τ_s*-sg closed sets in Topological spaces

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Abstract

The aim of this paper is to introduce a new class of sets called τ_s^*-sg closed sets and τ_s^*-sg open sets in topological spaces and study some of their properties.

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Key Words:scl**- operator,τ_s^*- topology,τ_s^*-sg closed sets and τ_s^*-sg open sets.

1.Introduction

In 1970, Levine[6] introduced the concepts of generalized closed sets as a generalization of closed sets in topological spaces. Using generalized closed sets, Dunham[5] introduced the concept of the closure operator cl* and a new topology τ^* and studied some of their properties. P. Bhattacharya and B.K. Lahiri[3] introduced the concept of semi generalized closed sets in topological spaces. J. Dontchev[4] introduced generalized semi-open sets, H. Maki, R. Devi and K. Balachandran[9] introduced generalized α-closed sets in topological spaces.

In this paper, we obtain a new semi generalization of closed sets in the topological space (X,τ_s^*). Throughout this paper X and Y are topological spaces in which no separation axioms areassumed unless otherwise explicitly stated. For a subset A of a topological space X, int(A), cl(A), scl**(A) and A^c denote the interior, closure, semi generalized closureand complement of A respectively.

2. Preliminaries

Definition: 2.1

A subset A of a topological space (X,τ) is called

(i) Generalized closed(briefly g-closed)[6] if cl(A)$\subseteq$G whenever A $\subseteq$ G and G is open in X.

(ii) Semi-generalized closed(briefly sg-closed)[3] if scl(A)$\subseteq$ G whenever A$\subseteq$ G and G is semi-open in X.

(iii)Generalized semi-closed(briefly gs-closed)[2] if scl(A)$\subseteq$ G whenever A$\subseteq$G and G is openin X.

(iv)α-closed[8] if cl(int(cl(A)))$\subseteq$ A.

(v) α - generalized closed(briefly αg-closed)[9] if αcl(A)$\subseteq$G whenever A$\subseteq$ G and G is open inX.

(vi) Generalized α - closed(briefly gα –closed)[12]if αcl(A)$\subseteq$ G whenever A$\subseteq$ G and G is α-open in X.

(vii) Generalized semi-pre closed(briefly gsp-closed)[2] if spcl(A)$\subseteq$ G whenever A$\subseteq$ G and Gis open in X.

(viii) Pre closed[11] if cl(int(A))$\subseteq$ A.

(ix) Semi-closed[7] if int(cl(A))$\subseteq$ A.

(x) Semi-pre closed(briefly sp-closed)[1] if int(cl(int(A))) $\subseteq$A.

The complements of the above mentioned sets are called their respective open sets.

Definition: 2.2

For the subset A of a topological space X, the semi generalized closure operator scl**(A) is defined by the intersection of all sg-closed sets containing A.

Definition: 2.3

For a topological space X, the topology τ_s* is defined by

τ_s* = {G :scl**(G^c) = G^c }

Example:

Let X={a,b,c}and τ={X,Φ,{a},{b},{a,b}}.Thenthe collection of subsets {X,Φ, {a},{b},{a,b}} is a τ_s* - topology on X.

3. τ_s*- sg closed sets in topological spaces

In this section, we introduce the concept ofτ_s*-sg closed sets in topological spaces.

Definition: 3.1

A subset A of a topological space X is called τ_s*- semi generalized closed set(briefly

τ_s*-sgclosed) if scl**(A) $\subseteq$G whenever A$\subseteq$G and G is τ_s*- semi open.

The complement of τ_s*- semi generalized closed set is called the τ_s*- semi generalizedopen set(briefly τ_s*-sg-open).

Example:

Let X={a,b,c}and τ = {X,Φ,{a},{b},{a,b}}.Then τ_s*-sg-closed sets are {X,Φ, {a},{b},{a,b}}.

Theorem: 3.2

Every closed set in X is τ_s*-sg closed.

Proof:

Let A be a closed set.

Let A $\subseteq$ G,since A is closed,

Cl(A) = A $\subseteq$ G, where G is τ_s*- semi open.

But scl**(A)$\subseteq$cl(A) $\subseteq$ G

Then scl**(A)⊆G , whenever A⊆ G and G is τ_s*- semi open.

Hence A is τ_s*- sg closed.

Theorem: 3.3

Every τ_s* - closed set in Xis τ_s* - sg closed.

Proof:

Let A be a τ_s* - closed set.

Let A ⊆ G, where G is τ_s* - semi open.

Since A is τ_s* - closed, scl**(A) = A ⊆ G

scl**(A) ⊆ G.

Hence A is τ_s* - sg closed.

Theorem: 3.4

Every sg - closed set in X is τ_s*- sg closed but not conversely.

Proof:

Let A be a sg - closed set.

Assume that A⊆G, G is τ_s* - semi open in X,

Since A is sg-closed,scl(A)⊆ G.

But scl**(A)⊆scl(A)⊆ G

⇒scl**(A)⊆G.

Hence A is τ_s* - sg-closed.

The converse of the above theorem need not be true as seen from the following example.

Example: 3.6

Let X = {a,b,c} and Y = {a,b,c,d} be the two non-empty sets.

(i) Consider the topology τ = {X,Φ}. Then the sets{{a},{b},{c},{a,b},{b,c},{a,c}}are τ_s* -sg- closed but not closed.

(ii) Consider the topology τ = {X,Φ}. Then the sets{{a},{b}.{c},{a,b},{b,c},{a,c}} are τ_s* -sg- closed but not α-closed.

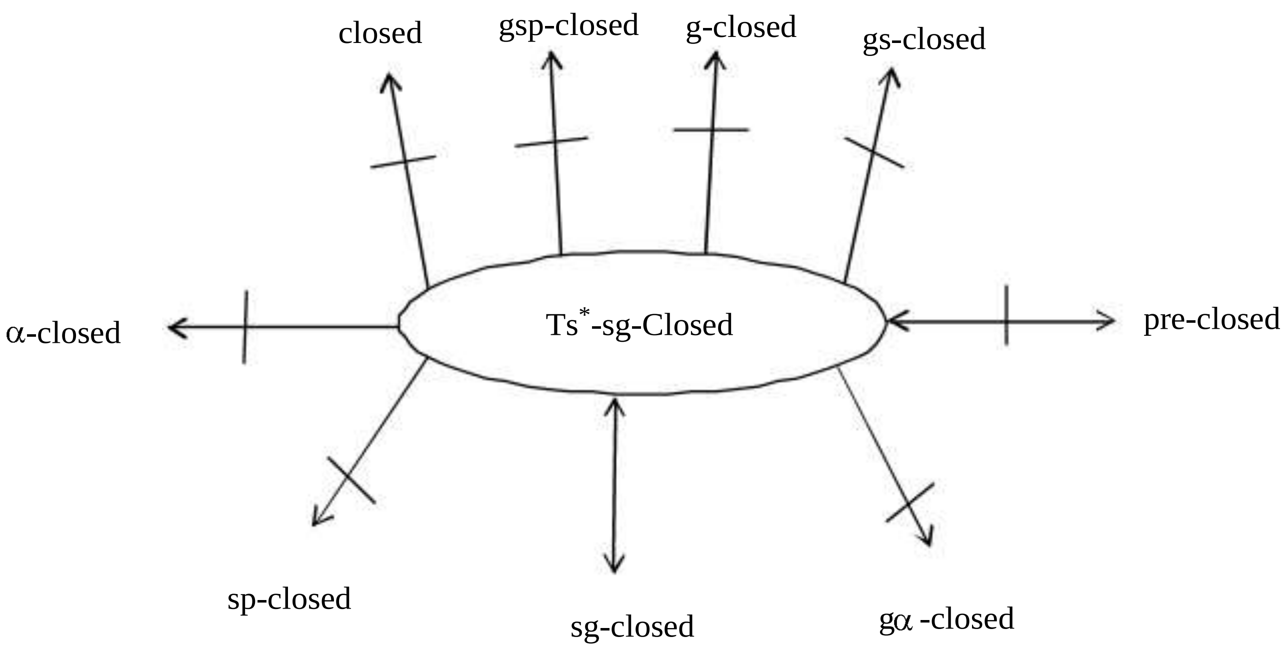
(iii) Consider the topology τ = {X,Φ,{a}}. Then the sets {{a},{a,b},{a,c}} are not sp-closed but τ_s* -sg- closed.

(iv) Consider the topology τ = {X,Φ,{a}}. Then the set {a} is not gsp-closed but τ_s* - sg- closed.

(v) Consider the topology $\tau = \{X,\Phi,\{a\}\}$. Then the set $\{a\}$ is not g- closed but τ_s* -sg- closed.
(vi) Consider the topology $\tau = \{X,\Phi,\{a\}\}$. Then the set $\{a\}$ is not gs- closed but τ_s* -sg- closed.
(vii) Consider the topology $\tau = \{X,\Phi,\{b\},\{a,b\}\}$.Then the sets$\{\{c\},\{b,c\},\{a,c\}\}$ arenot pre-closed but τ_s* - sg- closed.
(viii) Consider the topology $\tau = \{X,\Phi,\{a\}\}$. Then the sets $\{\{a,b\},\{a,c\}\}$ are not gα-closed but τ_s* -sg- closed.
(ix) Consider the topology $\tau = \{X,\Phi,\{a\},\{b\},\{a,b\}\}$. Then the set$\{\{a\},\{b\}\}$ are both sg-closed and τ_s* -sg- closed.
(x)Consider the topology $\tau = \{X,\Phi,\{b\},\{a,b\}\}$.Then the set$\{a\}$is pre-closed but not τ_s* -sg- closed.

Remark: 3.7

From the above discussion, we obtain the following implications.



Theorem: 3.8

For any two sets A and B,

Scl**(A∪B) = scl**(A)∪scl**(B).

Proof:
Since A⊆A∪B and B ⊆ A∪B then

scl**(A) ⊆scl**(A∪ B) and

scl**(B) ⊆scl**(A∪B)

Hence scl**(A) ∪scl**(B) ⊆scl**(A∪B) ⟶ (1)

scl**(A) and scl**(B) are sg- closed sets.

⟹scl**(A) ∪scl**(B) is alsosg-closed set.

Again, A ⊆scl**(A) and B⊆scl**(B)

⟹A ∪ B ⊆scl**(A) ∪scl**(B).

Thus scl**(A) ∪scl**(B) is a sg - closed set containing A∪B.

Since scl**(A ∪ B) is the smallest sg - closed set containing A ∪ B

We have scl**(A ∪ B) ⊆scl**(A) ∪scl**(B)⟶ (2)

From (1) and (2),scl**(A ∪ B) = scl**(A) ∪scl**(B).

Theorem: 3.9

Union oftwo τ_s*- sg closed sets in X is a τ_s*-sg- closed set in X.

Proof:

Let A and B be two τ_s*-sg closed sets.

Let A∪ B⊆G, where G is τ_s *- semi open.

Since A and B are τ_s*-sg- closed sets,

then scl**(A) ⊆ G and scl**(B)⊆ G.

scl**(A) ∪scl**(B)⊆ G.

But by the above theorem,

scl**(A)∪αcl*(B) = scl**(A∪B)

scl**(A∪B) ⊆ G, where G is τ_s *- semi open.

Hence A∪B isaτ_s*-sg closed set.

Theorem: 3.10

A subset A of X is τ_s*-sg-closed if and only if scl**(A)-A contains no non-empty

τ_s* - closed set in X.

Proof:

Let A be a τ_s* - sg- closed set.

Suppose that F is a non-empty τ_s* - closed subset of scl**(A) – A.

Now, F$\subseteq$scl**(A) – A.

Then F $\subseteq$scl**(A) $\cap$ A^c

Since scl**(A) – A = scl**(A)$\cap$ A^c

F $\subseteq$scl**(A) and F $\subseteq A^c$

A $\subseteq$ F^c

Since F^c is a τ_s* - open set and A is τ_s*-sg- closed.

scl**(A) $\subseteq$ F^c

(i.e.,) F $\subseteq$ $[scl^{**}(A)]^c$.

Hence, F$\subseteq$scl**(A) $\cap$ $[scl^{**}(A)]^c = \Phi$

(i.e.,) F = Φ

Which is a contradiction.

scl**(A)-A contains no non-empty τ_s* - closed set in X.

Conversely, assume that scl**(A) – A contains no non-empty τ_s* - closed set.

Let A$\subseteq$ G, G is τ_s* - open.

Suppose that scl**(A) is not contained in G then scl**(A)$\cap G^c$ is a non-empty τ_s* - closed set of scl**(A) – A, which is a contradiction.

scl**(A)$\subseteq$ G, G is τ_s* - semi open.

Hence A is τ_s* - sg closed.

Corollary: 3.11

A subset A of X is τ_s* - sg- closed if and only if scl**(A) – A contain no non-empty closed set in X.

Proof:

The proof follows from the theorem 3.10 and the fact that every closed set is τ_s*-sg- closed set in X.

Theorem: 3.12

If a subset A of X is τ_s* - sg- closed and A $\subseteq$ B$\subseteq$scl**(A) then B is τ_s* -sg- closed set in X.

Proof:

Let A be a τ_s*-sg- closed set such that A⊆ B⊆scl**(A). Let U be a τ_s* - semi open set of X such that B⊆ U.

Since A is τ_s* - sg- closed,

We have scl**(A) ⊆ U.

Now, scl**(A)⊆scl**(B)⊆scl**[scl**(A)] = scl**(A) ⊆ U.

scl**(B) ⊆ U, U is τ_s* - semi open set.

B is τ_s* - sg- closed set in X.

Theorem: 3.13

Let A be a τ_s* - sg- closed set in (X,τ_s*). Then A is sg- closed if and onlyifscl**(A) is τ_s* - open.

Proof:

Suppose A is sg – closed in X.

Then scl**(A) = A and so scl**(A) – A = Φ which is τ_s* - open in X.

Conversely, suppose scl**(A) – A is τ_s* - open in X.

Since A is τ_s*-sg-closed, by theorem: 3.10, scl**(A) – A contains no non-empty τ_s* - closed set in X.

Then scl**(A) – A = Φ.

Hence A is sg - closed.

Theorem: 3.14

For x∈ X, the set X-{x} is τ_s* -sg-closed or τ_s* - open.

Proof:

Suppose X – {x} is not τ_s* - open. Then X is the only τ_s* - open set containing X – {x}. This implies scl**(X – {x}) ⊆ X.

Hence X – {x} is a τ_s* - sg-closed in X.

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