCTION

A Set of positive integers ($a_1, a_2, a_3, \ldots\ldots a_m$) is said to have the property D(n), $n \in z - 0$, if $a_i a_j + n$ is a perfect square for all $1 \le i \lhd j \le m$ and such a set is called a Diophantine –m-tuple with property D(n). Many mathematicians considered the problem of the existence of Diophantine quadruples with the property D(n) for any arbitrary integer n[1] and also for any linear polynomial in n. Further, various authors considered the connections of the problems of Diophantus, Davenport and Fibonacci numbers in(2-27).

In this communication, we have present a quadruple (a, b, c_n, c_{n+1}) such that the product of any two members minus the some of the same members and increased by 17

II. METHOD OF ANALYSIS:

Let $a=1$ and $b=15$ be two integers such that ab-(a+b)+17 is a perfect square
Therefore (a,b) is the special dio-2-tuple with property D(17)
If 'c' is any non-zero integer different from (a,b) such that (a,b,c) is a special dio-3-tuple, then c has to satisfy the equation

$$14c+2=p^2 \tag{1}$$

Note that $c=c_0=1, p=p_0=4$ satisfies (1) and c=a
Since 'c' has to be different from 'a and b', we try for the other values of 'c' satisfying (1).
Let $c_1=c_0+h; p_1=h-p_0$ be the second solution of (1)
After simplification,we get $h=14+2p_0$ and thus $c_1=23$
The repetition of the above process leads to the general value of 'c', satisfying (1) to be

$$c_n = 14n^2 + 8n + 1 \tag{2}$$

Let ‘ c_{n+1} ’ be any non-zero integer such that

$$14c_{n+1} + 2 = \beta^2 \tag{3}$$

$$(14n^2 + 8n)c_{n+1} - (14n^2 + 8n - 16) = \gamma^2 \tag{4}$$

Eliminating ‘ c_{n+1} ’ from (3) and (4), we obtain

$$(14n^2 + 8n)(\beta^2) - 14(\gamma^2) = 224n^2 + 128n - 224 \tag{5}$$

Using the linear transformations

$$\beta = X + 14T \tag{6}$$

$$\gamma = X + (14n^2 + 8n)T \tag{7}$$

in (5), it leads to the pell equation

$$X^2 = 14(14n^2 + 8n)T^2 + 16 \tag{8}$$

Let $T_0 = 1$ and $X_0 = 14n + 14$ be the initial solution of (8). Thus (6) yields $\beta_0 = 14n + 18$ And using (3), we get

$$c_{n+1} = 14n^2 + 36n + 23$$

Hence (a, b, c_n, c_{n+1}) represents special dio-quadruple with property D(17)

A few numerical examples are presented below

n	Quadruple (a, b, c_n, c_{n+1}) with property $D(17)$
0	(1,15,1,23)
1	(1,15,23,73)
2	(1,15,7,151)
3	(1,15,151,257)
4	(1,15,257,391)
5	(1,15,391,553)

Some properties are presented below

1. $c_{(pr_{n+\_1})} - c_{pr_n} - 28pr_n = 22$
2. $(c_{\alpha+1} - c_\alpha - 8)^2 - 14^2 = 14^2(8t_{3,\alpha})$
3. $c_{n(2n^2-1)+1} - (b + 7) = 28SO_n$
4. $\frac{1}{2}(c_{n(2n^2-1)+1} - c_{n(2n^2-1)}) - 11 = 14SO_n$
5. $7[c_{\alpha^2+1} - c_{\alpha^2} - (a + b + 6)]$ is a perfect square
6. $42[c_{\alpha^2+1} - c_{\alpha^2} - (a + b + 6)]$ is a Nasty number

III. CONCLUSION

In this paper, we have extended the special dio-2-tuple with property D(17) to special Diophantine quadruple with the same property. To conclude, One may attempt for the extendibility starting with a different-2-tuple.

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REFERENCES:

[1]. A.Baker,H.Davenport,The equations $3x^2 - 2 = y^2$ and $8x^2 - 7 = z^2$, Quart.J. Math.Oxford ser.20(2) (1969),129-137.

[2]. V.E.Hoggatt,G.E.Bergum, A Problem of Fermat and the Fibonacci sequence, Fibonacci Quart, 15(1977),323-330.

[3]. A.F.Horadam, Generalization of a result of Morgudo,Portugaliae Math., 44, (1987),131-136.

[4]. B.W.Jones, Asecond variation on a problem of diophantus and davenport, Fibonacci Quart,16, (1978),155-165.

[5]. C.Long, G.E.Bergum, On a problem of Diophantus in: Application of Fibonacci numbers, (A.N.Philippou,A.F.Horadam and G.E.Bergum,eds),Kluwer,Dordrecht, Vol.2,(1988)183-191.

[6]. J.Morgado, Generalization of a result of Hoggatt and bergum on Fibonacci numbers , Portugaliae Math., 42,(1983-1984),441-445.

[7]. Tao Liqun "On the property P_{-1} ",Electronic Journal of combinatorial number theory 7(2007),#A47.

[8]. Y.Fujita,"The extensibility of Diophantine pairs (k-1,k+1)",J.number theory 128 , (2008), 322-353.

[9]. H.Gupta and K.Singh, "On k-triad Sequences", Internet.J.Math.Sci., 5(1985), 799-804.

[10]. A.F.Beardon and M.N.Deshpande, "Diophantine triples", The Mathematical Gazette, 86 (2002),253-260.

[11]. E.Brown,"Sets in which xy+k is always a square", Math.Comp.45(1985), 613-620.

[12]. M.N.Deshpande,"Families of Diophantine Triplets",Bulletin of the Marathawada Mathematical Society, 4(2003),19-21.

[13]. Y.Bugeaud,A.Dujella and Mignotte,"On the family of Diophantine triples $(k-1, k+1, 16k^3 - 4k)$ ",Glasgow Math.J.49(2007),333-344.

[14]. Gopalan.M.A,G.Srividhya," Diophantine quadruples for pell no's with property D(1)",Antarctica J.Math.,vol-1,no-3,july 2010,357-362.

[15]. Gopalan.M.A,S.Mallika, S.Vidhyalakshmi,"On the extendibility of gaussian Diophantine 2-tuple to Gaussian diophantine quadruple withproperty D(1)",IJISM,Vol-2,issue 3,310-311,2014.

[16]. G.Srividhya,"Diophantine Quadruples for Fibbonacci numbers with property D(1)",Indian Journal of Mathematics and Mathematical Science,Vol.5, No.2,(December 2009),57-59.

[17]. Gopalan.M.A ,V.Pandichelvi, "The Non Extendibility of the Diophantine Triple $(4(2m-1)^2 n^2, 4(2m-1)n+1, 4(2m-1)^4 n^4 - 8(2m-1)^3 n^3)$, Impact.J.sci.Tech, 5(1),25-28, 2011.

[18]. Yasutsugu Fujita,Alain Togbe, "Uniqueness of the extension of the $D(4k^2)$ -triple $(k^2-4, k^2, 4k^2-4)$ " NNTDM 17 (2011),4,42-49.

[19]. Gopalan.M.A , G.Srividhya,"Some non-extendable P_{-5} sets ", Diophantus J.Math.,1(1) , (2012), 19-22.

[20]. Gopalan.M.A, G.Srividhya," Two Special Diophantine Triples ", Diophantus J.Math.,1(1),(2012), 23-27.

[21]. Gopalan.M.A , G.Srividhya," Diophantine Quadruple for Fionacci and Lucas Numbers with property D(4) ", Diophantus J.Math.,1(1),(2012),1`5-18.

[22]. Andrej Dujella,Zagreb,Croatia,"The problem of Diophantus and Davenport for Gaussian Integers",Glas.Mat.Ser.III,32(1997),1-10.

[23]. S.Vidhyalakshmi,M.A.Gopalan and K.Lakshmi,"Gaussian Diophantine quadruples with property D(1)", IOSR Journal of Mathematics, Vol 10, issue 3, Ver.II,(may-jun 2014),12-14.

[24]. M.A.Gopalan, S.Vidhyalakshmi E.Premalatha and R.Presenna ," On the extendibility of 2-tuple to 4-tuple withproperty D(4)", Bulletin of Mathematical Sciences and Applications,Vol 3, issue 2, (2014),100-104.

[25]. M.A.Gopalan ,G.Sumathi , S.Vidhyalakshmi ,"Special Dio-quadruple involviong Jacobsthal and Jacobsthal lucas Numbers with property $D(k^2+1)$ ",IJMSEA, Vol 8, no 3,2014,221-225.

[26]. K.Meena, S.Vidhyalakshmi, M.A.Gopalan,G.Akila and R.Presenna,"Formation of special diophantine quadruples with property $D(6kpq)^2$ "The IJST,Vol 2,issue 2,2014,11-14.

[27]. Andrej Dujella, Vinko petricevic,"Strong Diophantine triples",Experimental Math,Vol 17,No 1 (2008),83-89.