Degree Sequence and Clique Number of Bi-Factograph and Tri-Factograph

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Abstract— **By the theorem of unique factorization for integers, every positive integer z can be written in the form $z = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_r^{\alpha_r}$, where $p_1, p_2, \dots p_r$ are distinct primes, $\alpha_1, \alpha_2, \dots \alpha_r$ are positive integers. We can construct a graph G which is associated with this z. Positive integral divisors of z being a vertex set V and two distinct vertices of V are adjacent in G if their product is in V. In z, when $r = 1$ then the corresponding graph is called the perfect factograph. Here we extend the concept to $r = 2, 3$ and the corresponding graphs are called Bi-factograph and Tri-factograph respectively. In this paper we attempt to find the degree sequence and clique number of Bi-factograph and Tri-factograph.**

Keywords— **Factograph, Perfect factograph, Bi-factograph, Tri-factograph, clique number.**

I. INTRODUCTION

By a graph, we mean a finite undirected, non - trivial graph without loops and multiple edges. The order and size of a graph is denoted by p and q respectively. For terms not defined, we refer to Frank Harary [4]. The concept of factograph and perfect factograph was introduced by E. Giftin Vedha Merly and N. Gnanadhas [1],[2]. In this paper we extend the concept to Bi-factograph and Tri-factograph. For a positive integer z, we define a factograph as $G = (V, E)$ where $V = \{v_i / v_i \text{ is a factor of } z\}$ and two distinct vertices v_i and v_j are adjacent if and only if their product is in V. A clique of a graph G is a complete subgraph of G. A clique of G is the maximal clique, if it is not properly contained in another clique of G. Number of vertices in maximal clique of G is called the clique number of G and is denoted by $\omega(G)$. For $v \in V, d(v)$ is the number of edges incident with v. A factograph G with $z = p_1^{\alpha_1}$, where p is a prime and α is any positive integer is called a perfect factograph.

II. DEGREE SEQUENCE AND CLIQUE NUMBER OF BI - FACTOGRAPH.

A. Definition

A factograph G with $z = p_1^{\alpha_1} p_2^{\alpha_2}$ where p_1, p_2 are distinct primes and α_1, α_2 are positive integers is called a Bi - factograph.

B. Theorem

Let α_1 and α_2 be two positive integers, p_1 and p_2 be two distinct primes. A Bi-factograph G with $z = P_1^{\alpha_1} P_2^{\alpha_2}$ has order $(\alpha_1 + 1)(\alpha_2 + 1)$ and the degree sequence of G is given by

i) $s_1: (\alpha_1 + 1)(\alpha_2 + 1) - 1, (\alpha_1 + 1)\alpha_2 - 1, \dots$
$(\alpha_1 + 1)\frac{\alpha_2}{2}, \dots \alpha_1 + 1,\ \alpha_1(\alpha_2 + 1) - 1,$
$\alpha_1\alpha_2 - 1, \dots \frac{\alpha_1\alpha_2}{2}, \dots\ \alpha_1,$
$\dots \frac{\alpha_1}{2}(\alpha_2 + 1), \frac{\alpha_1\alpha_2}{2}, \dots \frac{\alpha_1\alpha_2}{4}, \dots \frac{\alpha_1}{2}, \dots\ \alpha_2 + 1, \alpha_2, \dots \frac{\alpha_2}{2}, \dots 1,$
where α_1 and α_2 are even.

ii) $s_2: (\alpha_1 + 1)(\alpha_2 + 1) - 1,$

$(\alpha_1+1)\alpha_2-1, \ldots (\alpha_1+1)\left\lfloor\frac{\alpha_2}{2}\right\rfloor, \ldots \alpha_1+1,$

$\alpha_1(\alpha_2+1)-1,\ \alpha_1\alpha_2-\ 1, \ldots\ \alpha_1\left\lfloor\frac{\alpha_2}{2}\right\rfloor, \ldots \alpha_1,$

$\ldots\left\lfloor\frac{\alpha_1}{2}\right\rfloor(\alpha_2+1), \left\lfloor\frac{\alpha_1}{2}\right\rfloor\alpha_2, \ldots\left\lfloor\frac{\alpha_1}{2}\right\rfloor\left\lfloor\frac{\alpha_2}{2}\right\rfloor,$

$\ldots\left\lfloor\frac{\alpha_1}{2}\right\rfloor, \ldots \alpha_2+1, \alpha_2, \ldots\left\lfloor\frac{\alpha_2}{2}\right\rfloor, \ldots 1,$ where α_1 and α_2 are odd.

iii) $s_3:(\alpha_1+1)(\alpha_2+1)-1, (\alpha_1+1)\alpha_2-1, \ldots$

$(\alpha_1+1)\frac{\alpha_2}{2}, \ldots\ \alpha_1+1,\ \alpha_1(\alpha_2+1)-1,$

$\alpha_1\alpha_2-1, \ldots\frac{\alpha_1\ \alpha_2}{2}, \ldots\ \alpha_1, \ldots$

$\left\lfloor\frac{\alpha_1}{2}\right\rfloor(\alpha_2+1), \left\lfloor\frac{\alpha_1}{2}\right\rfloor\alpha_2, \ldots\left\lfloor\frac{\alpha_1}{2}\right\rfloor\frac{\alpha_2}{2}, \ldots\left\lfloor\frac{\alpha_1}{2}\right\rfloor \ldots \alpha_2+1, \alpha_2, \ldots\ \frac{\alpha_2}{2}, \ldots 1.$

where α_1 is odd and α_2 is even.

iv) $s_4:(\alpha_1+1)(\alpha_2+1)-1, (\alpha_1+1)\alpha_2-1, \ldots$

$(\alpha_1+1)\left\lfloor\frac{\alpha_2}{2}\right\rfloor, \ldots \alpha_1+1, \alpha_1(\alpha_2+1)-1,$

$\alpha_1\alpha_2-1, \ldots\ \alpha_1\left\lfloor\frac{\alpha_2}{2}\right\rfloor, \ldots \alpha_1, \ldots$

$\frac{\alpha_1}{2}(\alpha_2+1), \frac{\alpha_1}{2}\alpha_2, \ldots\frac{\alpha_1}{2}\left\lfloor\frac{\alpha_2}{2}\right\rfloor, \ldots\frac{\alpha_1}{2}, \ldots \alpha_2+1, \alpha_2, \ldots\left\lfloor\frac{\alpha_2}{2}\right\rfloor, \ldots 1,$

where α_1 is even and α_2 is odd.

Proof:

i) When α_1 and α_2 are even, let $G=(V,E)$ be a Bi-factograph. If $z=\ p_1^{\alpha_1}\ p_2^{\alpha_2} \ldots p_r^{\alpha_r}$ then the number of positive divisors of z is $\prod_{i=1}^{r}(\alpha_i+1)$.Therefore, number of positive divisors of $z=p_1^{\alpha_1}p_2^{\alpha_2}$ is $(\alpha_1+1)(\alpha_2+1)$ so that the order of G is $(\alpha_1+1)(\alpha_2+1)$. Let $V=\{\,p_1^0\,p_2^0,\ \ p_1^0\,p_2^1, \ldots\ p_1^{\alpha_1}\ p_2^{\alpha_2}\}$.

This vertex set can be represented by the $(\alpha_1\ +1)\times(\alpha_2+1)$ array form as follows

$$\begin{pmatrix} p_1^0\,p_2^0, & p_1^0\,p_2^1, & p_1^0\,p_2^2, & \ldots\ p_1^0\,p_2^{\alpha_2} \\ p_1^1\,p_2^0, & p_1^1\,p_2^1, & p_1^1\,p_2^2, & \ldots\ p_1^1\,p_2^{\alpha_2} \\ \vdots & \vdots & & \vdots \\ p_1^{\alpha_1}\,p_2^0, & p_1^{\alpha_1}\,p_2^1, & p_1^{\alpha_1}\,p_2^2 & \ldots p_1^{\alpha_1}\,p_2^{\alpha_2} \end{pmatrix}_{(\alpha_1+1)\times(\alpha_2+1)}.$$

In a Bi-factograph G, we observe that for $i\neq k$ and $j\neq l$ then the vertex $p_1^i\,p_2^j$ is adjacent to $p_1^k\,p_2^l$ in G if and only if $i+k\leq\alpha_1$ and $j+l\leq\alpha_2$.Consider the first (α_2+1) vertices, $p_1^0\,p_2^0,\ p_1^0\,p_2^1,\ p_1^0\,p_2^2, \ldots,\ p_1^0\,p_2^{\alpha_2}$. By our factograph condition, it is obvious that $p_1^0\,p_2^0$ is adjacent to rest of the vertices implies $d(p_1^0\,p_2^0)=(\alpha_1+1)(\alpha_2+1)-1$. Pick the second vertex $p_1^0\,p_2^1$, exactly $(\alpha_1+1)\alpha_2$ combination of verities satisfy the condition and among that $p_1^0\,p_2^1$ is adjacent to remaining vertices which implies $d(p_1^0\,p_2^1)=(\alpha_1+1)\,\alpha_2-1$. Consider the vertex $p_1^0\,p_2^{\frac{\alpha_2}{2}+1}$, exactly $(\alpha_1+1)\left(\frac{\alpha_2}{2}\right)$ combination of vertices satisfy the condition of adjacency, among that the vertex $p_1^0\,p_2^{\frac{\alpha_2}{2}+1}$ is not in that combination which gives $d\left(p_1^0p_2^{\frac{\alpha_2}{2}+1}\right)=(\alpha_1+1)\left(\frac{\alpha_2}{2}\right)$. Continuing this way, the vertex $p_1^0\,p_2^{\alpha_2}$ is adjacent to (α_1+1) vertices and $d(p_1^0p_2^{\alpha_2})=(\alpha_1+1)$. Consider $\left(\frac{\alpha_1}{2}+2\right)^{\text{th}}$ row of α_2+1 vertices $p_1^{\frac{\alpha_2}{2}+1}p_2^0, p_1^{\frac{\alpha_2}{2}+1}p_2^1, \ldots\ldots\ldots p_1^{\frac{\alpha_2}{2}+1}p_2^{\alpha_2}$ have the degree sequence $d\left(p_1^{\frac{\alpha_1}{2}+1}p_2^0\right)=\left(\frac{\alpha_1}{2}\right)(\alpha_2+1), d\left(p_2^{\frac{\alpha_1}{2}+1}p_2^1\right)=\left(\frac{\alpha_1}{2}\right)\alpha_2, \ldots\ d\left(p_2^{\frac{\alpha_1}{2}+1}p_2^{\alpha_2}\right)=\frac{\alpha_1}{2}$. Finally, $(\alpha_1+1)^{\text{th}}$ row of (α_2+1) vertices $p_1^{\alpha_1}\,p_2^0,\ p_1^{\alpha_1}\,p_2^1,\ p_1^{\alpha_1}\,p_2^2 \ldots\ldots,\ p_1^{\alpha_1}\,p_2^{\alpha_2}$ have degree sequence

$d(p_1^{\alpha_1}p_2^0)=\alpha_2+1,\ d(p_1^{\alpha_1}p_2^1)=\alpha_2, \ldots\ldots, d(p_1^{\alpha_1}p_2^{\alpha_2})=1.$

Therefore, G has $(\alpha_1+1)(\alpha_2+1)$ vertices and has the degree sequence,

$s_1:(\alpha_1+1)(\alpha_2+1)-1, (\alpha_1+1)\alpha_2-1, \ldots$

$(\alpha_1+1)\frac{\alpha_2}{2}, \ldots\ \alpha_1+1,\ \alpha_1(\alpha_2+1)-1,\ \alpha_1\alpha_2-1,$

$\ldots\ \frac{\alpha_1\alpha_2}{2}, \ldots\ \alpha_1, \ldots\frac{\alpha_1}{2}(\alpha_2+1), \frac{\alpha_1\alpha_2}{2}, \ldots\ \frac{\alpha_1\alpha_2}{4}, \ldots\ \frac{\alpha_1}{2},$

$\ldots \alpha_2+1, \alpha_2, \ldots\ \frac{\alpha_2}{2}, \ldots 1.$

Similar manner we can prove case (ii) (iii) and (iv).

C. Theorem

The clique number of a Bi-factograph G is

i) $\omega(G)=\left(\frac{\alpha_1}{2}+1\right)\left(\frac{\alpha_2}{2}+1\right)$ when α_1 and α_2 are even.

ii) $\omega(G)=\left(\left\lfloor\frac{\alpha_1}{2}\right\rfloor+1\right)\left(\left\lfloor\frac{\alpha_2}{2}\right\rfloor+1\right)+2$

when α_1 and α_2 are odd.

Proof:

i) If α_1 and α_2 are even, let $G=(V,E)$ be a Bi-factograph and $V=\{p_1^0\,p_2^0,\ p_1^0\,p_2^1, \ldots\ldots\ldots\ldots,\ p_1^{\alpha_1}\,p_2^{\alpha_2}\}$ be the vertex set of G. We consider the set $S=\left\{p_1^x p_2^y/0\leq x\leq\frac{\alpha_1}{2}\ and\ 0\leq y\leq\frac{\alpha_2}{2}\right\}$ which is a proper subset of V. We seek to prove that the subgraph of G induced by S is the maximal clique of G.

In G, we observe that for $i \neq k$ and $j \neq l$, the vertex $p_1^i\, p_2^j$ is adjacent to $p_1^k\, p_2^l$ in G if and only if $i + k \leq \alpha_1$ and $j + l \leq \alpha_2$. We have to prove that every pair of distict vertices in S are adjacent. Take any two arbitrary vertices $p_1^a\, p_2^b$ and $p_1^c\, p_2^d$ in S. Since $a + c \leq \alpha_1$ and $b + d \leq \alpha_2$, we have $p_1^a\, p_2^b$ is adjacent to $p_1^c\, p_2^d$. Therefore each pair of distinct vertices in S are adjacent, which implies $\langle S\rangle$ is a clique of G. It remains to show that $\langle S\rangle$ is the maximal clique of G. Suppose we take any arbitrary vertex v in V which is not in S.

Since Bi-factograph, a vertex v cannot be adjacent with all vertices of S implies $\langle S + \{v\}\rangle$ cannot be the clique of G. Therefore $\langle S\rangle$ is the maximal clique of G and $\omega(G) = |S| = \left(\frac{\alpha_1}{2} + 1\right)\left(\frac{\alpha_2}{2} + 1\right)$.

Similarly we can prove case (ii).

D. Remark

(i) When α_1 is odd and α_2 is even, then the clique number of a Bi-factograph G is

$$\omega(G) = \left(\left\lfloor\frac{\alpha_1}{2}\right\rfloor + 1\right)\left(\frac{\alpha_2}{2} + 1\right) + 1.$$

(ii) When α_2 is odd and α_1 is even, then the clique number of a Bi-factograph G is

$$\omega(G) = \left(\frac{\alpha_1}{2} + 1\right)\left(\left\lfloor\frac{\alpha_2}{2}\right\rfloor + 1\right) + 1.$$

E. Example

Consider a Bi-factograph G with $z = p_1^2\, p_2^2$. Here order of G is 9.

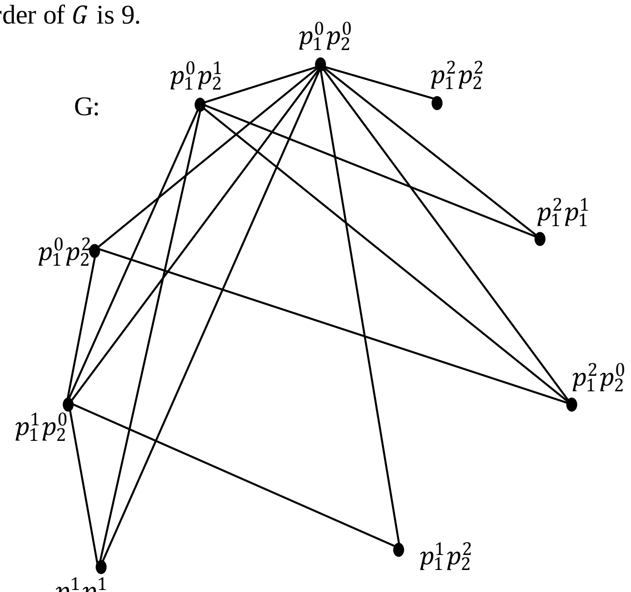

Figure:1

We observe that, the degree sequence of G is s: 8,5,3,5,3,2,3,2,1 and $\omega(G)$ is 4.

III. Degree Sequence and Clique Number of Tri-Factograph

A. Definition

A factograph G with $z = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}$, where p_1, p_2, p_3 are distinct primes and $\alpha_1, \alpha_2, \alpha_3$ any positive integers is called a Tri-factograph.

B. Theorem

Let α_1, α_2 and α_3 be any positive integers, p_1, p_2 and p_3 be any distinct primes, G is a Tri-factograph with $z = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}$, then G is of order $(\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1)$ and the degree sequence of G is given by,

(i) When $\alpha_1, \alpha_2, \alpha_3$ are even

$s_1: (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1) - 1,$

$\alpha_1(\alpha_2 + 1)(\alpha_3 + 1) - 1, \ldots \frac{\alpha_1}{2}(\alpha_2 + 1)(\alpha_3 + 1), \ldots,$

$(\alpha_2 + 1)(\alpha_3 + 1), (\alpha_1 + 1)\alpha_2(\alpha_3 + 1) - 1,$

$\alpha_1\alpha_2(\alpha_3 + 1) - 1, \ldots, \frac{\alpha_1}{2}\alpha_2(\alpha_3 + 1), \ldots, \alpha_2(\alpha_3 + 1), \ldots$

$(\alpha_1 + 1)\frac{\alpha_2}{2}(\alpha_3 + 1), \alpha_1\frac{\alpha_2}{2}(\alpha_3 + 1), \ldots,$

$\frac{\alpha_1}{2}\frac{\alpha_2}{2}(\alpha_3 + 1), \ldots, \frac{\alpha_2}{2}(\alpha_3 + 1), \ldots (\alpha_1 + 1), \alpha_1, \ldots, \frac{\alpha_1}{2}, \ldots, 1$

(ii) When α_1, α_2, α_3 are odd

$s_2 : (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1) - 1,$

$\alpha_1(\alpha_2 + 1)(\alpha_3 + 1) - 1, \ldots, \left\lfloor\frac{\alpha_1}{2}\right\rfloor(\alpha_2 + 1)(\alpha_3 + 1), \ldots,$

$(\alpha_2 + 1)(\alpha_3 + 1)$, $(\alpha_1 + 1)\alpha_2(\alpha_3 + 1) - 1,$

$\alpha_1\alpha_2(\alpha_3 + 1) - 1, \ldots, \left\lfloor\frac{\alpha_1}{2}\right\rfloor\alpha_2(\alpha_3 + 1), \ldots, \alpha_2(\alpha_3 + 1), \ldots$

$(\alpha_1 + 1)\left\lfloor\frac{\alpha_2}{2}\right\rfloor(\alpha_3 + 1), \alpha_1\left\lfloor\frac{\alpha_2}{2}\right\rfloor(\alpha_3 + 1), \ldots,$

$\left\lfloor\frac{\alpha_1}{2}\right\rfloor\left\lfloor\frac{\alpha_2}{2}\right\rfloor(\alpha_3 + 1), \ldots, \left\lfloor\frac{\alpha_2}{2}\right\rfloor(\alpha_3 + 1),$

$\ldots(\alpha_1 + 1), \alpha_1, \ldots, \left\lfloor\frac{\alpha_1}{2}\right\rfloor, \ldots, 1.$

(iii) When exactly one α_i is odd, where $i = 1,2,3$.

$s_3 : (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1) - 1,$

$\alpha_1(\alpha_2 + 1)(\alpha_3 + 1) - 1, \ldots, \left\lfloor\frac{\alpha_1}{2}\right\rfloor(\alpha_2 + 1)(\alpha_3 + 1), \ldots,$

$(\alpha_2 + 1)(\alpha_3 + 1)$, $(\alpha_1 + 1)\alpha_2(\alpha_3 + 1) - 1,$

$\alpha_1\alpha_2(\alpha_3+1)-1,\dots,\left\lfloor\frac{\alpha_1}{2}\right\rfloor\alpha_2(\alpha_3+1),\dots,\alpha_2(\alpha_3+1),\dots$

$(\alpha_1+1)\frac{\alpha_2}{2}(\alpha_3+1),\alpha_1\frac{\alpha_2}{2}(\alpha_3+1),\dots,$

$\left\lfloor\frac{\alpha_1}{2}\right\rfloor\frac{\alpha_2}{2}(\alpha_3+1),\dots,\frac{\alpha_2}{2}(\alpha_3+1),\dots\ (\alpha_1+1),\ \alpha_1,\dots,\left\lfloor\frac{\alpha_1}{2}\right\rfloor,\dots,1$

and here α_1 is odd.

(iv) When exactly one α_i is even, where $i=1,2,3$.

$S_4:(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)-1,$

$\alpha_1(\alpha_2+1)(\alpha_3+1)-1,\dots,\frac{\alpha_1}{2}(\alpha_2+1)(\alpha_3+1),\dots,$

$(\alpha_2+1)(\alpha_3+1),(\alpha_1+1)\alpha_2(\alpha_3+1)-1,$

$\alpha_1\alpha_2(\alpha_3+1)-1,\dots,\frac{\alpha_1}{2}\alpha_2(\alpha_3+1),\dots,\alpha_2(\alpha_3+1),\dots$

$(\alpha_1+1)\left\lfloor\frac{\alpha_2}{2}\right\rfloor(\alpha_3+1),\alpha_1\left\lfloor\frac{\alpha_2}{2}\right\rfloor(\alpha_3+1),\dots,$

$\frac{\alpha_1}{2}\left\lfloor\frac{\alpha_2}{2}\right\rfloor(\alpha_3+1),\dots,\left\lfloor\frac{\alpha_2}{2}\right\rfloor(\alpha_3+1),\dots$

$(\alpha_1+1),\ \alpha_1,\dots,\frac{\alpha_1}{2},\dots,1$ and here α_1 is even.

Proof:

(i) Let $G=(V,E)$ be the Tri-factograph. We have the number of positive divisors of $p_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}$ is $(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)$ and hence the order of G is $(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)$.

Let $V(G)$={ $\{p_1^0p_2^0p_3^0,p_1^1p_2^0p_3^0,\dots p_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}\}$. This vertex set can be represented by the $(\alpha_2+1)(\alpha_3+1)\times(\alpha_1+1)$ array form as follows

$$\begin{pmatrix} p_1^0p_2^0p_3^0,p_1^1p_2^0p_3^0,\dots,p_1^{\alpha_1}p_2^0p_3^0 \\ p_1^0p_2^1p_3^0,p_1^1p_2^1p_3^0,\dots,p_1^{\alpha_1}p_2^1p_3^0 \\ \vdots \quad \vdots \quad \vdots \\ p_1^0p_2^{\alpha_2}p_3^{\alpha_3},p_1^1p_2^{\alpha_2}p_3^{\alpha_3},\dots,p_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}\end{pmatrix}_{(\alpha_2+1)(\alpha_3+1)\times(\alpha_1+1)}.$$

In a Tri-factograph G, we observe that for $i\neq l,j\neq m$ and $k\neq n$, the vertex $p_1^ip_2^jp_3^k$ is adjacent to $p_1^lp_2^mp_3^n$ in G if and only if $i+l\le\alpha_1,j+m\le\alpha_2$ and $k+n\le\alpha_3$. Consider the first (α_1+1) vertices $p_1^0p_2^0p_3^0,p_1^1p_2^0p_3^0,\dots,p_1^{\alpha_1}p_2^0p_3^0$, by our definition, $p_1^0p_2^0p_3^0$ adjacent with rest of the vertices which implies $d(p_1^0p_2^0p_3^0)=(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)-1$. We pick the second vertex $p_1^1p_2^0p_3^0$, exactly $\alpha_1(\alpha_2+1)(\alpha_3+1)$ combination of vertices satisfy our condition and among that $p_1^1p_2^0p_3^0$ adjacent to remaining vertices, which implies $d(p_1^1p_2^0p_3^0)=\alpha_1(\alpha_2+1)(\alpha_3+1)-1$. Consider $\left(\frac{\alpha_1}{2}+2\right)^{th}$ vertex $p_1^{\frac{\alpha}{2}+1}p_2^0p_3^0$, exactly $\frac{\alpha_1}{2}(\alpha_2+1)(\alpha_3+1)$ combination of vertices satisfy our adjacency condition, also $p_1^{\frac{\alpha_1}{2}+1}p_2^0p_3^0$ adjacent to $\frac{\alpha_1}{2}(\alpha_2+1)(\alpha_3+1)$ vertices. Therefore, $d\left(p_1^{\frac{\alpha_1}{2}+1}p_2^0p_3^0\right)=\frac{\alpha_1}{2}(\alpha_2+1)(\alpha_3+1)$. Continuing like that $p_1^{\alpha_1}p_2^0p_3^0$ adjacent with $(\alpha_2+1)(\alpha_3+1)$ vertices implies $d(p_1^{\alpha_1}p_2^0p_3^0)=(\alpha_2+1)(\alpha_3+1)$. Consider the second (α_1+1) vertices $p_1^0p_2^1p_3^0,\ p_1^1p_2^1p_3^0,\ p_1^2p_3^1p_3^0,\dots,\ p_1^{\alpha_1}p_3^1p_3^0$ have degree sequence as follows

$d(p_1^0p_2^0p_3^0)=(\alpha_1+1)\alpha_2(\alpha_3+1)-1,$

$d\ (p_1^1p_2^1p_3^0)\ =\ \alpha_1\alpha_2(\alpha_3+1)-1,\dots,d\left(p_1^{\frac{\alpha_1}{2}+1}\ p_2^1p_3^0\right)=$
$\frac{\alpha_1}{2}\ \alpha_2\ (\alpha_3+1),\dots,d\ (p_1^{\alpha_1}p_2^1p_3^0)=\alpha_2(\alpha_3+1).$

Take $\left(\frac{\alpha_2}{2}+2\right)^{th}$ (α_1+1) vertices have degree sequence $d\left(p_1^0p_2^{\frac{\alpha_2}{2}+1}p_3^0\right)$

$=(\alpha_1+1)\frac{\alpha_2}{2}(\alpha_3+1),d\left(p_1^1p_2^{\frac{\alpha_2}{2}+1}p_3^0\right)$

$=\alpha_1\frac{\alpha_2}{2}(\alpha_3+1),\dots,d\left(p_1^{\alpha_1}p_2^{\frac{\alpha_2}{2}+1}p_3^0\right)=\frac{\alpha_2}{2}(\alpha_3+1).$

Also $(\alpha_2+1)^{th}$ (α_1+1) vertices have degree sequence $d(p_1^0p_2^{\alpha_2}p_3^0)=(\alpha_1+1)(\alpha_3+1),d(p_1^1p_2^{\alpha_2}p_3^0)=\alpha_1(\alpha_3+1),\dots,d(p_1^{\alpha_1}p_2^{\alpha_2}p_3^0)=(\alpha_3+1).$

Finally, $(\alpha_2+1)(\alpha_3+1)^{th}$ (α_1+1) vertices have degree sequence $d(p_1^0p_2^{\alpha_2}p_3^{\alpha_3})=(\alpha_1+1),d(p_1^1p_2^{\alpha_2}p_3^{\alpha_3})=\alpha_1,\dots,$ $d(p_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3})=1.$

Hence the theorem. Similarly we can prove (ii), (iii) and (iv) parts.

C. Theorem

The clique number of a Tri-factograph G is

(i) $\omega(G)=\left(\frac{\alpha_1}{2}+1\right)\left(\frac{\alpha_2}{2}+1\right)\left(\frac{\alpha_3}{2}+1\right)$

when $\alpha_1,\alpha_2,\alpha_3$ are even.

(ii) $\omega(G)=\left(\left\lfloor\frac{\alpha_1}{2}\right\rfloor+1\right)\left(\left\lfloor\frac{\alpha_2}{2}\right\rfloor+1\right)\left(\left\lfloor\frac{\alpha_3}{2}\right\rfloor+1\right)+3$

when $\alpha_1,\alpha_2,\alpha_3$ are odd.

Proof:

Let $G=(V,E)$ be a Tri-factograph with

$z = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}$ and $V = \{p_1^0 p_2^0 p_3^0, p_1^1 p_2^0 p_3^0, \dots, p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}\}$ be the vertex set of G. We consider the set $S = \left\{p_1^x p_2^y p_3^z / 0 \le x \le \frac{\alpha_1}{2}, 0 \le y \le \frac{\alpha_2}{2} \text{ and } 0 \le z \le \frac{\alpha_3}{2}\right\}$ proper subset of V. We seek to prove that the subgraph induced by S is the maximal clique of G.

Claim : $< S >$ is a clique of G.

In G, we observe that for $i \neq l, j \neq m$ and $j \neq m$, the vertex $p_1^i p_2^j p_3^k$ is adjacent with $p_1^l p_2^m p_3^n$ in G if and only if $i + l \le \alpha_1, j + m \le \alpha_2$ and $k + m \le \alpha_3$.It is enough to prove that every pair of distinct vertices in S is adjacent. Take any two arbitrary vertices in S such as $p_1^a p_2^b p_3^c$ and $p_1^d p_2^e p_3^f$. Since $a + d \le \alpha_1$, $b + e \le \alpha_2$ and $c + f \le \alpha_3$ implies $p_1^a p_2^b p_3^c$ and $p_1^d p_2^e p_3^f$ are adjacent. Which gives each pair of distinct vertices in S is adjacent implies $< S >$ is a clique of G. It remains to show that $< S >$ is the maximal clique of G. Suppose we take any arbitrary vertex v in V which is not in S, by our factograph condition $< S + \{v\} >$ cannot be a clique of G. Therefore, $< S >$ is the maximal clique of G. Also $\omega(G) = |S| = \left(\frac{\alpha_1}{2} + 1\right)\left(\frac{\alpha_2}{2} + 1\right)\left(\frac{\alpha_3}{2} + 1\right)$.

(ii) When $\alpha_1, \alpha_2, \alpha_3$ are odd.

Consider the set $S = \left\{p_1^x p_2^y p_3^z / 0 \le x \le \left\lfloor\frac{\alpha_1}{2}\right\rfloor, 0 \le y \le\right.$ *α22* and *0≤z≤α32∪p1α12p20p30, p10p2α22 p30, p10p20p3α32* proper subset of V. We seek to prove that the subgraph of G induced by S is the maximal clique of G. To prove that every pair of distinct vertices in S is adjacent, take any two arbitrary vertices $p_1^a p_2^b p_3^c$ and $p_1^d p_2^e p_3^f$ in S. Since $a + d \le \alpha_1$, $b + e \le \alpha_2$ and $c + f \le \alpha_3$ is the condition, $p_1^a p_2^b p_3^c$ and $p_1^d p_2^e p_3^f$ are adjacent.

Also, $< S + \{v\} >$ where $v \in V \backslash S$ in not a clique of G implies $< S >$ is the maximal clique of G and $\omega(S) = |S| = \left(\left\lfloor\frac{\alpha_1}{2}\right\rfloor + 1\right)\left(\left\lfloor\frac{\alpha_2}{2}\right\rfloor + 1\right)\left(\left\lfloor\frac{\alpha_3}{2}\right\rfloor + 1\right) + 3$.

D. Remark

When exactly one of α_i is odd where $i = 1,2,3$, then the clique number of a Tri-factograph G is

$\omega(G) = \left(\left\lfloor\frac{\alpha_1}{2}\right\rfloor + 1\right)\left(\frac{\alpha_2}{2} + 1\right)\left(\frac{\alpha_3}{2} + 1\right) + 1$, here α_1 is odd .

E. Example

Consider G with $z = p_1^1 p_2^1 p_3^2$. Then the order of G is 12.

G:

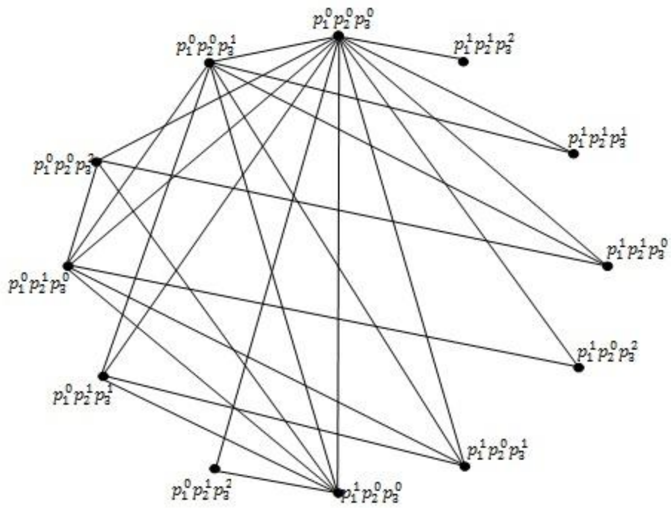

Figure:2

We observe that, the degree sequence of G is s: 11, 7, 4, 6, 4, 2, 6, 4, 2, 3, 2, 1 and $\omega(G)$ is 4.

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