FLC Modeling of Classical EEG Signals Model using the Technique of TSK - Fuzzy Inference Rules and its Generalization

Prakash N. Kamble

Department of Mathematics,
Dr. BabasahebAmbedkarMarathwada University,
Aurangabad – 431004 (Maharashtra), India.

Abstract: In this paper we use, the idea of "modeling of a model". That is, we reform the classical mathematical model of EEG signals model to fuzzy model utilizing the modeling technique of Takagi-Sugeno-Kang (TSK) fuzzy rule base.. We design the model using exactly same inputs and their values (sensor readings) as that of used in designing classical mathematical model of EEG signal and achieve the desired output result. Further we generalize this model by making $\pm 10\%$, $\pm 20\%$, variations in the input sensor readings and also achieve expected output results. Further it is to be noted that the efforts required to work out the fuzzy model are more feasible as compare to that of the classical mathematical model of EEG signals.

2000 Mathematics Subject Classification: Primary: 03B52, Secondary: 68T27, 68T37.

Key words: Mathematical model of EEG signals, I/Ps-O/P linguistic variables, Mamdani fuzzy inference rules, TSK fuzzy inference rules, weighted average formula.

1. Introduction

The very earliest and most popular direct fuzzy reasoning technique is as of the Mamdani method. To improve upon this method, we need to attempt to develop its natural extension by means of "Takagi-Sugeno-Kang (TSK) architecture". The main motivation for developing this model is to reduce the number of rules required by Mamdani model (the large number of rules create error to fire the rules and reduce the accuracy of the result). This can be done by inserting linear equations of the **input (I/P)** –variables in the consequence (then part) of the Mamdani fuzzy inference rules. In this method the overall **output (o/p)** obtained using "weighted average formula" which is numeric (and not fuzzy). This avoids time consuming process of defuzzification required in Mamdani model.

TSK FLC can be used for controlling a process (i.e. plant) for which it is inconvenient to use classical control model. Also it is ease of describing human knowledge expressed in imprecise linguistic terms. In the classical mathematical EEG signal model I/Ps: -intensity (I), duration (τ) and the O/P:-membrane current ($(I_{memb}$) are linguistic terms which are imprecise or inconvinient in nature, which produce major uncertainty to build up the model. Hence TSK fuzzy control is a technique is used for deriving control law when control information is expressed in linguistic terms.

Thus to overcome all such inconveniences TSK FLC is considered to be better methodology because it provides superior architecture to those obtained in the conventional algorithm mathematical EEG signal model.

2. Acquaintance with EEG Signals

Acquiring signals and images from the human body become vital for early diagnosis of diseases. There are various electro – biological signals among them we study EEG – it is an equipment for processing human activities. More precisely, an EEG signals are measurements of current flow during synaptic excitation of the dendrites of many pyramidal neurons in the cerebral cortex.

2.1. Mechanism of Mathematical Modeling of EEG Signals

This EEG signal model is based on the Hodgkin - Huxley **Nobel prize** winning model for the squid axon published in1952[6].

The electrical excitation arises from the effect of membrane potential on the movement of ions, and from interaction of the potential with the opening and closing of voltage activated membrane channels. The membrane potential increases when the membrane polarized with a net negative charges lining in the inner surface and equal but apposite net positive charge on the outer surface. This potential (E) may be related to the amount of electrical charge (Q), using the relation,

$$E = \frac{Q}{c_m}, \quad (1)$$

where E, electrical potential is measured in the unit of volts; Q, electrical charge is measured in terms of coulombs/cm^2; and Cm, is the measure of capacity of membrane in units of farad/cm^2. The Hodgkin-Huxley model is shown in Figure 1.

In this ***Figure 1,*** I_{memb} is the result of positive charges flowing out of cell. This current consist of three currents namely, sodium (Na), potassium (K) and leak currents (the leak current is due to fact that the inner and outer Na and K ions are not exactly equal). Hodgkin and Huxley estimated the activation and inactivation functions for the Na and K currents and derived a mathematical model to describe an action potential (AP) similar to that of a giant squid. The model is neuron model that usages voltage gated channels. This model describes the change in membrane potential (E) with respect to time. The overall membrane current is the sum of capacity current and ionic current as follows,

$$I_{memb} = c_m \frac{dE}{dt} + I_i \quad (2)$$

where I_i, is the ionic current as indicated in ***Figure 1***. It consists of the sum of three individual components as follows,

$$I_i = I_{Na} + I_k + I_{leak} \quad (3)$$

Where I_{Na}, can be related to the maximal conductance $\bar{g}_{Na}$; activation variable a_{Na}; inactivation variable h_{Na} and driving force $(E - E_{Na})$ through

$$I_{Na} = \bar{g}_{Na} h_{Na} (E - E_{Na}) a^3_{Na} \quad (4)$$

Similarly I_k and I_{leak} can be described.

The change in the variables a_{Na}, a_k and h_{Na} vary from 0 to 1 (time in ms) according to the following equations:

$$\frac{d}{dt}(a_{Na}) = \lambda_t \left[\alpha_{Na}(E)(1-a_{Na}) - \beta_{Na}(E) a_{Na}\right] \quad (5)$$

where, $\alpha(E)$ and $\beta(E)$ are forward and backward rate functions respectively and λ_t is a temperature dependent factor.

Similarly, $\frac{d}{dt}(h_{Na})$ and $\frac{d}{dt}(a_k)$ can be described. The forward and backward parameters were empirically estimated by Hodgkin and Huxley as follows:

$$\alpha_{Na}(E) = \frac{3.5+0.1E}{1-e^{-(3.5+0.1E)}} \quad \beta_{Na}(E) = 4e^{\frac{-(E+60)}{80}} \text{ ,etc.}$$
(6)

As stated in the simulator for neural network and action potential (SNNPA) $literature$[8]. The parameters $\alpha(E)$ and $\beta(E)$ have been converted from the original Hodgkin-Huxley version to a version agreeing with physiological practice taking depolarization of the membrane as positive. Resting potential has been shifted to -60mV (from original 0mV). A simulated action potential is illustrated in ***Figure 1.*** For this model, the parameters are set to be, $c_m = 1.1\ \mu F/cm^2$, $\bar{g}_{Na} = 100 ms/cm^2$, $\bar{g}_k = 35\ ms/cm^2$, $\bar{g}_l = 0.35\ ms/cm^2$, d$E_{Na}$=60mV.

Using the values of $c_m, \bar{g}_k, g_l$ etc in the above related equations (1)-(6), one gets $\boldsymbol{I_{memb} = 80\mu A/cm^2}$, which is shown in ***Figure 2*** of neuron model.

This model is complex due to imprecise linguistic I/P-variables and coupling of different parameters. The technique of TSK-fuzzy controllers on EEG signal modeling is more convenient under these conditions.

2.2. TSK Fuzzy controller on EEG signal modeling

As the system of the classical EEG signal model consist of two fuzzy I/ Ps ‘intensity (I)’ and ‘duration (τ)’ and one fuzzy o/p membrane current (I_{memb}) to be computed. In this context we elaborate a general scheme for controlling a desired value by the technique of TSK-FLC over the classical EEG signal model is as shown in ***Figure 3***

The general inference process based on the TSK FLC excutes in three steps:

Step a) Construction of fuzzy sets and fuzzifications.

Step b) Formation of fuzzy inference rules from Mamdani to TSK.

Step c) Compositions of fuzzy inference rules.

Step a) Construction of fuzzy sets and fuzzifications:-After identifying the relevant I/Ps and O/p variables of the classical controller, our first step in designing the FLC should be to characterize the range of values for the I/Ps and O/P variables. **Since the duration of the action potential of a nerve system in the classical controller is in the range of 5 to 10 ms**, so that we have chosen the range of values for the both I/P- variables: 'intensity' and 'duration' in the time interval of 0 to 10ms in FLC. **And since final injected current in EEG signal model is, I_{memb} = 80μA/cm^2,** accordingly we have chosen range of values for O/P-variable ' membrane current' as 0 to 100 μA / cm^2 $in\ FLC$. Also we have to select "meaningful linguistic states (adjectives) and their corresponding numerical descriptions" for each of the three I/Ps and O/P linguistic variables: Negative Large (NL) for about and below 0.13; Negative Medium (NM) for about 0.26; Negative Slow (NS) for about 0.39; *Almost zero (AZ)* for about 0.52*;* Positive Slow (PS) for about 0.65; Positive Medium (PM) for about 0.78 and Positive Large (PL) for about and above 0.91.

Representing these seven linguistic states and their corresponding numerical fuzzy numbers of I/Ps and O/P linguistic variables by triangular shape fuzzy numbers as in Figures 4 and 5.

Fuzzification of I/P-variables:-The main purpose of the fuzzification is to interpret measurement of I/P -variables and to express the associated measurement uncertainties as described below.

A fuzzification process (function) applied to the I/P variable 'intensity' (I), we denote it by f_I. Then the fuzzification function has the form $f_I: [0,1] \to R$, where R denote the set of all fuzzy numbers. Then $f_I(x_0 = 0.40)$ is a fuzzy number chosen by f_I as a fuzzy approximation of the measurement (sensor reading) intensity (I) at x_0 = 0.40. The computation of fuzzy membership values from **Figure 4**, for which $f_I(x_0 = 0.40) \neq 0$, is as below and shown in **Figure 6.**

NS(0.40sec)= $\frac{0.40-0.52}{0.39-0.52} = \frac{0.12}{0.13} = 0.92$;

AZ$(0.40sec) = \frac{0.40-0.39}{0.52-0.39} = \frac{0.01}{0.13} = 0.08$.

Remaining all fuzzy membership values from **Figure 4** are zero such as,

NL(0.40) = NM(0.40) = PS(0.40)=PM(0.40)=PL(0.40)=0.

The computation of fuzzy membership values from **Figure 5,** for which $f_I(x_0 = 0.40) \neq 0$, is as below and is as shown in **Figure 7.**

The membership values for fuzzy sets NL are computed as, $NL(0.10)$ = 1.

All other remaining memberships' values from **Figure 5** are zero. Such as NS (0.10) = AZ (0.10) = PL (0.10) = PM (0.10) = PS (0.10) = NM (0.10) = 0. This shows that only one rule fires, namely NL (0.10) = 1.

Step b) Formation of fuzzy inference rules from Mamdani to TSK:- The knowledge pertaining to the given control problem is formulated in terms of a set of fuzzy inference rules. To elicit fuzzy inference rules, for the I/P-variables intensity (I), duration (τ) and O/P - variable membrane current (I_{memb}) in our problem, Mamdani fuzzy inference rules have the canonical form of the following type,

If I = A and τ = B then $I_{memb} =$ C,

where A, B and C are fuzzy numbers chosen from the set of fuzzy numbers , that represent the linguistic states NL, NM, NS, AZ, PM, PS and PL. Since each I/P-variable has, seven linguistic states, the total number of possible non- conflicting fuzzy inference rules are $7^2 = 49$. In practice, instead of these 49 rules, a small subset of all possible fuzzy inference rules is often sufficient to obtain acceptable performance of the fuzzy controllers.

An appropriate subset of fuzzy rules derived intuitively by common sense reasoning is as follows:

Rule (1): If I is AZ and τ is NL then I_{memb} is PL.

Rule (2): If I is NS and τ is NL then I_{memb} is PM.

Rule (3): If I is NM and τ is NL then I_{memb} is NS.

Rule (4): If I is NM and τ is AZ then I_{memb} is AZ.

Rule (5): If I is NS and τ is PS then I_{memb} is PL.

Rule (6): If I is PS and τ is NS then I_{memb} is PS.

Rule (7): If I is PL and τ is AZ then I_{memb} is PL.

Rule (8): If I is AZ and τ is NS then I_{memb} is PS.

Rule (9): If I is AZ and τ is NM then I_{memb} is PM.

The fuzzy sets used in this set of rules are shown in Figure 4 and Figure 5. If we replace these fuzzy sets with practical fuzzy numbers such as, Negative slow = about 0.13; Negative medium = about 0.26 etc.We can rewrite the above rules as follows:

Rule (1): If I is AZ = about 0.52 and τ is NL = about 0.13 then I_{memb} is PL = about 91.

Rule (2): If I is NS = about 0.39 and τ is NL = about 0.13 then I_{memb} is PM = about 78.

Rule (3): If I is NM = about 0.26 and τ is NL = about 0.13 then I_{memb} is NS = about 39.

Rule (4): If I is NM = about 0.26 and τ is AZ = about 0.52 then I_{memb} is AZ = about 52.

Rule (5): If I is NS = about 0.39 and τ is PS = about 0.65 then I_{memb} is PL = about 91.

Rule (6): If I is PS = about 0.65 and τ is NS = about 0.39 then I_{memb} is PS = about 65.

Rule (7): If I is PL = about 0.91 and τ is AZ = about 0.52 then I_{memb} is PL= about 91.

Rule (8): If I is AZ = about 0.52 and τ is NS = about 0.39 then I_{memb} is PS = about 65.

Rule (9): If I is AZ = about 0.52 and τ is NM = about 0.26 then I_{memb} is PM = about78.

The I/Ps – O/P relationship of the TSK model on the XY- plane from the above nine rules is as in Figure 8.From the Figure 8 we obtain the equation of a plane passing through three points (or is called three points form equation of the plane). We know formula for the equation of a plane is,

$$a\,(x - x_1) + b(y - y_1) + c(z - z_1) = 0,$$

For the three points A (0.26, 0.52, 52), B (0.26, 0.13, 39) and C (0.39, 0.65, 91) equation of plane is,

$$z_1 = \frac{80}{3}x + \frac{10}{3}y + \frac{130}{3}.$$

And for the three points A (.52, 0.26, 78), B (0.65, 0.39, 65) and C (0.91, 0.52, 91) equation of plane is,

$$z_2 = 30\,x - 40\,y + 72.80.$$

Thus Mamdani fuzzy inference rules were nine but it has been reduced to two in the simplified TSK method. We express the I/Ps - O/P relations using these two linear equations as,

From Figure 8, when the I/P variables x *and* y lie in the region low then we can write, If x *and* y are low then $z_1 = \frac{80}{3}x + \frac{10}{3}y + \frac{130}{3}$. (7)

Next when the I/P variables x *and* y lie in the region high then we get, If x *and* y are high then

$$z_2 = 30x - 40\,y + 72.80. \qquad (8)$$

TSK Method:

TSK method is used when the consequence part is given as a linear function of I/P – variables, such as,

R_i : "If x *is* A_i *and* y *is* B_i *then* z *is* $f(x,y)$",

where, A_i *and* B_i are fuzzy sets and $z = f(x,y)$ is a crisp linear function of the I/ P variable x *and* y expressed as, $z = f(x,y) = ax + by + c$, wherein a, b and c are real numerical constants. We note that this method works when I/Ps are given as a singleton values and it is called fuzzy singleton. Thus in view of derivation of equations (7), (8) and rule R_i we note that the inference performed by the TSK - model is an interpolation of the relevant linear models. The degree of relevance of linear model is determined by the degree of I/P data belonging to the fuzzy subspaces associated with the linear model. These degrees of relevance become the weight in the interpolation process. The total O/P of the fuzzy model is given by the equation below, where α_i is the matching of R_i, which is analogous to the matching degree of the Mamdani model expressed as,

$$z_0 = \frac{\sum_{i=1}^{L} \alpha_i f_i(x_1, x_2, \dots, x_r)}{\sum_{i=1}^{L} \alpha_i},$$

$$= \frac{\sum_{i=1}^{L} \alpha_i (b_{i0} + b_{i1}\mu_{A_{i1}}(a_{i1}) x_1 + \dots + b_{ir} x_r)}{\sum_{i=1}^{L} \alpha_i}, \qquad (9)$$

where L is a finite positive integer.

The I/Ps of TSK – model are crisp numbers. Therefore degree of I/Ps is typically computed by "min"operator given by,

$$\alpha_i = \min(\mu_{A_{i_1}}(a_{i1}),\ \mu_{A_{i_2}}(a_{i2}), \ldots, \mu_{A_{ir}}(a_{ir})).$$

Step c) Compositions of fuzzy inference: - The inferred values of the control action from the rule (7) and the rule (8) are $f_1(x_0, y_0)$ and $f_2(x_0, y_0)$, respectively, wherein x_0, y_0 are I/Ps sensor readings. The matching degrees α_1 *from* R_1 and α_2 *from* R_2 are the same with the Mamdani matching degree. These matching degrees are shown in Figure 9.

In the view of (7) and (8), (9) takes the form

$$z_0 = \frac{\alpha_1 f_1(x_0, y_0) + \alpha_2 f_2(x_0, y_0)}{\alpha_1 + \alpha_2} = \frac{\alpha_1 z_1 + \alpha_2 z_2}{\alpha_1 + \alpha_2}. \qquad (10)$$

From Figure 9 matching values are, $\alpha_1 = 0.08$ and $\alpha_2 = 0.92$.

We calculate the inferred value of the control action from the first rule is $f_1(x_0, y_0)$ where x_0, y_0 I / Ps sensor readings as given below,

$$z_1 = f_1(x_0, y_0) = \frac{80}{3}x + \frac{10}{3}y + \frac{130}{3}.$$

$f_1(x_0, y_0) = f_1(0.40, 0.10) = \frac{80}{3}$ *0.40 + $\frac{10}{3}$ * 0.10+ $\frac{130}{3}$. = 54.32.

And from the second rule $f_2(x_0, y_0)$ as,

$z_2 = f_2(x_0, y_0) = 30\,x - 40\,y + 72.80$

$f_2(x_0, y_0)$ = 30 * 0.40 – 40 * 0.10 + 72.80 = 80.80

Now using these values in (10) we get, desired O/P result "membrane current(I_{memb})"

$$I_{memb} = z_0 = \frac{0.08 * 54.32 + 0.92 * 80.80}{0.08 + 0.92} = 78.68.$$

3 Generalization of TSK - fuzzy logic control model

In order to examine the sensitivity and validity of TSK - fuzzy logic control model. We design TSK - fuzzy logic control models for distinct I/P values (sensor readings) of linguistic variables intensity (I) and duration (τ) and study responses of the O/P results "membrane current" of the respective model. This is to be carried out by repeating the same preceding three as listed below:

Step a_1) Construction of fuzzy sets and fuzzifications.

Step b_1) Formation of fuzzy inference rules from Mamdani to TSK.

Step c_1) Compositions of fuzzy inference rules.

Step a_1) Construction of fuzzy sets and fuzzifications:

The computation of fuzzy membership values from Figure 5 for which $f_I(x_0 = 0.40) \neq 0$, is already calculated in **step a)** and its Pictorial representation is also shown in Figure 10.

In order to examine the sensitivity responses of O/P results of fuzzy controller, we calculate the membership values for the respective fuzzy sets by varying ±*10%* of the above sensor reading $x_0 = 0.40$ as follows.

First maximizing 10% of 0.40 we get 0.44. The determination of the membership values for NS and AZ is as below and is shown in Figure 10.

NS (0.44) = $\frac{0.44-0.52}{0.39-0.52} = \frac{0.08}{0.13}$ = 0620;

AZ (0.44) = $\frac{0.44-0.39}{0.52-0.39} = \frac{0.05}{0.13}$ = 0.380.

Remaining all fuzzy membership values are zero such as, N L (0.44) = N M (0.44) = P S (0.44) = P M (0.44) = PL (0.44) = 0.

Next by minimizing 10% of 0.40 we get 0.36. The determination of the membership values for NS and NM is as below and is shown in Figure 10.

NS (0.36) = $\frac{0.36-0.26}{0.39-0.26} = \frac{0.10}{0.13}$ = 0.770;

NM (0.36) = $\frac{0.36-0.39}{0.26-0.39} = \frac{0.03}{0.13}$ = 0.230.

Remaining all fuzzy membership values are zero such as, N L (0.36) = AZ (0.36) = P S (0.36) = P M (0.36) = PL (0.36) = 0.

Proceeding similar to above. The computation of fuzzy membership values for $f_\tau(y_0 = 0.10)$ is carried out using only that part of Figure 5 for which $f_\tau(y_0 = 0.10) \neq 0$, as below and is shown in Figure 11.

,

$$NL(0.10) = 1.$$

Remaining all memberships values from Figure 5 are zero such as, NS (0.10) = AZ (0.10) = PL (0.10) = PM (0.10) = PS (0.10) =NM (0.10) = 0. This shows that only one rule fires, namely NL (0.10) = 1.

In order to examine the sensitivity and validity of O/P results of fuzzy controller, we calculate the membership values for the respective fuzzy sets by varying ±10% of the above sensor reading $y_0 = 0.10$ as follows.

First by maximizing 10% of 0.10 we get 0.11. The determination of the membership values for the fuzzy set NL is as below and is shown in Figure 11

NL (0.11) = 1.

Remaining all memberships values from Figure 5 are zero such as, NS (0.11) = AZ (0.11) = PL (0.11) = PM (0.11) = PS (0.11) = NM (0.11) = 0. This shows that only one rule fires, namely NL (0.11) = 1.
Secondly by minimizing 10% of 0.10 we get 0.09. The determination of the membership values for the fuzzy set NL is as below and is shown in Figure 11.

NL (0.09) = 1.

Remaining all memberships values from Figure 5 are zero such as NS (0.09) = AZ (0.09) = PL (0.09) = PM (0.09) = PS (0.09) = NM (0.09) = 0. This shows that only one rule fires, namely NL (0.09) = 1.

Step b_1) Formation of fuzzy inference rules from Mamdani to TSK: This step is similar to **step (b).**

Step c_1) Compositions of TSK – fuzzy inference rules:

We note that the composition of TSK – fuzzy inference rules for sensor readings (x_0, y_0) = (0.40, .10) is already carried out **Step c.**

Now we proceed for the calculation of ± 10 variations of sensor reading (0.40, 0.10). For maximizing 10% of (0.40, 0.10) we get (0.44, 0.11). The matching degrees are computed analogous to Mamdani model using min operator as shown in Figure 12.

In the sence of TSK – inference rules the aggregated result is given by weighted average formula,

$$z_0 = \frac{\alpha_1 f_1(x_0, y_0) + \alpha_2 f_2(x_0, y_0)}{\alpha_1 + \alpha_2}$$

$$= \frac{\alpha_1 z_1 + \alpha_2 z_2}{\alpha_1 + \alpha_2}.$$

From Figure 12, matching values are, $\alpha_1 = 0.380$ and $\alpha_2 = 0.620$
We calculate the inferred value of the control action from the first rule is $f_1(x_0, y_0)$ where x_0, y_0 I/Ps sensor readings as are,

$$z_1 = f_1(x_0, y_0) = \frac{80}{3}x + \frac{10}{3}y + \frac{130}{3}.$$

$$f_1(x_0, y_0) = f_1(0.44,\ 0.11) = \frac{80}{3}*0.44 + \frac{10}{3}*0.11 + \frac{130}{3}.$$

$= 55.43.$

And from the second rule $f_2(x_0, y_0)$ as,

$$z_2 = f_2(x_0, y_0) = 30\,x - 40\,y + 72.80$$

$f_2(x_0, y_0) = 30 * 0.44 – 40 * 0.11 + 72.80 = 90.40.$

Now using these values in weighted average formula, TSK -fuzzy logic control gives desired O/P result linguistic variable membrane current (I_{memb}).

$$z_0 = \frac{\alpha_1 f_1(x_0, y_0) + \alpha_2 f_2(x_0, y_0)}{\alpha_1 + \alpha_2}$$

$$I_{memb} = z_0 = \frac{0.380*55.43 + 0.620*90.40}{0.380 + 0.620} = 77.04$$

For minimizing 10% of (0.40,0.10) we get 0.36, 0.09). the matching degrees are computed analogous to to Mamdani model using min operator as shown in Figure 13.

In the sence of TSK – inference rules (i) and(ii) the aggregated result is given by weighted average formula,

$$z_0 = \frac{\alpha_1 f_1(x_0, y_0) + \alpha_2 f_2(x_0, y_0)}{\alpha_1 + \alpha_2}$$

$$= \frac{\alpha_1 z_1 + \alpha_2 z_2}{\alpha_1 + \alpha_2}.$$

From fig (10), matching values are, $\alpha_1 = 0.380$ and $\alpha_2 = 0.620$
We calculate the inferred value of the control action from the first rule is $f_1(x_0, y_0)$ where x_0, y_0I /Ps sensor readings as are,

$$z_1 = f_1(x_0, y_0) = \frac{80}{3}x + \frac{10}{3}y + \frac{130}{3}.$$

$$f_1(x_0, y_0) = f_1(0.36,\ 0.09) = \frac{80}{3} * 0.36 + \frac{10}{3} * 0.09 + \frac{130}{3} = 53.22$$

And from the second rule $f_2(x_0, y_0)$ as,

$$z_2 = f_2(x_0, y_0) = 30\,x - 40\,y + 72.80$$

$f_2(x_0, y_0) = f_2(0.36,\ 0.09)$ = 30 * 0.36 – 40 * 0.09 + 72.80 = 80.00

Now using these values in weighted average formula, TSK -fuzzy logic control gives desired O/P result linguistic variable membrane current(I_{memb}),

$$z_0 = \frac{\alpha_1 f_1(x_0, y_0) + \alpha_2 f_2(x_0, y_0)}{\alpha_1 + \alpha_2}$$

$$I_{memb} = z_0 = \frac{0.77*53.22+0.23*80.00}{0.77+0.23} = 68.40$$

The comparative study of the Classical EEG signal model and our designed TSK fuzzy logic controlled models for the suitable choice of input sensor readings and their desired output results are given in the following table.

Models	Input Sensor Readings	Output Results
Classical EEG Signals Model	$(x_0, y_0) = (0.40, 0.10)$	80.00
EEG Signals Model	$(x_0, y_0) = (0.40, 0.10)$	78.68
EEG Signals Model	$(x_0, y_0) = (0.44, 0.11)$	77.04
EEG Signals Model	$(x_0, y_0) = (0.36, 0.09)$	68.40

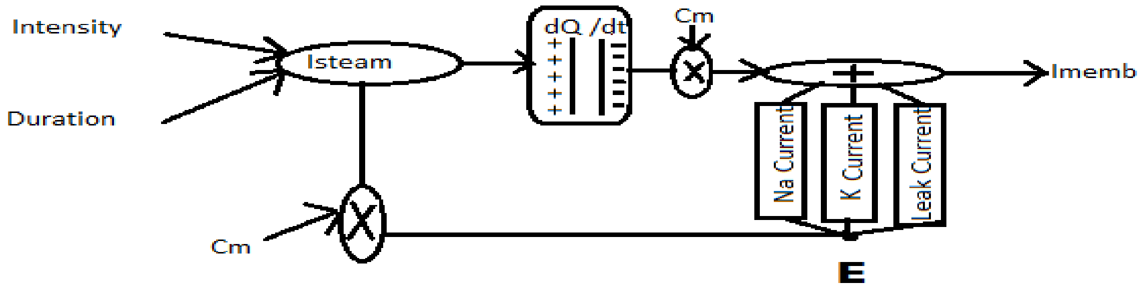


Figure 1: Hodgkin-Huxley excitation model.

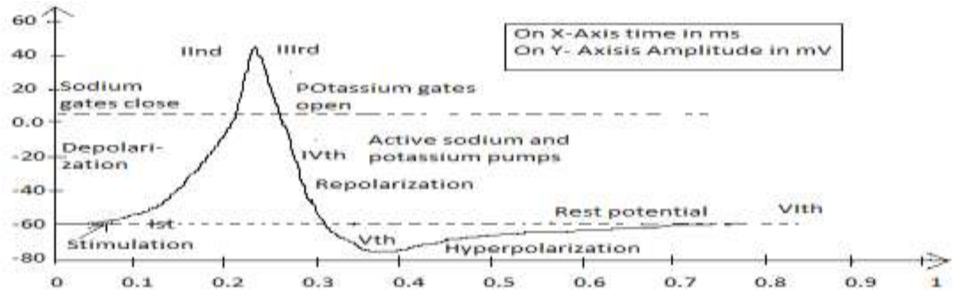


Figure 2.: A single AP in response to a transient stimulation based on Hodgkin –Huxley model.

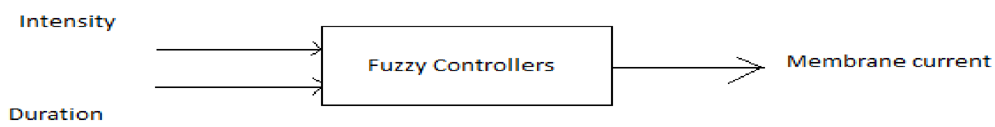


***Figure 3*: A general scheme of TSK- FLC for controlling desired value**

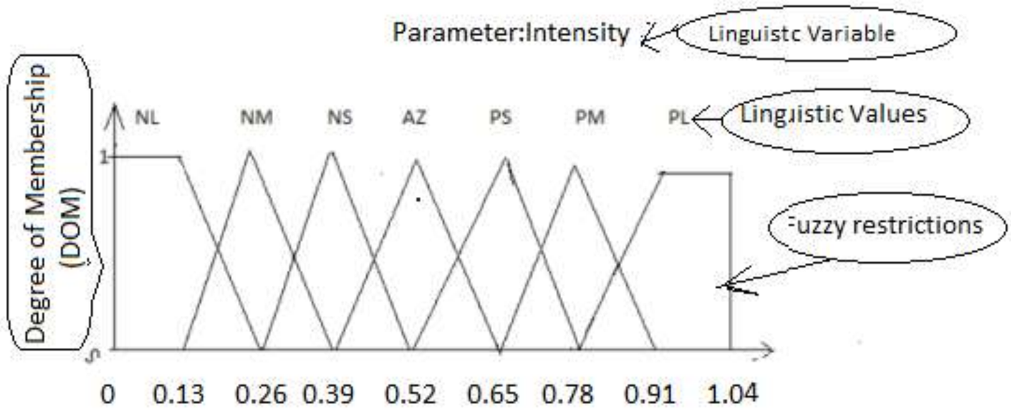


Figure 4: Fuzzy sets and decomposition for I/P variable intensity/ duration over the range [0, 1]-is the time in ms.

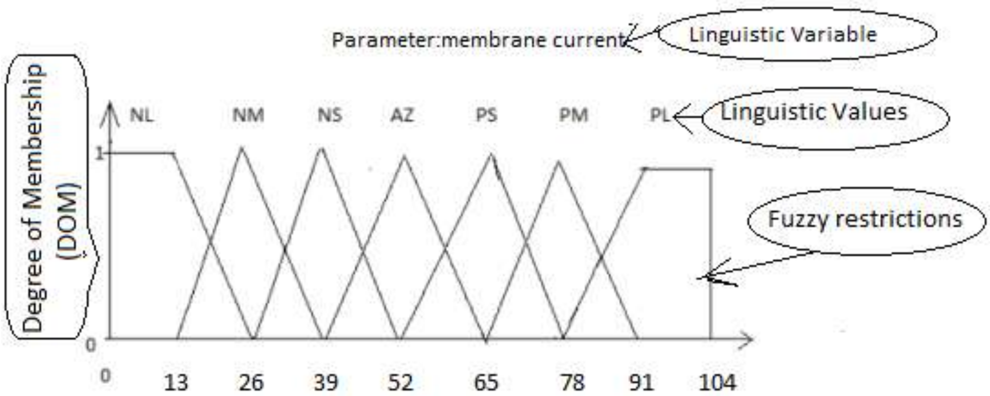
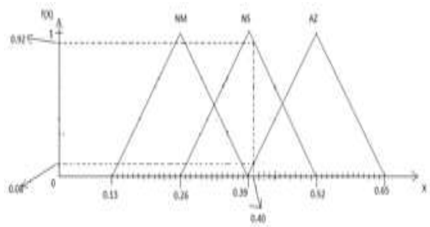
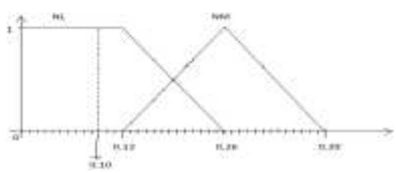


Figure 5: Fuzzy sets and decomposition for O/P variable 'membrane current' (I_{memb}) over the range [0,100] is the injected current in $\mu A/cm^2$.



Figures 6



Figures 7

Figures (6 and 7): Fuzzification of I/P variable intensity for $x_0 = 0.40$.

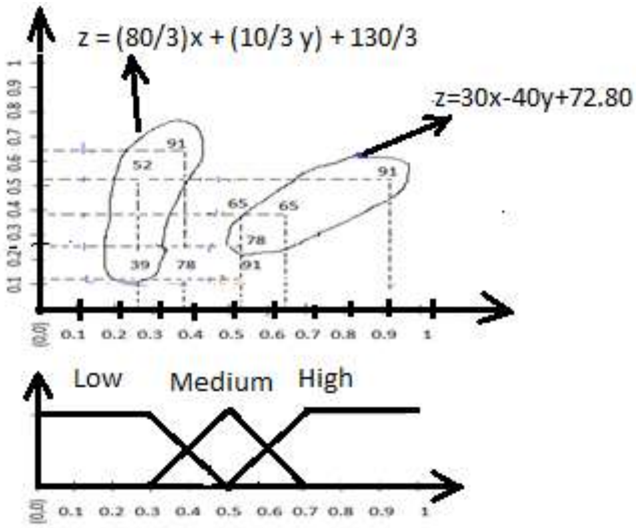
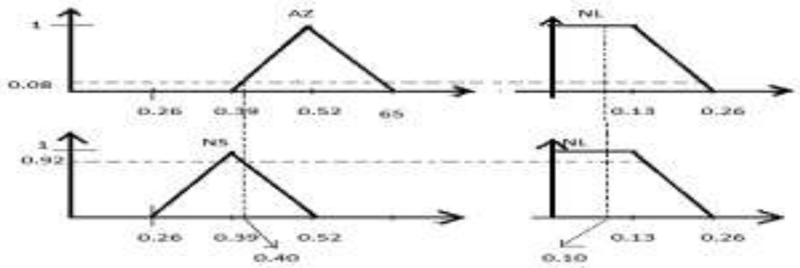


Figure 8. I/Ps -O/P relationship of simplified TSK fuzzy model.

$$\alpha_1 = min(\mu_{A_1}(0.40), \mu_{B_1}(0.13)) = 0.08.$$



$$\alpha_2 = min(\mu_{A_2}(0.40),\ \mu_{B_2}(0.10)) = 0.92.$$

Figure 9: Graphical representation of TSK method for sensor reading (x_0, y_0) = (0.40, 0.10).

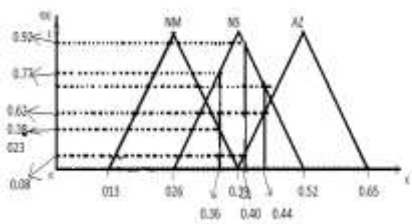


Figure 10

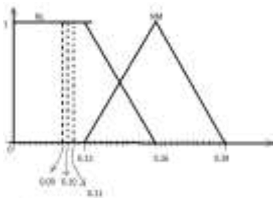
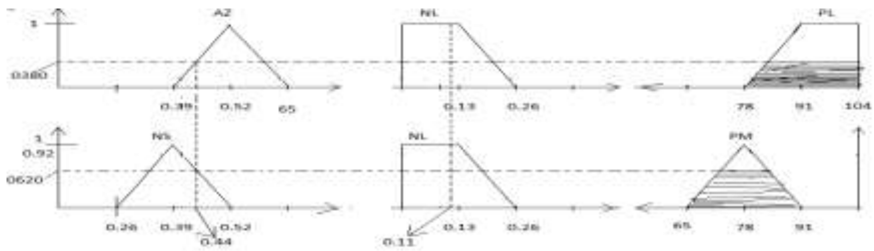


Figure 11

Figures (10 and 11): ***The fuzzification of I/P- variables intensity (at $x_0 = 0.40$ and its ±10% variations) and duration (at*** **$y_0 = 0.10$ and at its ±10% variations)** ***is shown in*** **Figure 10*and* Figure 11** ***respectively.***

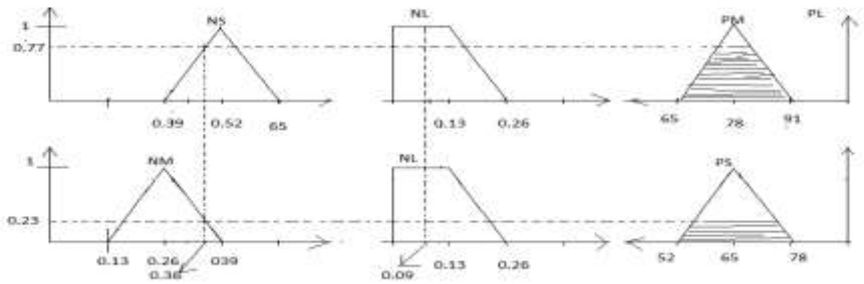
$$\alpha_1 = min(\mu_{A_1}(0.44), \mu_{B_1}(0.11)) = 0.380.$$



$$\alpha_2 = min(\mu_{A_2}(0.44), \ \mu_{B_2}(0.11)) = 0.620$$

Figure 12: Graphical representation of TSK method for 10% maximization of sensor reading (x_0, y_0) = (0.40, 0.10).

$$\alpha_1 = min(\mu_{A_1}(0.36), \mu_{B_1}(0..09)) = 0.77$$



$$\alpha_2 = min(\mu_{A_2}(0.36), \mu_{B_2}(0.09)) = 0.23$$

Figure 13: Graphical representation of TSK method for 10% minimization of sensor reading (x_0, y_0) = (0.40, 0.10).

4 Concluding Remark

We conclude that the traditional classical EEG signal mathematical model may appear simpler and perhaps more economical but we should not easily make this assumption due to its complex PID model and time consuming factor. In fact TSK fuzzy logic control are often easily prototyped and implemented, very simpler to describe and verify, can be maintained and embedded with higher degree of accuracy in less time and generally have an equivalent output result with the O/P of the classical EEG signal model provided I/Ps (Sensor reading) of the Linguistic variable used for both models must be the same.

5 Future scope

The method can be extended for more general applications. Accordingly our future attempt will be to do work to develop "Neuro - fuzzy controller over mathematical modeling of EEG signals".

References:

(1) George J. Klir and Bo Yuan, *Fuzzy sets and fuzzy logic (Theory and applications)*, PHI, New Delhi, 2006.

(2) John Yen and Reza Langari, *Fuzzy Logic (Intelligence, Control and Information)*, Pearson Education, 1999.

(3) Kamble P.N., Mherotra S.C and Dhakne M.B., *Superiority Of Fuzzy Controllers over Mathematical Modeling of EEG Signals*, Proceedings Of The International Conference On Mathematical Sciences In Honour of A. M. Mathai, pp. 175-190, January- 2011.

(4) Kamble P.N., Mherotra S.C and Dhakne M.B., *TSK - fuzzy Controlled Modeling via Classical Mathematical EEG Signals Modeling*, International Journal of Mathematical Archive, PP.147-154, October, 2013.

(5) Kazuo Tanaka, *An introduction to fuzzy logic for practical applications,* Springer, 1997.

(6) Metin Akay, Editor, *Nonlinear Biomedical Signal processing Volume - I (Fuzzy Logic, Neural Networks and New Algorithms),* IEEE press products, 2000

(7) Saeid Sanei and J. A. *Chambers, EEG signal processing*, John Wiley and sons Ltd, 2007.

(8) Simulator for Neural Networks and Action potentials (SNNAP) Tutorial, The University of Texas-Houston Medical School, 2003,http//snap.uth.tmc.edu.

(9) Terrance J. Senijnowski *Towardas, Brain computer Interfacing*, The MIT Press Cambridge, London, England, 2007.