

Degree Sequence and Clique Number of Bi-Factograph and Tri-Factograph

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Abstract— By the theorem of unique factorization for integers, every positive integer z can be written in the form $z = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_r^{\alpha_r}$, where p_1, p_2, \dots, p_r are distinct primes, $\alpha_1, \alpha_2, \dots, \alpha_r$ are positive integers. We can construct a graph G which is associated with this z . Positive integral divisors of z being a vertex set V and two distinct vertices of V are adjacent in G if their product is in V . In z , when $r = 1$ then the corresponding graph is called the perfect factograph. Here we extend the concept to $r = 2, 3$ and the corresponding graphs are called Bi-factograph and Tri-factograph respectively. In this paper we attempt to find the degree sequence and clique number of Bi-factograph and Tri-factograph.

Keywords— Factograph, Perfect factograph, Bi-factograph, Tri-factograph, clique number.

I. INTRODUCTION

By a graph, we mean a finite undirected, non-trivial graph without loops and multiple edges. The order and size of a graph is denoted by p and q respectively. For terms not defined, we refer to Frank Harary [4]. The concept of factograph and perfect factograph was introduced by E. Giftin Vedha Merly and N. Gnanadhas [1],[2]. In this paper we extend the concept to Bi-factograph and Tri-factograph. For a positive integer z , we define a factograph as $G = (V, E)$ where $V = \{v_i/v_i \text{ is a factor of } z\}$ and two distinct vertices v_i and v_j are adjacent if and only if their product is in V . A clique of a graph G is a complete subgraph of G . A clique of G is the maximal clique, if it is not properly contained in another clique of G . Number of vertices in maximal clique of G is called the clique number of G and is denoted by $\omega(G)$. For $v \in V, d(v)$ is the number of edges incident with v . A factograph G with $z = p_1^{\alpha_1}$, where p is a prime and α is any positive integer is called a perfect factograph.

II. DEGREE SEQUENCE AND CLIQUE NUMBER OF BI-FACTOGRAPH.

A. Definition

A factograph G with $z = p_1^{\alpha_1} p_2^{\alpha_2}$ where p_1, p_2 are distinct primes and α_1, α_2 are positive integers is called a Bi-factograph.

B. Theorem

Let α_1 and α_2 be two positive integers, p_1 and p_2 be two distinct primes. A Bi-factograph G with $z = P_1^{\alpha_1} P_2^{\alpha_2}$ has order $(\alpha_1 + 1)(\alpha_2 + 1)$ and the degree sequence of G is given by

$$\begin{aligned} \text{i) } s_1: & (\alpha_1 + 1)(\alpha_2 + 1) - 1, (\alpha_1 + 1)\alpha_2 - 1, \dots \\ & (\alpha_1 + 1)\frac{\alpha_2}{2}, \dots, \alpha_1 + 1, \alpha_1(\alpha_2 + 1) - 1, \\ & \alpha_1\alpha_2 - 1, \dots, \frac{\alpha_1\alpha_2}{2}, \dots, \alpha_1, \\ & \dots, \frac{\alpha_1}{2}(\alpha_2 + 1), \frac{\alpha_1\alpha_2}{2}, \dots, \frac{\alpha_1\alpha_2}{4}, \dots, \frac{\alpha_1}{2}, \dots, \alpha_2 + 1, \alpha_2, \dots, \frac{\alpha_2}{2}, \dots, 1, \end{aligned}$$

where α_1 and α_2 are even.

$$\text{ii) } s_2: (\alpha_1 + 1)(\alpha_2 + 1) - 1,$$

$$\begin{aligned}
 &(\alpha_1 + 1)\alpha_2 - 1, \dots, (\alpha_1 + 1) \left\lfloor \frac{\alpha_2}{2} \right\rfloor, \dots, \alpha_1 + 1, \\
 &\alpha_1(\alpha_2 + 1) - 1, \alpha_1\alpha_2 - 1, \dots, \alpha_1 \left\lfloor \frac{\alpha_2}{2} \right\rfloor, \dots, \alpha_1, \\
 &\dots \left\lfloor \frac{\alpha_1}{2} \right\rfloor (\alpha_2 + 1), \left\lfloor \frac{\alpha_1}{2} \right\rfloor \alpha_2, \dots, \left\lfloor \frac{\alpha_1}{2} \right\rfloor \left\lfloor \frac{\alpha_2}{2} \right\rfloor, \\
 &\dots \left\lfloor \frac{\alpha_1}{2} \right\rfloor, \dots, \alpha_2 + 1, \alpha_2, \dots, \left\lfloor \frac{\alpha_2}{2} \right\rfloor, \dots, 1, \text{ where } \alpha_1 \text{ and } \alpha_2 \text{ are odd.}
 \end{aligned}$$

iii) $s_3 : (\alpha_1 + 1)(\alpha_2 + 1) - 1, (\alpha_1 + 1)\alpha_2 - 1, \dots$

$$(\alpha_1 + 1) \frac{\alpha_2}{2}, \dots, \alpha_1 + 1, \alpha_1(\alpha_2 + 1) - 1,$$

$$\begin{aligned}
 &\alpha_1\alpha_2 - 1, \dots, \frac{\alpha_1\alpha_2}{2}, \dots, \alpha_1, \dots \\
 &\left\lfloor \frac{\alpha_1}{2} \right\rfloor (\alpha_2 + 1), \left\lfloor \frac{\alpha_1}{2} \right\rfloor \alpha_2, \dots, \left\lfloor \frac{\alpha_1}{2} \right\rfloor \frac{\alpha_2}{2}, \dots, \left\lfloor \frac{\alpha_1}{2} \right\rfloor \dots, \alpha_2 + 1, \alpha_2, \dots, \frac{\alpha_2}{2}, \dots, 1.
 \end{aligned}$$

where α_1 is odd and α_2 is even.

iv) $s_4 : (\alpha_1 + 1)(\alpha_2 + 1) - 1, (\alpha_1 + 1)\alpha_2 - 1, \dots$

$$(\alpha_1 + 1) \left\lfloor \frac{\alpha_2}{2} \right\rfloor, \dots, \alpha_1 + 1, \alpha_1(\alpha_2 + 1) - 1,$$

$$\alpha_1\alpha_2 - 1, \dots, \alpha_1 \left\lfloor \frac{\alpha_2}{2} \right\rfloor, \dots, \alpha_1, \dots$$

$$\frac{\alpha_1}{2}(\alpha_2 + 1), \frac{\alpha_1}{2}\alpha_2, \dots, \frac{\alpha_1}{2} \left\lfloor \frac{\alpha_2}{2} \right\rfloor, \dots, \frac{\alpha_1}{2}, \dots, \alpha_2 + 1, \alpha_2, \dots, \left\lfloor \frac{\alpha_2}{2} \right\rfloor, \dots, 1,$$

where α_1 is even and α_2 is odd.

Proof:

i) When α_1 and α_2 are even, let $G = (V, E)$ be a Bi-factograph. If $z = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_r^{\alpha_r}$ then the number of positive divisors of z is $\prod_{i=1}^r (\alpha_i + 1)$. Therefore, number of positive divisors of $z = p_1^{\alpha_1} p_2^{\alpha_2}$ is $(\alpha_1 + 1)(\alpha_2 + 1)$ so that the order of G is $(\alpha_1 + 1)(\alpha_2 + 1)$. Let $V = \{ p_1^0 p_2^0, p_1^0 p_2^1, \dots, p_1^{\alpha_1} p_2^{\alpha_2} \}$.

This vertex set can be represented by the $(\alpha_1 + 1) \times (\alpha_2 + 1)$ array form as follows

$$\begin{pmatrix} p_1^0 p_2^0, & p_1^0 p_2^1, & p_1^0 p_2^2, & \dots & p_1^0 p_2^{\alpha_2} \\ p_1^1 p_2^0, & p_1^1 p_2^1, & p_1^1 p_2^2, & \dots & p_1^1 p_2^{\alpha_2} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ p_1^{\alpha_1} p_2^0, & p_1^{\alpha_1} p_2^1, & p_1^{\alpha_1} p_2^2, & \dots & p_1^{\alpha_1} p_2^{\alpha_2} \end{pmatrix}_{(\alpha_1+1) \times (\alpha_2+1)}.$$

In a Bi-factograph G , we observe that for $i \neq k$ and $j \neq l$ then the vertex $p_1^i p_2^j$ is adjacent to $p_1^k p_2^l$ in G if and only if $i + k \leq \alpha_1$ and $j + l \leq \alpha_2$. Consider the first $(\alpha_2 + 1)$ vertices, $p_1^0 p_2^0, p_1^0 p_2^1, p_1^0 p_2^2, \dots, p_1^0 p_2^{\alpha_2}$. By our factograph condition, it is obvious that $p_1^0 p_2^0$ is adjacent to rest of the vertices implies $d(p_1^0 p_2^0) = (\alpha_1 + 1)(\alpha_2 + 1) - 1$. Pick the second

vertex $p_1^0 p_2^1$, exactly $(\alpha_1 + 1)\alpha_2$ combination of vertices satisfy the condition and among that $p_1^0 p_2^1$ is adjacent to remaining vertices which implies $d(p_1^0 p_2^1) = (\alpha_1 + 1)\alpha_2 - 1$.

Consider the vertex $p_1^0 p_2^{\frac{\alpha_2}{2}+1}$, exactly $(\alpha_1 + 1) \left(\frac{\alpha_2}{2}\right)$ combination of vertices satisfy the condition of adjacency, among that the vertex $p_1^0 p_2^{\frac{\alpha_2}{2}+1}$ is not in that combination which gives $d(p_1^0 p_2^{\frac{\alpha_2}{2}+1}) = (\alpha_1 + 1) \left(\frac{\alpha_2}{2}\right)$. Continuing this

way, the vertex $p_1^0 p_2^{\alpha_2}$ is adjacent to $(\alpha_1 + 1)$ vertices and $d(p_1^0 p_2^{\alpha_2}) = (\alpha_1 + 1)$. Consider $\left(\frac{\alpha_1}{2} + 2\right)^{\text{th}}$ row of $\alpha_2 + 1$ vertices $p_1^{\frac{\alpha_2}{2}+1} p_2^0, p_1^{\frac{\alpha_2}{2}+1} p_2^1, \dots, \dots, p_1^{\frac{\alpha_2}{2}+1} p_2^{\alpha_2}$ have the degree sequence $d(p_1^{\frac{\alpha_2}{2}+1} p_2^0) = \left(\frac{\alpha_1}{2}\right)(\alpha_2 + 1), d(p_1^{\frac{\alpha_2}{2}+1} p_2^1) =$

$\left(\frac{\alpha_1}{2}\right)\alpha_2, \dots, d(p_1^{\frac{\alpha_2}{2}+1} p_2^{\alpha_2}) = \frac{\alpha_1}{2}$. Finally, $(\alpha_1 + 1)^{\text{th}}$ row of $(\alpha_2 + 1)$ vertices $p_1^{\alpha_1} p_2^0, p_1^{\alpha_1} p_2^1, p_1^{\alpha_1} p_2^2, \dots, \dots, p_1^{\alpha_1} p_2^{\alpha_2}$ have degree sequence

$$d(p_1^{\alpha_1} p_2^0) = \alpha_2 + 1, d(p_1^{\alpha_1} p_2^1) = \alpha_2, \dots, \dots, d(p_1^{\alpha_1} p_2^{\alpha_2}) = 1.$$

Therefore, G has $(\alpha_1 + 1)(\alpha_2 + 1)$ vertices and has the degree sequence,

$$\begin{aligned}
 s_1 : &(\alpha_1 + 1)(\alpha_2 + 1) - 1, (\alpha_1 + 1)\alpha_2 - 1, \dots \\
 &(\alpha_1 + 1) \frac{\alpha_2}{2}, \dots, \alpha_1 + 1, \alpha_1(\alpha_2 + 1) - 1, \alpha_1\alpha_2 - 1, \\
 &\dots, \frac{\alpha_1\alpha_2}{2}, \dots, \alpha_1, \dots, \frac{\alpha_1}{2}(\alpha_2 + 1), \frac{\alpha_1\alpha_2}{2}, \dots, \frac{\alpha_1\alpha_2}{4}, \dots, \frac{\alpha_1}{2}, \\
 &\dots, \alpha_2 + 1, \alpha_2, \dots, \frac{\alpha_2}{2}, \dots, 1.
 \end{aligned}$$

Similar manner we can prove case (ii) (iii) and (iv).

C. Theorem

The clique number of a Bi-factograph G is

i) $\omega(G) = \left(\frac{\alpha_1}{2} + 1\right) \left(\frac{\alpha_2}{2} + 1\right)$ when α_1 and α_2 are even.

ii) $\omega(G) = \left(\left\lfloor \frac{\alpha_1}{2} \right\rfloor + 1\right) \left(\left\lfloor \frac{\alpha_2}{2} \right\rfloor + 1\right) + 2$ when α_1 and α_2 are odd.

Proof:

i) If α_1 and α_2 are even, let $G = (V, E)$ be a Bi-factograph and $V = \{ p_1^0 p_2^0, p_1^0 p_2^1, \dots, \dots, \dots, p_1^{\alpha_1} p_2^{\alpha_2} \}$ be the vertex set of G . We consider the set $S = \{ p_1^x p_2^y / 0 \leq x \leq \frac{\alpha_1}{2} \text{ and } 0 \leq y \leq \frac{\alpha_2}{2} \}$ which is a proper

subset of V . We seek to prove that the subgraph of G induced by S is the maximal clique of G .

In G , we observe that for $i \neq k$ and $j \neq l$, the vertex $p_1^i p_2^j$ is adjacent to $p_1^k p_2^l$ in G if and only if $i + k \leq \alpha_1$ and $j + l \leq \alpha_2$. We have to prove that every pair of distinct vertices in S are adjacent. Take any two arbitrary vertices $p_1^a p_2^b$ and $p_1^c p_2^d$ in S . Since $a + c \leq \alpha_1$ and $b + d \leq \alpha_2$, we have $p_1^a p_2^b$ is adjacent to $p_1^c p_2^d$. Therefore each pair of distinct vertices in S are adjacent, which implies $\langle S \rangle$ is a clique of G . It remains to show that $\langle S \rangle$ is the maximal clique of G . Suppose we take any arbitrary vertex v in V which is not in S .

Since Bi-factograph, a vertex v cannot be adjacent with all vertices of S implies $\langle S + \{v\} \rangle$ cannot be the clique of G . Therefore $\langle S \rangle$ is the maximal clique of G and $\omega(G) = |S| = \left(\frac{\alpha_1}{2} + 1\right) \left(\frac{\alpha_2}{2} + 1\right)$.

Similarly we can prove case (ii).

D. Remark

(i) When α_1 is odd and α_2 is even, then the clique number of a Bi-factograph G is

$$\omega(G) = \left(\left\lfloor \frac{\alpha_1}{2} \right\rfloor + 1\right) \left(\frac{\alpha_2}{2} + 1\right) + 1.$$

(ii) When α_2 is odd and α_1 is even, then the clique number of a Bi-factograph G is

$$\omega(G) = \left(\frac{\alpha_1}{2} + 1\right) \left(\left\lfloor \frac{\alpha_2}{2} \right\rfloor + 1\right) + 1.$$

E. Example

Consider a Bi-factograph G with $z = p_1^2 p_2^2$. Here order of G is 9.

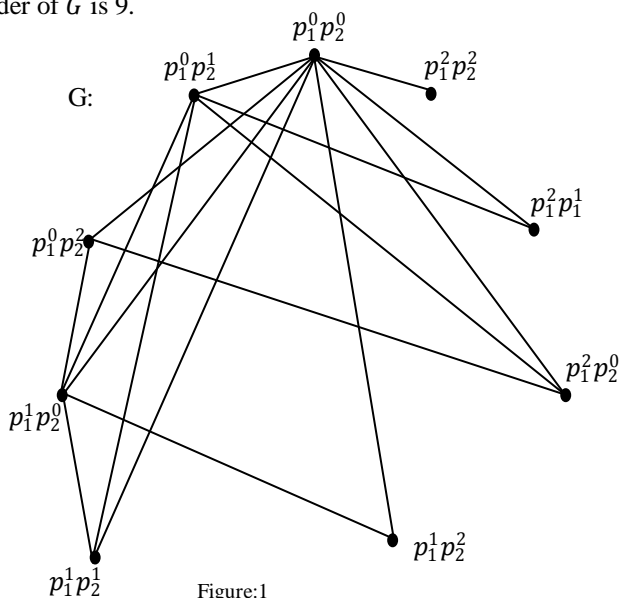


Figure:1

We observe that, the degree sequence of G is $s: 8,5,3,5,3,2,3,2,1$ and $\omega(G)$ is 4.

III. DEGREE SEQUENCE AND CLIQUE NUMBER OF TRI-FACTOGRAPH

A. Definition

A factograph G with $z = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}$, where p_1, p_2, p_3 are distinct primes and $\alpha_1, \alpha_2, \alpha_3$ any positive integers is called a Tri-factograph.

B. Theorem

Let α_1, α_2 and α_3 be any positive integers, p_1, p_2 and p_3 be any distinct primes, G is a Tri-factograph with

$z = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}$, then G is of order

$(\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1)$ and the degree sequence of G is given by,

(i) When $\alpha_1, \alpha_2, \alpha_3$ are even

$$s_1 : (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1) - 1, \alpha_1(\alpha_2 + 1)(\alpha_3 + 1) - 1, \dots, \frac{\alpha_1}{2}(\alpha_2 + 1)(\alpha_3 + 1), \dots, (\alpha_2 + 1)(\alpha_3 + 1), (\alpha_1 + 1)\alpha_2(\alpha_3 + 1) - 1, \alpha_1\alpha_2(\alpha_3 + 1) - 1, \dots, \frac{\alpha_1}{2}\alpha_2(\alpha_3 + 1), \dots, \alpha_2(\alpha_3 + 1), \dots, (\alpha_1 + 1)\frac{\alpha_2}{2}(\alpha_3 + 1), \alpha_1\frac{\alpha_2}{2}(\alpha_3 + 1), \dots, \frac{\alpha_1}{2}\frac{\alpha_2}{2}(\alpha_3 + 1), \dots, \frac{\alpha_2}{2}(\alpha_3 + 1), \dots, (\alpha_1 + 1), \alpha_1, \dots, \frac{\alpha_1}{2}, \dots, 1$$

(ii) When $\alpha_1, \alpha_2, \alpha_3$ are odd

$$s_2 : (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1) - 1, \alpha_1(\alpha_2 + 1)(\alpha_3 + 1) - 1, \dots, \left\lfloor \frac{\alpha_1}{2} \right\rfloor(\alpha_2 + 1)(\alpha_3 + 1), \dots, (\alpha_2 + 1)(\alpha_3 + 1), (\alpha_1 + 1)\alpha_2(\alpha_3 + 1) - 1, \alpha_1\alpha_2(\alpha_3 + 1) - 1, \dots, \left\lfloor \frac{\alpha_1}{2} \right\rfloor\alpha_2(\alpha_3 + 1), \dots, \alpha_2(\alpha_3 + 1), \dots, (\alpha_1 + 1)\left\lfloor \frac{\alpha_2}{2} \right\rfloor(\alpha_3 + 1), \alpha_1\left\lfloor \frac{\alpha_2}{2} \right\rfloor(\alpha_3 + 1), \dots, \left\lfloor \frac{\alpha_1}{2} \right\rfloor\left\lfloor \frac{\alpha_2}{2} \right\rfloor(\alpha_3 + 1), \dots, \left\lfloor \frac{\alpha_2}{2} \right\rfloor(\alpha_3 + 1), \dots, (\alpha_1 + 1), \alpha_1, \dots, \left\lfloor \frac{\alpha_1}{2} \right\rfloor, \dots, 1.$$

(iii) When exactly one α_i is odd, where $i = 1,2,3$.

$$s_3 : (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1) - 1, \alpha_1(\alpha_2 + 1)(\alpha_3 + 1) - 1, \dots, \left\lfloor \frac{\alpha_1}{2} \right\rfloor(\alpha_2 + 1)(\alpha_3 + 1), \dots, (\alpha_2 + 1)(\alpha_3 + 1), (\alpha_1 + 1)\alpha_2(\alpha_3 + 1) - 1,$$

$\alpha_1 \alpha_2 (\alpha_3 + 1) - 1, \dots, \left\lfloor \frac{\alpha_1}{2} \right\rfloor \alpha_2 (\alpha_3 + 1), \dots, \alpha_2 (\alpha_3 + 1), \dots$
 $(\alpha_1 + 1) \frac{\alpha_2}{2} (\alpha_3 + 1), \alpha_1 \frac{\alpha_2}{2} (\alpha_3 + 1), \dots,$
 $\left\lfloor \frac{\alpha_1}{2} \right\rfloor \frac{\alpha_2}{2} (\alpha_3 + 1), \dots, \frac{\alpha_2}{2} (\alpha_3 + 1), \dots (\alpha_1 + 1), \alpha_1, \dots, \left\lfloor \frac{\alpha_1}{2} \right\rfloor, \dots, 1$
 and here α_1 is odd.

(iv) When exactly one α_i is even, where $i = 1, 2, 3$.

$S_4 : (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1) - 1,$
 $\alpha_1 (\alpha_2 + 1)(\alpha_3 + 1) - 1, \dots, \frac{\alpha_1}{2} (\alpha_2 + 1)(\alpha_3 + 1), \dots,$
 $(\alpha_2 + 1)(\alpha_3 + 1), (\alpha_1 + 1)\alpha_2 (\alpha_3 + 1) - 1,$
 $\alpha_1 \alpha_2 (\alpha_3 + 1) - 1, \dots, \frac{\alpha_1}{2} \alpha_2 (\alpha_3 + 1), \dots, \alpha_2 (\alpha_3 + 1), \dots$
 $(\alpha_1 + 1) \left\lfloor \frac{\alpha_2}{2} \right\rfloor (\alpha_3 + 1), \alpha_1 \left\lfloor \frac{\alpha_2}{2} \right\rfloor (\alpha_3 + 1), \dots,$
 $\frac{\alpha_1}{2} \left\lfloor \frac{\alpha_2}{2} \right\rfloor (\alpha_3 + 1), \dots, \left\lfloor \frac{\alpha_2}{2} \right\rfloor (\alpha_3 + 1), \dots$
 $(\alpha_1 + 1), \alpha_1, \dots, \frac{\alpha_1}{2}, \dots, 1$ and here α_1 is even.

Proof:

(i) Let $G = (V, E)$ be the Tri-factograph. We have the number of positive divisors of $p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}$ is $(\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1)$ and hence the order of G is $(\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1)$.

Let $V(G) = \{p_1^0 p_2^0 p_3^0, p_1^1 p_2^0 p_3^0, \dots, p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}\}$. This vertex set can be represented by the

$(\alpha_2 + 1)(\alpha_3 + 1) \times (\alpha_1 + 1)$ array form as follows

$$\begin{pmatrix} p_1^0 p_2^0 p_3^0, p_1^1 p_2^0 p_3^0, \dots, p_1^{\alpha_1} p_2^0 p_3^0 \\ p_1^0 p_2^1 p_3^0, p_1^1 p_2^1 p_3^0, \dots, p_1^{\alpha_1} p_2^1 p_3^0 \\ \vdots \\ p_1^0 p_2^{\alpha_2} p_3^0, p_1^1 p_2^{\alpha_2} p_3^0, \dots, p_1^{\alpha_1} p_2^{\alpha_2} p_3^0 \end{pmatrix}_{(\alpha_2+1)(\alpha_3+1) \times (\alpha_1+1)}$$

In a Tri-factograph G , we observe that for $i \neq l, j \neq m$ and $k \neq n$, the vertex $p_1^i p_2^j p_3^k$ is adjacent to $p_1^l p_2^m p_3^n$ in G if and only if $i + l \leq \alpha_1, j + m \leq \alpha_2$ and $k + n \leq \alpha_3$. Consider the first $(\alpha_1 + 1)$ vertices $p_1^0 p_2^0 p_3^0, p_1^1 p_2^0 p_3^0, \dots, p_1^{\alpha_1} p_2^0 p_3^0$, by our definition, $p_1^0 p_2^0 p_3^0$ adjacent with rest of the vertices which implies $d(p_1^0 p_2^0 p_3^0) = (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1) - 1$. We pick the second vertex $p_1^1 p_2^0 p_3^0$, exactly $\alpha_1 (\alpha_2 + 1)(\alpha_3 + 1)$ combination of vertices satisfy our condition and among that $p_1^1 p_2^0 p_3^0$ adjacent to remaining vertices, which implies $d(p_1^1 p_2^0 p_3^0) = \alpha_1 (\alpha_2 + 1)(\alpha_3 + 1) - 1$. Consider $\left(\frac{\alpha_1}{2} + 2\right)^{th}$

vertex $p_1^{\frac{\alpha_1}{2}+1} p_2^0 p_3^0$, exactly $\frac{\alpha_1}{2} (\alpha_2 + 1)(\alpha_3 + 1)$ combination of vertices satisfy our adjacency condition, also $p_1^{\frac{\alpha_1}{2}+1} p_2^0 p_3^0$ adjacent to $\frac{\alpha_1}{2} (\alpha_2 + 1)(\alpha_3 + 1)$ vertices. Therefore,

$$d\left(p_1^{\frac{\alpha_1}{2}+1} p_2^0 p_3^0\right) = \frac{\alpha_1}{2} (\alpha_2 + 1)(\alpha_3 + 1).$$

Continuing like that $p_1^{\alpha_1} p_2^0 p_3^0$ adjacent with $(\alpha_2 + 1)(\alpha_3 + 1)$ vertices implies $d(p_1^{\alpha_1} p_2^0 p_3^0) = (\alpha_2 + 1)(\alpha_3 + 1)$. Consider the second $(\alpha_1 + 1)$ vertices $p_1^0 p_2^1 p_3^0, p_1^1 p_2^1 p_3^0, p_1^2 p_2^1 p_3^0, \dots, p_1^{\alpha_1} p_2^1 p_3^0$ have degree sequence as follows

$$d(p_1^0 p_2^1 p_3^0) = (\alpha_1 + 1)\alpha_2 (\alpha_3 + 1) - 1,$$

$$d(p_1^1 p_2^1 p_3^0) = \alpha_1 \alpha_2 (\alpha_3 + 1) - 1, \dots, d\left(p_1^{\frac{\alpha_1}{2}+1} p_2^1 p_3^0\right) = \frac{\alpha_1}{2} \alpha_2 (\alpha_3 + 1), \dots, d\left(p_1^{\alpha_1} p_2^1 p_3^0\right) = \alpha_2 (\alpha_3 + 1).$$

Take $\left(\frac{\alpha_2}{2} + 2\right)^{th}$ $(\alpha_1 + 1)$ vertices have degree sequence

$$d\left(p_1^0 p_2^{\frac{\alpha_2}{2}+1} p_3^0\right) = (\alpha_1 + 1) \frac{\alpha_2}{2} (\alpha_3 + 1), d\left(p_1^1 p_2^{\frac{\alpha_2}{2}+1} p_3^0\right) = \alpha_1 \frac{\alpha_2}{2} (\alpha_3 + 1), \dots, d\left(p_1^{\alpha_1} p_2^{\frac{\alpha_2}{2}+1} p_3^0\right) = \frac{\alpha_2}{2} (\alpha_3 + 1).$$

Also $(\alpha_2 + 1)^{th}$ $(\alpha_1 + 1)$ vertices have degree sequence $d(p_1^0 p_2^{\alpha_2} p_3^0) = (\alpha_1 + 1)(\alpha_3 + 1), d(p_1^1 p_2^{\alpha_2} p_3^0) = \alpha_1 (\alpha_3 + 1), \dots, d(p_1^{\alpha_1} p_2^{\alpha_2} p_3^0) = (\alpha_3 + 1)$.

Finally, $(\alpha_2 + 1)(\alpha_3 + 1)^{th}$ $(\alpha_1 + 1)$ vertices have degree sequence $d(p_1^0 p_2^{\alpha_2} p_3^{\alpha_3}) = (\alpha_1 + 1), d(p_1^1 p_2^{\alpha_2} p_3^{\alpha_3}) = \alpha_1, \dots, d(p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}) = 1$.

Hence the theorem. Similarly we can prove (ii), (iii) and (iv) parts.

C. Theorem

The clique number of a Tri-factograph G is

- (i) $\omega(G) = \left(\frac{\alpha_1}{2} + 1\right) \left(\frac{\alpha_2}{2} + 1\right) \left(\frac{\alpha_3}{2} + 1\right)$ when $\alpha_1, \alpha_2, \alpha_3$ are even.
- (ii) $\omega(G) = \left(\left\lfloor \frac{\alpha_1}{2} \right\rfloor + 1\right) \left(\left\lfloor \frac{\alpha_2}{2} \right\rfloor + 1\right) \left(\left\lfloor \frac{\alpha_3}{2} \right\rfloor + 1\right) + 3$ when $\alpha_1, \alpha_2, \alpha_3$ are odd.

Proof:

Let $G = (V, E)$ be a Tri-factograph with

$z = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}$ and $V = \{p_1^0 p_2^0 p_3^0, p_1^1 p_2^0 p_3^0, \dots, p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}\}$ be the vertex set of G . We consider the set $S = \{p_1^x p_2^y p_3^z / 0 \leq x \leq \frac{\alpha_1}{2}, 0 \leq y \leq \frac{\alpha_2}{2} \text{ and } 0 \leq z \leq \frac{\alpha_3}{2}\}$ proper subset of V . We seek to prove that the subgraph induced by S is the maximal clique of G .

Claim : $\langle S \rangle$ is a clique of G .

In G , we observe that for $i \neq l, j \neq m$ and $j \neq m$, the vertex $p_1^i p_2^j p_3^k$ is adjacent with $p_1^l p_2^m p_3^n$ in G if and only if $i + l \leq \alpha_1, j + m \leq \alpha_2$ and $k + n \leq \alpha_3$. It is enough to prove that every pair of distinct vertices in S is adjacent. Take any two arbitrary vertices in S such as $p_1^a p_2^b p_3^c$ and $p_1^d p_2^e p_3^f$. Since $a + d \leq \alpha_1, b + e \leq \alpha_2$ and $c + f \leq \alpha_3$ implies $p_1^a p_2^b p_3^c$ and $p_1^d p_2^e p_3^f$ are adjacent. Which gives each pair of distinct vertices in S is adjacent implies $\langle S \rangle$ is a clique of G . It remains to show that $\langle S \rangle$ is the maximal clique of G . Suppose we take any arbitrary vertex v in V which is not in S , by our factograph condition $\langle S + \{v\} \rangle$ cannot be a clique of G . Therefore, $\langle S \rangle$ is the maximal clique of G . Also $\omega(G) = |S| = \left(\frac{\alpha_1}{2} + 1\right) \left(\frac{\alpha_2}{2} + 1\right) \left(\frac{\alpha_3}{2} + 1\right)$.

(ii) When $\alpha_1, \alpha_2, \alpha_3$ are odd.

Consider the set $S = \{p_1^x p_2^y p_3^z / 0 \leq x \leq \lfloor \frac{\alpha_1}{2} \rfloor, 0 \leq y \leq \lfloor \frac{\alpha_2}{2} \rfloor \text{ and } 0 \leq z \leq \lfloor \frac{\alpha_3}{2} \rfloor\}$ proper subset of V . We seek to prove that the subgraph of G induced by S is the maximal clique of G . To prove that every pair of distinct vertices in S is adjacent, take any two arbitrary vertices $p_1^a p_2^b p_3^c$ and $p_1^d p_2^e p_3^f$ in S .

Since $a + d \leq \alpha_1, b + e \leq \alpha_2$ and $c + f \leq \alpha_3$ is the condition, $p_1^a p_2^b p_3^c$ and $p_1^d p_2^e p_3^f$ are adjacent.

Also, $\langle S + \{v\} \rangle$ where $v \in V \setminus S$ is not a clique of G implies $\langle S \rangle$ is the maximal clique of G and $\omega(S) = |S| = \left(\left\lfloor \frac{\alpha_1}{2} \right\rfloor + 1\right) \left(\left\lfloor \frac{\alpha_2}{2} \right\rfloor + 1\right) \left(\left\lfloor \frac{\alpha_3}{2} \right\rfloor + 1\right) + 3$.

D. Remark

When exactly one of α_i is odd where $i = 1, 2, 3$, then the clique number of a Tri-factograph G is

$$\omega(G) = \left(\left\lfloor \frac{\alpha_1}{2} \right\rfloor + 1\right) \left(\frac{\alpha_2}{2} + 1\right) \left(\frac{\alpha_3}{2} + 1\right) + 1, \text{ here } \alpha_1 \text{ is odd.}$$

E. Example

Consider G with $z = p_1^1 p_2^1 p_3^2$. Then the order of G is 12.

G :

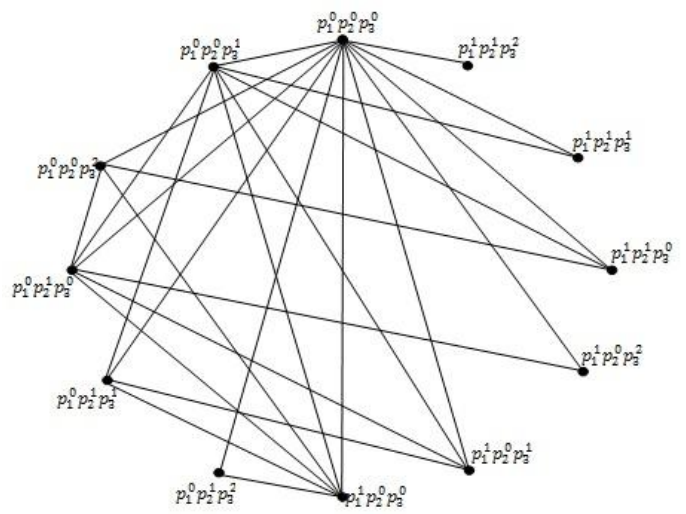


Figure:2

We observe that, the degree sequence of G is: 11, 7, 4, 6, 4, 2, 6, 4, 2, 3, 2, 1 and $\omega(G)$ is 4.

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