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On b-Separation Axioms in intuistionistic Fuzzy Topological Spaces

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ABSTRACT.

The aim of this paper is to introduce *some new types of b- separation axioms (b- T_i space, for i = 0, 1, 2) in intuitionistic fuzzy topological spaces by using the concept of an intuitionistic fuzzy b-open (fuzzy b-closed) sets. Moreover the topological property and relationships between these separation axioms are investigated.*

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1.Introduction

In 1996, D. Adnadjevic [1] introduced the concept of *b*-open sets in general topology, and Hakeem A. Othman [21]extend that concept to fuzzy topological space.

Several authors introduced the concepts of fuzzy separation axioms using the notion of fuzzy open set see (Ghanim, Kerre and Mashhour [16]) Singal and Rajvanshi [25], G.Balasubramanian [7], F. S. Mahmoud, M. A. Fath Alla and S. M. Abd Ellah [20] and Hakeem A. Othman and S. Latha [22] by using the notions of fuzzy regular open sets, fuzzy β-open sets, fuzzy α-open sets and fuzzy semi α-open sets respectively.

Singal and Prakash [24] have introduced the concept of fuzzy pre-separation axioms. H.Al-Qahtani and Abdulgawad Al-Qubati [2] have introduced and studied new kinds of fuzzy pre-separation axioms .
Several notaions based on fuzzy pre-separation axioms have been studied.
After defining the concept of intuitionistic fuzzy set by K. T.Atanassov[3] and *intuitionistic fuzzy topological spaces,* by D- Coker in [15],some authors studied the concept of separation axioms in *intuitionistic fuzzy topological spaces* .
In (2001) S. Bayhan and D. Coker **[** 8] gave some characterizations of T_1 and T_2 separation axioms in intuitionistic topological spaces, they gave interrelations between several types of separation axioms and some counterexamples
In 2003[20] F. Gallego. Lupianez defined new notions of Hausdorffness in the intuitionistic fuzzy topological spaces.
In (2005)[10], S- Bayhan and D- Coker studied Pairwise separation axioms in double intuitionistic topological spaces, for more studies we can find them in (**[**7],[9],[13],[15],[18],[23]**)**.

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In this paper , *by using the concept of fuzzy b-open (fuzzy b-closed) sets ,we* introduce *some new types of b- separation axioms (b T_i - space, for i = 0, 1, 2) in intuitionistic fuzzy topological spaces, and we study the topological property and hereditary property of these spaces.*
Relationships between these separation axioms are investigated. Some counter examples are given to show that the inverse of those relations are not true in general.

2. Preliminaries

Throughout this paper by (X,τ) or simply by X we mean an intuitionistic fuzzy topological space (Ifts, for short**).**

Definition 2.1. [3,4,5,6] Let X be a nonempty fixed set. An intuitionistic fuzzy set A (IFS for short) in X is an object having the form $A=\{<x, \mu_A(x),\gamma_A(x)> : x\in X\}$ where the functions $\mu_A:X\to I$ and $\gamma_A: X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x\in X$ to the set A, respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$ **.**

Every fuzzy set A on a non-empty set X is obviously an IFS having the form $A =\{ <x, \mu_A(x),1- \mu_A(x) > :x\in X\}$.

Definition 2.2: [4] Let X be a nonempty set and the form $A =\langle x, \mu_A(x),\gamma_A(x) \rangle : x\in X\}$, $B = \langle x, \mu_B(x),\gamma_B(x) \rangle : x\in X\}$and let$\{ A_\lambda : \lambda\in\Lambda\}$be an arbitrary family of intuitionistic fuzzy sets in X. Then

(a) $A\subseteq B$ if $\forall x \in X [\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)]$;

(b) $A = B$ if $A\subseteq B$ and $B\subseteq A$ (c) $A^c = \langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X$

(d) $A\bigcap B = \langle x, \mu_A(x)\wedge \mu_B(x), \gamma_A(x)\vee \gamma_B(x)\rangle : x \in X$

(e) $A\bigcup B = \langle x, \mu_A(x)\vee \mu_B(x), \gamma_A(x)\wedge \gamma_B(x)\rangle : x \in X$

(f) $0^{\sim}_X = \{\langle x, 0, 1\rangle: x \in X\}$ and $1^{\sim}_X = \{\langle x, 1, 0\rangle: x \in X\}$

Definition 2.3[4] Let $\{A_i: i\in I \}$be an arbitrary family of IFS in X. Then

$$(a)\bigcap A_i = \langle x, \wedge\mu_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X$$

$$(b)\bigcup A_i = \langle x : \vee\mu_{A_i}(x), \wedge\gamma_{A_i}(x) : x \in X$$

Definition 2.4 [12] : Let $\alpha,\beta\in[0,1], \alpha + \beta \le 1$.An intuitionistic fuzzy point (IFP for short) of nonempty set X is an IFS of X denoted by $p = x_{(\alpha,\beta)}$ and defined by

$$p = x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha,\beta) & if \quad x = y \\ (0,1) & if \quad x \neq y \end{cases}$$

In this case, x is called the support of $x_{(\alpha,\beta)}$ and α,β are called the value and no value of $x_{(\alpha,\beta)}$ respectively.

Clearly an intuitionistic fuzzy point can be represented by an ordered pair of fuzzy point as follows:

$x_{(\alpha,\beta)} = (x_\alpha, 1 - x_{(1-\beta)})$

In IFP $x_{(\alpha,\beta)}$is said to belong to an IFS $A=\{\langle x, \mu_A(x),\gamma_A(x) \rangle : x\in X\}$denoted by $x_{(\alpha,\beta)} \in A$ (*or* $p \subseteq A$), if $\alpha \leq \mu_A(x)$ and $\beta \geq \gamma_A(x)$.

We identify a fuzzy point x_r in X by the intuitionistic fuzzy point $x_{(r,(1-r))}$ in X.

Proposition 2.5. *An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points belonging to A.*

The proof is on similar lines as in [19] .

Definition 2.6[3]. Let X and Y be two nonempty sets and

$f : X \to Y$ be a function. Then

(a) If $B = \{\langle y, \mu_B(y),\gamma_B(y) \rangle : y \in Y\}$ is an intuitionistic fuzzy set in Y,

then the preimage of B under f denoted by $f^{-1}(B)$ is the IFS in X defined by

$f^{-1}(B) = \langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) : x \in X\}$

(b) If $A = \{\langle x, \lambda_A(x), \nu_A(x) \rangle : x \in X\}$is an intuitionistic fuzzy set in X,

then the image of A under$f$ denoted by $f(A)$ is the intuitionistic

fuzzy set in Y defined by

$f(A) = \{\langle y, f(\lambda_A)(y), 1 - f(1 - \nu_A)(y) \rangle : y \in Y\}$

Where ,

$$f(\lambda_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \{\lambda_A(x)\} & if \quad f^{-1}(y) \neq \emptyset, \\ 0 & ,otherwise \end{cases}$$

$$1-f(1-\upsilon_A)(y)=\begin{cases}\inf\limits_{x\in f^{-1}(y)}\{\nu_A(x)\} & if \quad f^{-1}(y)\neq\varnothing,\\ 1 & ,otherwise\end{cases}$$

Proposition2.7[5]: Let A, A_i ($i \in I$) be IFSs in X ,B, B_j ($j \in J$) IFSs in Y and f: X → Y a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,
(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,
(c) $A \subseteq f^{-1}((f(A))$ and if f is injective, then $A = f((f^{-1}((A))$,
(d) $f(f^{-1}((B)) \subseteq B$ and if f is surjective, then $f((f^{-1}((B)) = B$,
(e) $f^{-1}(1_{\tilde{X}}) = 1_{\tilde{X}}$,
(f) $f^{-1}(0_{\tilde{X}}) = 0_{\tilde{X}}$
(g) $f(1_{\tilde{X}}) = 1_{\tilde{X}}$ if f is surjective .
(h) $f(0_{\tilde{X}}) = 0_{\tilde{X}}$

Replacing fuzzy sets [26] by intuitionistic fuzzy sets in Chang's definition of a fuzzy topological space[11], we get the following.

Definition 2.8[14]. An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family of intuitionistic fuzzy sets in X satisfy the following axioms:

(T_1) $0_{\tilde{X}}, 1_{\tilde{X}} \in \tau$

(T_2) If $A_1, A_2 \in \tau$, then $A_1 \cap A_2 \in \tau$

(T_3) If $A_\lambda \in \tau$ for each λ in Λ, then $\bigcup_{\lambda\in\Lambda} A_\lambda \in \tau$.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space (IFTS for short)and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set (IFOS for short) of X ,and the complement an intuitionistic fuzzy closed set (IFCS for short).

Example 2.9[14] Let X = {a, b, c} and M_1, M_2 and M_3 be an *intuitionistic* fuzzy sets on X defined as follows:

$M_1 = \langle x: (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.4}))\rangle$
$M_2 = \langle x:((\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.2}), (\frac{a}{0.5}, \frac{b}{0.3}, \frac{c}{0.3}))\rangle$
$M_3 = \langle x:((\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.4}), (\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}))\rangle$
$M_4 = \langle x:((\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.2}), (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.4}))\rangle$,Then the family $\tau = \{0_{\tilde{X}}, 1_{\tilde{X}}, M_1, M_2, M_3, M_4\}$is an IFT on X.

Definition 2.10. [14] An IFS A of an IFTS (X,τ) is an
(i) *intuitionistic fuzzy b-open set* (IFbOS in short) if
$A \subseteq int(cl(A)) \cup cl(int(A))$,

(ii) *intuitionistic fuzzy* b-*closed set* (IFbCS in short) if
$cl(int(A)) \cap int(cl(A)) \subseteq A$.

Defintion2.11[14] *Let A any intuitionistic fuzzy set. Then,*
$Ibcl(A) = \cap\{ F : A \subseteq F$ is an intuitionistic fuzzy b-closed set of X } is called intuitionistic fuzzy b-closure.
$Ibint(A) = \cup\{ U : U \subseteq A$, U is an intuitionistic fuzzy b-open set of X } is called intuitionistic fuzzy bInterior

Example2.12 . In example (2.9) If
$A = \langle x\, (\frac{a}{0.55}, \frac{b}{0.55}, \frac{c}{0.45}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.3}))\rangle$ then
$int(A) = \cup\{U : U \subseteq A$, U is an IFOS in X and $U \subseteq A\} = M_4$,and
$cl(A) = \cap\{F : F$ is an IFCS in X and $A \subseteq F \} = 1_{\tilde{X}}$

Corollary 2.13. *[1-20] Any union of intuitionistic fuzzy b-open sets is a fuzzy b-open set.*

Remark 2.14*[1-20] The intersection of intuitionistic fuzzy b-open sets need not be fuzzy b-open set.*

Definition 2.15*[20]. A mapping $f : (X, \tau) \to (Y, \rho)$ from an intuitionistic fuzzy topological space (X, τ) to another intuitionistic fuzzy topological space (Y, ρ) is said to be intuitionistic fuzzy b-continuous if $f^{-1}(M)$ is an intuitionistic fuzzy b-open set in X for each intuitionistic fuzzy open set M in Y .*

intuitionistic fuzzy b^*-continuous if $f^{-1}(M)$ is intuitionistic fuzzy b-open set in X for each intuitionistic fuzzy b-open set M in Y .

Intuitionistic fuzzy b^{**}-continuous if $f^{-1}(M)$ isintuitionistic fuzzy open set in X for each intuitionistic fuzzy b-open set M in Y .

Definition 2.16. Let (X, τ) be an IFTS and $Y \subseteq X$. Then $(Y, \tau|Y)$ is called a subspace of (X, τ) where

$$\tau | Y = \{G | Y = (\mu_G | Y, \nu_G | Y) : G \in \tau\}.$$

Mean Results

Definition 3.1. *An intuitionistic fuzzy topological space (X, T) is said to be intuitionistic fuzzy b-T_0 (or in short IFb-T_0) if for every pair of intuitionistic fuzzy points $p = x_{(\alpha,\beta)}, q = y_{(\gamma,\eta)}$ with different supports, there exists an intuitionistic fuzzy b-open set M such that either .($p \subseteq M$, $q \not\subseteq M$) or.($q \subseteq M$, $p \not\subseteq M$).*

Theorem 3.2. *An intuitionistic fuzzy topological space (X, T) is IFb- T_0 if and only if any two crisp intuitionistic fuzzy points with different supports have disjoint intuitionistic fuzzy b-closure.*

Proof . *Let (X, τ) be an intuitionistic fuzzy b-T_0 and* $p = x_{(\alpha,\beta)}$, $q = y_{(\gamma,\eta)}$ *be two crisp intuitionistic fuzzy points with supports x, y respectively, where $x \neq y$. since (X,τ) is IFb-T_0, there exists an intuitionistic fuzzy b-open set M such that either.($p \subseteq M$, $q \not\subseteq M$)or.($q \subseteq M$, $p \not\subseteq M$)*

If $p \subseteq M$, $q \not\subseteq M$' this implies that $q \subseteq Ibcl(q)$, $Ibcl(q) \not\subset M$, since $p \not\subset M^c$, $p \not\subset (Ibcl(q))^c$. But $p \subseteq Ibcl(p)$ .Therefore $Ibcl(p) \neq Ibcl(q)$.

Conversely, Let p and q be any two *intuitionistic* fuzzy points with different supports x, y, respectively. Let p_1 ,q_1 be *intuitionistic* fuzzy points such that $p_1(x) = q_1(y) = 1$. By hypothesis $Ibcl(p_1) \neq bIcl(q_1)$

, but $p \le p_1$implies that $p^c \ge p_1^c \ge (Ibcl(p_1))^c$. Thus $(Ibcl(p_1))^c$ is an *intuitionistic* fuzzy b - open set such that $q \not\subset (Ibcl(p_1)$,$P \subseteq Ibcl(p_1)$ .Hence (X, τ) is IFB T_0 .■

Theorem 3.3 Let *f* be an injective, *intuitionistic* fuzzy b^*-continuous mapp*ing from an intuitionistic fuzzy topological space (X,τ) into fuzzy topological space (Y, ρ). If (Y,ρ) is an intuitionistic fuzzy b-T_0 space, then so is (X,τ).*

Proof. *Let* $p = x_{(\alpha,\beta)}$, *and* $q= y_{(\gamma,\eta)}$ *are intuitionistic fuzzy points with different supports in X, then f(p) and f(q) are two intuitionistic fuzzy points with different supports in Y. Since (Y,ρ) is an intuitionistic fuzzy b T_0 -space, then there exists an intuitionistic fuzzy b- open set M such that $f(p) \subseteq M$, $f(q)) \not\subset M$, or $f(q) \subseteq M$, $f(p)) \not\subset M$ .Consider the part .If $f(p) \subseteq M$, $f(q)) \not\subset M$ .Its follows that, $p \subseteq f^{-1}(M)$, $q \not\subset f^{-1}(M)$, where $f^{-1}(M)$ is an intuitionistic fuzzy b-open set in X. Hence (X,τ) is a intuitionistic fuzzy b- T_0 space.* ■

Theorem 3.4 *. If $f : (X,\tau) \to (Y, \rho)$ is an injective, intuitionistic fuzzy b-continuous mapping and $(Y,\rho)$is an intuitionistic fuzzy -T_0 space, then (X,τ) is intuitionistic fuzzy b- T_0 space.*

Proof Let *p* = $x_{(\alpha,\beta)}$, *and q*= $y_{(\gamma,\eta)}$ be an *intuitionistic* fuzzy points with different supports in X. Then *f*(p), *f*(q) are two *intuitionistic* fuzzy points with different supports in Y.

Since (Y, ρ) is an *intuitionistic* fuzzy T_0 - space, then there exists a *intuitionistic* fuzzy open set M such that $f(p) \subseteq M$, $f(q)) \not\subset M$ or $f(q) \subseteq M$, $f(p) \not\subset M$. Consider the part $f(q) \subseteq M$, $(f(p)) \not\subset M$. it follows that

$q \subseteq f^{-1}(M)$, $p \not\subset f^{-1}(M)$ .where $f^{-1}(M)$ is an *intuitionistic* fuzzy b-open set in X. Hence (X,τ) is an *intuitionistic* fuzzy b -T_0 space. ■

Theorem (3.5). *If $f : (X, \tau) \to (Y,\rho)$ is an injective, intuitionistic fuzzy b^{**}-continuous mapping and (Y, ρ) is an intuitionistic fuzzy b -T_0 space, then (X,τ) is an intuitionistic fuzzy T_0 space.*

Proof *Let* $p = x_{(\alpha,\beta)}$, *and* $q= y_{(\gamma,\eta)}$ *be an intuitionistic fuzzy points with different supports in X. Then f(p) and f(q) are two intuitionistic fuzzy points with different supports in Y. Since (Y, σ) is an intuitionistic fuzzy b- T_0 space, then there exists an intuitionistic fuzzy b-open set M such that $f(p) \subseteq M$,$f(q) \not\subset M$ or $f(q) \subseteq M$, $f(p) \not\subset M$. Consider the part $f(q) \subseteq M$,$f(p) \not\subset M$ .It follows that $q \subseteq f^{1}(M)$,$p \not\subset f^{1}(M)$, where*

f^{-1}(M) is an intuitionistic fuzzy open set in X. Hence (X,T) is a fuzzy T_0 -space. ■

Definition (3.6). An *intuitionistic* fuzzy topological space (X,T) is said to be a *intuitionistic* fuzzy b-T_1 (or in short IFb-T_1) if for every pair of *intuitionistic* fuzzy points $p = x_{(\alpha,\beta)}, q = y_{(\gamma,\eta)}$ with $x \neq y$, there exist a *intuitionistic* fuzzy b-open sets M and N such that $p \subseteq M$, $q \not\subset M$, and $q \subseteq N$, $p \not\subset N$.

Theorem (3.7). An *intuitionistic* fuzzy topological space (X, T) is An IFb-T_1 if and only if every crisp *intuitionistic* fuzzy point is an *intuitionistic* fuzzy b-closed set.

Proof. Let (X,T) be IFb - T_1 and $p_0 = x_{0(\alpha,\beta)}$ be an *intuitionistic* crisp fuzzy points with support x_0. Now, for any *intuitionistic* fuzzy point $p = x_{(\gamma,\eta)}$ with support x in X such that $x \neq x_0$, there exist an *intuitionistic* fuzzy b-open sets M and N such that such that $p_0 \subseteq M$, $p \not\subset M$ and $p \subseteq N$ $p_0 \not\subset N$. Since $p \not\subset N$,by proposition (2.5) every *intuitionistic* fuzzy set is considered as the union of *all intuitionistic* fuzzy points which are contained in it , we obtain in particular $p_0^c = \bigcup_{(p \subseteq p_0^c)} p$ from

$$p_0^c(x_0) = 1 - p_0(x_0) = 0$$

We deduce that $p_0^c = \bigcup_{(p \subseteq p_0^c)} N$ and thus p_0^c is an *intuitionistic* fuzzy b-open set. Then p_0 is an *intuitionistic* fuzzy b-closed set.

Conversely, let $p_1 = x_{1(\alpha 1,\beta 1)}$,and $p_2 = x_{2(\alpha 2,\beta 2)}$,are *intuitionistic* fuzzy points with different supports x_1 and x_2. Also let $q_1 = x_{1(\gamma 1,\eta 1)}$ and $q_2 = x_{2(\gamma 2,\eta 2)}$ be an *intuitionistic* fuzzy points with different supports x_1 and x_2, respectively and such that $q_1(x_1) = q_2(x_2) = 1$. The *intuitionistic* fuzzy sets q_1^c and q_2^c are *intuitionistic* fuzzy b-open *and satisfy the conditions* $p_1 \subseteq q_2^c, p_2 \not\subset q_2^c$ *and* $p_2 \subseteq q_1^c, p_1 \not\subset q_1^c$ *.Hence the space* (X, T) *is IFb-T_1* . ■

Remark (3.8). Every *intuitionistic* fuzzy b-T_1 space is obviously fuzzy b-T_0 space. But the converse need not be true.

Example (3.9) Let X = {a, b} and M_1, M_2 and M_3 be an *intuitionistic* fuzzy sets on X defined as follows:

$M_1 = \langle x: ((\frac{a}{0.0}, \frac{b}{0.1}), (\frac{a}{0.4}, \frac{b}{0.0}))\rangle$

$M_2 = \langle x:((\frac{a}{0.2}, \frac{b}{0.0}), (\frac{a}{0.3}, \frac{b}{0.4}))\rangle$

$M_3 = \langle x:((\frac{a}{0.0}, \frac{b}{0.2}), (\frac{a}{0.3}, \frac{b}{0.0}))\rangle$

Let $\tau = \{ 0_X^{\sim} , 1_X^{\sim} ,M_1, M_2, M_3 \}$. Then the space (X, τ) is an *intuitionistic* fuzzy b-T_0 space but not *intuitionistic* fuzzy b-T_1.

Remark 3.10 Every *intuitionistic* fuzzy T_1 space is obviously , *intuitionistic* fuzzy bT_1- space. But the converse need not be true.

Example 3.11 Let X = {a, b} and M_1, M_2 and M_3 be an *intuitionistic* fuzzy sets on X defined as follows:

$M_1 = \langle x: (\frac{a}{0.0}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0}))\rangle$

$M_2 = \langle x:(\frac{a}{0.4}, \frac{b}{0}), (\frac{a}{0.}, \frac{b}{0.2})\rangle$

$M_3 = \langle x:((\frac{a}{0.7}, \frac{b}{1.0}), (\frac{a}{0.8}, \frac{b}{0.6}))\rangle$

Let $\tau = \{ 0^{\sim}_{X}, 1^{\sim}_{X}, M_1, M_2, M_3 \}$. Then the space (X; τ) is an *intuitionistic* fuzzy b-T_1 space but not *intuitionistic* fuzzy-T_1 space.

Theorem (3.12). *Let f be an injective, intuitionistic fuzzy b*-continuous mapping from an intuitionistic fuzzy topological space (X,τ) into intuitionistic fuzzy topological space (Y,ρ). If (Y,ρ) is a intuitionistic fuzzy b- T_1 space, then so is X.*

Proof*. Let $p = x_{(\alpha,\beta)}$, and $q = y_{(\gamma,\eta)}$ are intuitionistic fuzzy points with different supports in X, then f(p) and f(q) are two intuitionistic fuzzy points with different supports in Y. Since (Y,ρ) is an intuitionistic fuzzy b T_1 -space, then there exists an intuitionistic fuzzy b- open sets M and N such that $f(p) \subseteq M$, $f(q) \not\subseteq M$ and $f(q) \subseteq N$ $f(p)) \not\subseteq N$ .Its follows that, $p \subseteq f^{1}(M)$, $q \not\subseteq f^{1}(M)$ and $q \subseteq f^{1}(N)$, $p \not\subseteq f^{1}(N)$, where $f^{1}(M)$ and $f^{1}(N)$ are intuitionistic fuzzy b-open sets in X. Hence (X,τ) is a intuitionistic fuzzy b- T_1 space.* ■

Theorem (3.13). *If $f: (X,\tau) \to (Y,\rho)$ is an injective, intuitionistic fuzzy b -continuous mapping and (Y,ρ) is an intuitionistic fuzzy T_1 space, then (X,τ) is intuitionistic fuzzy b-T_1 space.*

Proof *Similar to that of theorem (3.4).* ■

Theorem (3.14)*. If $f: (X,\tau) \to (Y,\rho)$ is an injective, intuitionistic fuzzy b^{**}-continuous mapping and (Y,ρ) is an intuitionistic fuzzy bT_1 -space, then (X,τ) is intuitionistic fuzzy T_1 space.*

Proof *. Similar to that of theorem(3.5)).* ■

Definition (3.15) A fuzzy topological space (X,τ) is said to be a *intuitionistic* fuzzy stronger-bT_1 (or in short IFb-Ts) if every *intuitionistic* fuzzy point is an *intuitionistic* fuzzy b-closed set.

Remark (3.16)*. Clearly an, intuitionistic fuzzy stronger -bT_1- space is an intuitionistic fuzzy b- T_1 but the converse need not be true as shown by the following example.*

Example 3.17 Let X = {a, b} and M_1, M_2 ,M_3 and M_4be an *intuitionistic* fuzzy sets on X defined as follows:

$M_1 = \langle x: ((\frac{a}{1.0}, \frac{b}{0.0}), (\frac{a}{0.0}, \frac{b}{1.0}))\rangle$

$M_2 = \langle x:((\frac{a}{0.0}, \frac{b}{1.0}), (\frac{a}{1.0.}, \frac{b}{0.0})\rangle$

$M_3 = \langle x:((\frac{a}{0.7}, \frac{b}{0.0}), (\frac{a}{0.8}, \frac{b}{0.6}))\rangle$

$M_4 = \langle x:((\frac{a}{0.8}, \frac{b}{1.0}), (\frac{a}{0.3}, \frac{b}{0.2}))\rangle$

Let $\tau = \{ 0_X^{\sim}, 1_X^{\sim}, M_1, M_2, M_3, M_4 \}$.All crisp *intuitionistic fuzzy points are intuitionistic fuzzy b-closed sets, so* the space (X,τ) is an *intuitionistic* fuzzy b-T_1 space but not *every intuitionistic* fuzzy points is *intuitionistic* fuzzy b closed sets.

Definition (3.18) An IFTS (X,τ) is said to be an *intuitionistic* Fuzzy b- Hausdorff (or in short IFb-T_2) if for every pair of *intuitionistic* fuzzy poins $p = x_{(\alpha,\beta)}$ and $q = y_{(\gamma,\eta)}$ with different supports, there exist two *intuitionistic* fuzzy b-open sets M and N such that $p \subseteq M$, $q \not\subset M$, $q \subseteq N$, $p \not\subset N$, and $M \not\subset N$.

Example 3.19 [23] Let X = {a, b} and Let $\tau = \{ 0_X^{\sim}, 1_X^{\sim}, M_1, M_2, \}$. where $M_1 = \langle x: (\frac{a}{1.0}, \frac{b}{0.0}), (\frac{a}{0.0}, \frac{b}{1.0}))\rangle$

$M_2 = \langle x:((\frac{a}{1.0}, \frac{b}{0.0}), (\frac{a}{0.0}, \frac{b}{1.0}))\rangle$

Then (X,τ) is an IFTS and it is bT_0,b T_1,b T_2 -spaces.

Proposition (3.20) Every subspace of IFb-T_2 space is IFb-T_2.

Proof: Let (X,τ) be a IFbT_2- space and Y be subspace of X, where $\tau_Y = \{G_Y = \{<x, \mu_{G|Y}(x), \nu_{G|Y}(x)\rangle, x\in Y, G\in\tau\}$ and $G = <x, \mu_G(x), \nu_G(x)>$ Let $p = x_{(\alpha,\beta)}$ and $q = y_{(\gamma,\eta)}$be two distinct IFP in Y, i.e., they have distinct supports. Then, clearly $x_{(\alpha,\beta)}$ and $q = y_{(\alpha,\beta)}$ are also distinct IFPs in X and as X is IFb- T_2, therefore there exist two *intuitionistic* fuzzy b-open sets M and N such that $p \subseteq M$, $q\not\subset M$,$q \subseteq N$, $p\not\subset N$. and $M\not\subset N$. Thus, there exist M_Y ,$N_Y \in \tau_Y$ such that $p \subseteq M_Y$, $q \subseteq N_Y$ and $M_Y \not\subset N_Y$. ■

Proposition(3.21). Let (X,τ) be an IFbT_2- space ,If (X,τ) is T_2, then (X,τ^*) is a fuzzy Hausdorff fts (where ,$\tau^* = \{\mu_G(x): G \in\tau\}$).

Proof. For any two fuzzy points x_r, y_s with distinct supports and $0 < r, s < 1$, we have that $p = x(r, 1 – r)$, $q = y(s, 1– s)$ are two distinct IFPs. Then, there exist IFbOs $M = <x, \mu_M(x), \nu_M(x)>$ and $N = <x, \mu_N(x), \nu_N(x)>$ containing p and q respectively such that $M\not\subset N$.This implies that $r < \mu_M(x)$, $s < \mu_N(y)$ and $x_r \in M$, $y_s \in N$ which are fuzzy b-open sets with $M \subseteq N^c$. ■

Theorem (3.22). A fuzzy topological space (X,τ) is a Fuzzy b-T_2 -space if for every pair of *intuitionistic* fuzzy points $p = x_{(\alpha,\beta)}$and $q = y_{(\gamma,\eta)}$with different supports ,there exists an *intuitionistic* fuzzy b-open set U such that $p \subseteq U \subseteq Ibcl(U) \subseteq q^c$.

Proof. Let $p = x_{(\alpha,\beta)}$ and $q = y_{(\alpha,\beta)}$ are *intuitionistic* fuzzy points with different supports. Since (X,τ) is an *intuitionistic* fuzzy b-T_2 space, then there exist two *intuitionistic* fuzzy b-open sets M and N such that $p \subseteq M$, $q \not\subset M$,$q \subseteq N$, $p \not\subset N$,and $M \not\subset N$. It follows that $p \subseteq M \subseteq Ibcl(M)$, $q \not\subset Ibcl(M)$ .Then

$Ibint(Ibcl(M)) \subseteq Ibcl(M)$. let $U = Ibint(Ibcl(M))$ is an *intuitionistic* fuzzy b-open set. Hence $p \subseteq U \subseteq Ibcl(U) \subseteq q^c$.■

Theorem (3.23) Let f be an injective, fuzzy b*-continuous mapping from an *intuitionistic* fuzzy topological space (X,τ) into *intuitionistic* fuzzy topological space (Y,ρ). If (Y,ρ) is an *intuitionistic* fuzzy b- T_2 space, then so is X.

Proof. *Let* $p = x_{(\alpha,\beta)}$, *and* $q = y_{(\gamma,\eta)}$ *are intuitionistic fuzzy points with different supports in X, then f(p) and f(q) are two intuitionistic fuzzy points with different supports in Y. Since (Y,ρ) is an intuitionistic fuzzy b* T_2 *-space, then there exists an intuitionistic fuzzy b- open sets M and N in (Y,ρ) such that* $f(p) \subseteq M$, $f(q) \not\subset M$, $f(q) \subseteq N$, $f(p)) \not\subset N$, *and* $M \not\subset N$. *.Its follows that,* $p \subseteq f^{-1}(M)$, $q \not\subset f^{-1}(M)$, $q \subseteq f^{-1}(N)$, $p \not\subset f^{-1}(N)$ and $f^{-1}(M) \not\subset f^{-1}(N)$., *where* $f^{-1}(M)$ *and* $f^{-1}(N)$ *are intuitionistic fuzzy b-open sets in X. Hence* (X,τ) *is a intuitionistic fuzzy b-* T_2 *space.* ■

Theorem (3.24) If $f : (X,\tau) \to (Y,\rho)$ is an injective, *intuitionistic* fuzzy b-continuous mapping and (Y,ρ) is an *intuitionistic* fuzzy T_2- space, then (X,τ) is *intuitionistic* fuzzy bT_2 -space.

Proof Similar to that of theorem (3.4) . ■

Theorem (3.25). If $f : (X,\tau) \to (Y,\rho)$ is an injective, *intuitionistic* fuzzy b^{**}-continuous mapping and (Y,ρ) is *intuitionistic* fuzzy b- T_2 space, then (X,τ) is *intuitionistic* fuzzy T_2 -space.

Proof .Similar to that of theorem (3.5) ■

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