# Non-Differentiable Fractional Programming Under Generalized $d, \rho, \eta, \theta$ - Type 1 Univex Function

*** Gayatri Devi, Rashmita Mohanty**

**Prof. in : CSE, ABIT College, CDA-1, Cuttack**

**Abstract:** Non-differentiable fractional duality is given and its weak duality and strong duality results are established under generalized $d, \rho, \eta, \theta$ type-1 univex function.

**Key Words:** Non differentiable fractional duality, generalized university, $d, \rho, \eta, \theta$ type – 1 univex function.

**INTRODUCTION:**

Several authors have shown their interest in developing optimality conditions and duality result for minimax programming problems. Bector at al [1] and Xu [2] gave a mixed type duality for fractional programming, established some sufficient condition and obtained various duality results between the mixed dual and primal problem. Zhou and Wang [3] introduced a class of nonsmooth multiobjective fractional mixed mixed dual programming. Mishra and Rautela [4] formulated a general dual and proved the duality results under generalized semi locally type – 1 univex and related function. To relax convexity assumption imposed on the function in theorems on optimality and duality , various generalized convexity concept have been proposed. Hanson and Mond [5] defined two new classes of function called type – I and type – II function. Rueda and Hanson [6] further extened type – I function to the class of pseudo type – I and quasi type – I function. Bector et al [7] introduce univex function. Mishra et al [8] introduced type – I univex, pseudo type – I univex, quasi type – I univex function and obtained optimality results for mathematical programs under generalized type – I univex function. Recently, Nahak and Mohapatra [9] obtained duality results for multiobjective programming problems under $d, \rho, \eta, \theta$ invexity assumption.

In his paper, we have introduced duality problems in non-differentiable fractional programming problem and established results under generalized class of $d, \rho, \eta, \theta$ - type – I univex function.

2. **Preliminaries and Definitions. :** Let $R^n$ be n-dimensional Euclidean space and $R^n_+$ be the non negative orthant. For vectors x and y in $R^n$, we denote $x < y \Rightarrow x_i < y_i$ for i = 1, 2, …..,n,

$x \leqq y \Leftrightarrow x_i \leqq y_i$, i = 1, 2, ….n, $x \leq y \Leftrightarrow x_i \leq y_i$, i = 1, 2, ….,n. Let $D \subseteq R^n$ be an invex set $\eta : D \times D \to R^n$, $x_0$ be an arbitrary point of D and $F : D \to R$, i = 1, 2, ….p, $\Psi : R \to R$, $b : D \times D \to R_+$.

**Definition 2.1 :** (Bector etal [7] A differentiable function f is said to be univex at $x_0 \in D$ of for all $x \in D$, we have $b(x, x_0)\Psi[F(x) - f(x_0)] \geq n(x, x_0)^T \nabla_x f(x_0)$.

**Definition 2.2 :** (f, h) is said to be $(d, \rho, \eta, \theta)$ - type I univex functions at $x_0 \in D$ if for all $x \in X$.

$$b_0(x, x_0)\psi_0[f(x) - f(x_0)] \geq f'(x_0, n(x, x_0)) + \rho_0 \| \theta(x, x_0)^2 \| ,$$

$$b_1(x, x_0)\psi[h(x_0)] \geq h'(x_0 in(x, x_0)) + \rho_1 \| \theta(x, x_0) \|^2 .$$

**Definition 2.3 :** (f, h) is said to be weak strictly pseudo $(d, \rho, \eta, \theta)$ - type – I univex functions at $x_0 \in D$ if for all $x \in X$.

$$b_0(x, x_0)\psi_0[f(x) - f(x_0)] \leq 0 \Rightarrow f'(x_0, n(x, x_0)) < -\rho_0 \| \theta(x, x_0) \|^2,$$

$$b_1(x, x_0)\psi_1[h(x_0)] \geq 0 \Rightarrow h'(x_0, \eta(x, x_0)) < -\rho_1 \| \theta(x, x_0) \|^2$$

**PRIMAL PROBLEM**

We consider the following nondifferentiable fractional programming problems.

(FP) minimize $\dfrac{f(x)}{g(x)}$

Subject to $h(x) \leq 0, x \in X$,

Where $f : x \to R$, $g : x \to R$ and $h : x \to R^m$ ….. (3.1)

We assume that $f(x) \geq 0$ and $g(x) > 0$ on $R^n$

**Dual Problem (FD)**

(FD) maximize $\dfrac{f(u)}{g(u)} + y^T h(u) = \nu$

Subject to $\left(f' + y_{J_1}^T h_{J_1}^{'}\right)(u; \eta(x, u)) - \nu g'(u; \eta(x, u)) + \sum_{j \in J_2} y_J^T; h_j^1(u; \eta(x, u)) \geq 0$ ….. (3.2)

$f(u) + y_{J_1}^T h(u) - \nu g(u) \geq 0$ ….. (3.3)

$y_j h_j(u) \geq 0 \; j \in J_2$ ….. (3.4)

$u \in D,\ y_{j_1} \geq 0,\ y_{J_2} \geq 0, \eta : x \to D \to R^n$ …… (3.5)

**Theorem – 3.1 (Weak duality theorem)**

Let x and $(u, y, \eta)$ be the feasible solution for (FP) and (FD) respectively. If

(i) $(f + y_{J_1}^T h_{J_1} - \nu g, h_{j_2})$ is weak strictly pseudo $(d, \rho, \eta, \theta)$ -type – I univex at u.

(ii) For any $a \in R, a \leq 0 \Rightarrow \psi_0(a) \leq 0$ & $b \in R, b \geq 0 \Rightarrow \psi(b) \geq 0, b_0(x, u) > 0, b_1(x, u) > 0$

(iii) $\rho_0 + \sum_j \rho_{1j} \geq 0,\ \rho_0, \rho_{1j} \in R,\ j \in J_2$

Then $\frac{f(x)}{g(x)} \nleq \nu$

**Proof :** Suppose to the contrary that $\frac{f(x)}{g(x)} \leq \nu$

$\Rightarrow f(x) - \nu g(x) \leq 0$

$\Rightarrow f(x) - \nu g(x) + y^T h(x) \leq 0$ from (3.1)

$\Rightarrow \left[f(x) - \nu g(x) + y^T h(x)\right] - \left[f(u) - \nu g(u) + y^T h(u)\right] \leq 0$ from (3.3).

From assumption (ii) of theorem 3.1, we get

$b_0(x, u)\psi_0\left[(f(x) - \nu g(x) + y^T h(x)) - (f(u) - \nu g(u) + y^T h(u))\right] < 0$ ….. (3.6)

and

$b_1(x, u)\psi_1\left[y_j h_j(u)\right] \geq 0,\ j \in J_2$ … (3.7)

Since hypothesis (i) of weak duality theorem satisfied and from above inequation (3.6) and (3.7), we get

$$(f' + y_{J_1}^T, h_{J_1}^1)(u; \eta(x, u)) - \nu g'(u; \eta(x, u)) < -\rho_0 \|\theta(x, u)\|^2,$$

$$\sum_{j \in J_2} (y_j h_j)'(u; \eta(x,u)) < -\sum_{j \in J_2} \rho_{ij} \|\theta(x, u)\|^2,\ j \in J_2$$

Adding above these two equations, we get

$$(f' + y_{J_1}^T h_{J_1}^{'})(u; \eta(x, u)) - \nu g(u; \eta(x, u)) + \sum_{j \in J_2} (y_j h_j^1)(u; \eta(x, u))$$

$$< -\left[\rho_0 \| \theta(x, u) \|^2\right] - \sum_{j \in J_2} \rho_{ij} \| \theta(x, u) \|^2 < -\left[\rho_0 + \rho_{1j}\right] \| \theta(x,u) \|^2 < 0$$

This is a contradiction.

**Theorem 3.2 (Strong duality) :**

Let $x_0$ be a weakly efficient solution of (FP) and $h_j$ continues at $x_0$ for $j \in J_2$ and $f - \nu g; h_j, j \in J_1$ are directionality differentiable at $x_0$. Also $f' - \nu g'(x_0, \eta(x, x_0)), i = 1,2,....k$ and $h_j'(x_0; \eta(x, x_0)), j \in J_i$ are pre-invex function on X. If h satisfies the generalized Slater's constraint qualification at $x_0$, $y \in R^k$ such that $(x_0, y, \eta)$ is feasible for (FD) and the objective function values of (FP) and (FD) are equal. Moreover, if theorem (3.1) holds, then $(x_0, y, \eta)$ is an efficient solution of (FD).

**Proof :** Since $x_0$ is weakly efficient solution of (FP), therefore by Karush – Kuhn Tucker constraints conditions, there exist $y \in R^k$ such that

$$\left[f'(x_0, \eta(x, x_0)) - \nu g'(x_0, \eta(x, x_0))\right] + \sum_{j=1}^{k} y_j, h_j'(x_0, \eta(x, x_0)) \geq 0, \forall x \in X \text{ and}$$

$$y_j h_j(x_0) = 0, \; j = 1,2,...k$$

It follows that $(x_0, y, \eta)$ is feasible for (FD) with $\nu = \left(f(x_0) + y_{j_1} h_{j_1}(x_0)\right) / g(x_0)$. From the proof of weak Duality theorem, we know that $(x_0, y, \eta)$ is an efficient solution for (FD) and the values of the objective function for (FP) and (FD) are equal at $x_0$ and $(x_0, y, \eta)$ respectively.

**Theorem 3.3 (Converse duality) :** Let $(u_0, y, \eta)$ be a feasible solution of (FD). Suppose that hypothesis of theorem (3.1) holds at $u_0$ with $y_{J_1}^T h_{J_1}(u_0) = 0$, $u_0$ is an efficient solution of (FP)

**Proof :** Suppose $u_0$ is not an efficient solution (FP). Then there exist $x_0 \in X$ such that

$$\frac{f(x_0)}{g(x_0)} \leq \frac{f(u_0)}{g(u_0)}$$

Since $y_{J_1}^T h_{J_1}(u_0) = 0$

$$\Rightarrow \frac{f(x_0)}{g(x_0)} \leq \frac{f(u_0) + y_{J_1} h_{J_1}(u_0)}{g(u_0)}$$

Therefore by theorem 3.1 we obtain a contradiction. Hence $u_0$ is an efficient solution of (FP).

**CONCLUSION :**

In this paper, we have established duality results for a non differentiable fractional programming problem under generalized class of $d, \rho, \eta, \theta$ - Type – I univex function

The duality results developed in this paper can be further extended for second order fractional programming problem.

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