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New Classes of Harmonic Mean Graphs

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ABSTRACT

A Graph G = (V, E) with p vertices and q edges is said to be Geometric mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2,….q+1 in such a way that when each edge e=uv is labeled with f(e=uv) = $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ (or) $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then the resulting edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper, we investigate Harmonic mean labeling for path and cycle related graphs.

Keywords: Graph, Path, Cycle, Harmonic mean labeling, Shadow graph, Splitting graph, Middle graph and Total graph.

1. INTRODUCTION

The graphs considered here are finite, undirected and simple. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively. A path of length n is denoted by P_n. A cycle of length n is denoted by C_n. For standard terminology and notations we follow Harary [2] and for the detailed survey of Graph labeling we follow J.A Gallian [1]. S.Somasundaram and R. Ponraj introduced the concept of mean labeling of graphs in [3]. S. Somasundram and S.S.Sandhya introduced the concept of Harmonic mean labeling of graphs in [4]. and its basic results was proved in [4]. In this paper we investigate Harmonic mean labeling behavior of some standard new graphs. The following definitions are used here.

Definition 1.1: A Graph G=(V, E) with p vertices and q edges is said to be a Geometric mean if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2,…, q+1 in such a way that when each edge e=uv is labeled with f(e=uv) = $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ (or) $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then the resulting edge labels are distinct. In this case f, is called a Harmonic mean labeling of G.

Definition 1.2: Let G be connected graph and G' be the copy of G then *shadow graph $D_2(G)$* is obtained by joining each vertex u in G to the neighbours of the corresponding vertex u′ in G′.

Definition 1.3: The *middle graph M(G)* of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it

Definition 1.4: The *total graph T(G)* of graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G.

Definition 1.5: The *splitting graph S′(G)* is obtained by adding new vertex v′ corresponding to each vertex v of G such that N(v) = N(v′) where N(v) and N(v′) are the neighbourhood sets of v and v′ respectively.

Definition 1.6: *Duplication* of a vertex v_k by a new edge $e=v_k' v_k''$ in a graph G produces a new graph G′ such that $N(v') \cap N(v_k'') = v_k$.

Definition 1.7: The *prism* D_n, $n \geq 3$ is a trivalent graph which can be defined as the Cartesian product $P_2 x C_n$ of a path on two vertices with a cycle on n vertices.

2. Results

Theorem 2.1: The graph $D_2(P_n)$ is a Harmonic mean graph

Proof: Let $u_1, u_2, \ldots u_n$ be the vertices of path P_n and $v_1, v_2, \ldots, v_n$ be the newly added vertices corresponding to the vertices $u_1, u_2, \ldots, u_n$ in order to obtain $D_2(P_n)$. Denoting $G = D_2(P_n)$ then $|V(G)|=2n$ and $|E(G)| = 4(n-1)$

We define $f: V(G) \rightarrow \{1,2,\ldots,q+1\}$

$f(u_1) = 1$

$f(u_i) = 4(i-1), 2 \leq i \leq n$

$f(v_1) = 3$

$$f(v_i) = \begin{cases} 4i-2, & 2 \leq i \leq n-1 \\ 4i-3, & i = n \end{cases}$$

Edges are labeled with

$f(u_i u_{i+1}) = 4i-3, \quad 1 \leq i \leq n-1$

$f(v_i v_{i+1}) = 4i, \quad 1 \leq i \leq n-1$

$f(u_i v_{i+1}) = 4i-2, \quad 1 \leq i \leq n-1$

$f(v_i u_{i+1}) = 4i-1, \quad 1 \leq i \leq n-1$

The above defined function f provides a Harmonic mean labeling for $D_2(P_n)$. That is, $D_2(P_n)$ is a Harmonic mean graph.

Example 2.2: Shadow graph of path P_5 and its Harmonic mean labeling is shown in the following figure

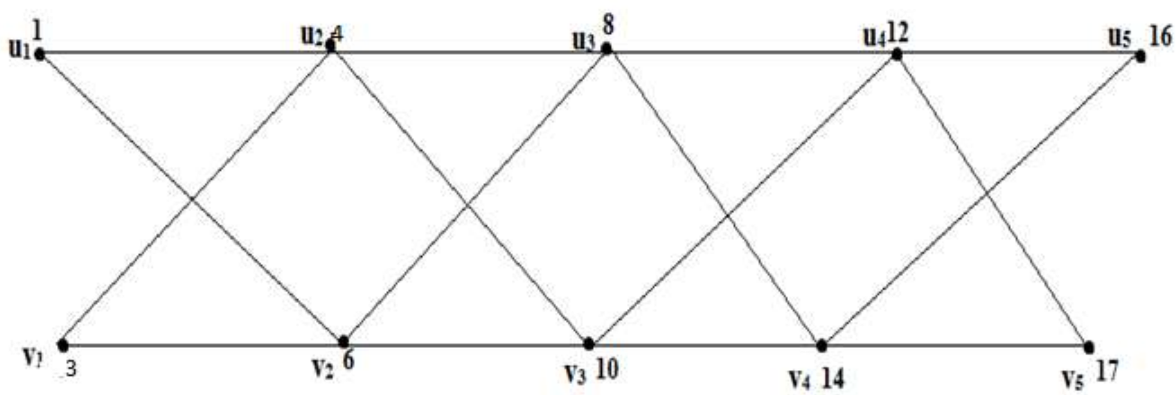

Figure: 1

Theorem 2.3: Middle graph of path P_n is a Harmonic mean graph

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices and $e_1, e_2, \ldots, e_{n-1}$ be the edges of path P_n and $G = M(P_n)$ be the middle graph of path P_n. According to the definition of middle graph $V(M(P_n)) = V(P_n) \cup E(P_n)$ and whose edge set is

$E(M(P_n)) = \{u_i e_i ; 1 \leq i \leq n-1, \quad u_i e_{i-1} ; 2 \leq i \leq n, \quad e_i e_{i+1} ; 1 \leq i \leq n-2\}$

Here |V(G)| = 2n-1 and |(E(G)| = 3n-4

we define f: V(G) $\rightarrow$ {1,2,…,q+1} by

$f(u_1) = 1$

$f(u_i) = 3i, \quad 2 \le i \le n$

$f(e_i) = 3i-1, \quad 1 \le i \le n-1$

Edges are labeled with

$f(u_i e_i) = 3i-2, \quad 1 \le i \le n-1$

$f(u_i e_{i-1}) = 3i-1, \quad 2 \le i \le n-1$

$f(e_i e_{i+1}) = 3i \quad ,1 \le i \le n-2$

The above defined function f provides an Harmonic mean labeling for $M(P_n)$. Hence, $M(P_n)$ is a Harmonic mean graph.

Example 2.4: $M(P_5)$ and its Harmonic mean labeling is shown is Figure 2.

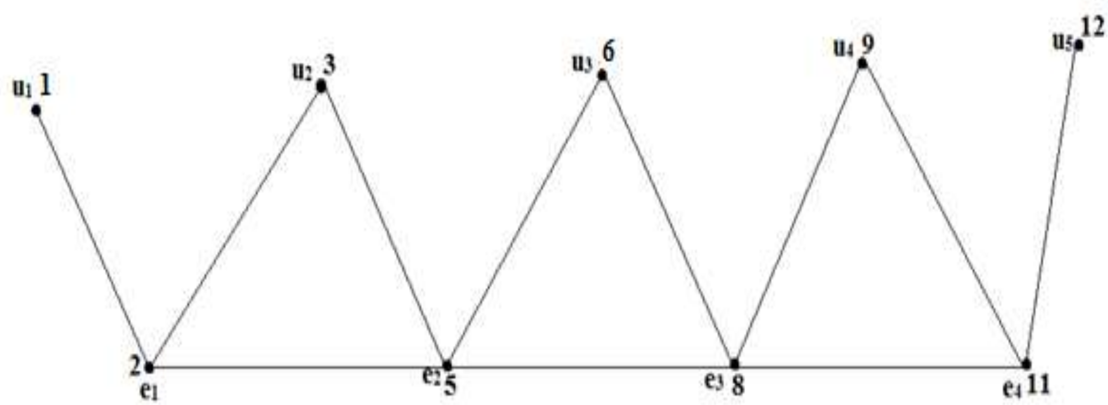

Figure: 2

Theorem 2.5: Total graph of path P_n is a Harmonic mean graph

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices and $e_1, e_2, \ldots, e_{n-1}$ be the edges of path P_n and G = $T(P_n)$ be the total graph of path P_n with $V(T(P_n)) = V(P_n) \cup E(P_n)$ and

$E[T(P_n)] = \{u_i u_{i+1}; 1 \le i \le n-1,\ u_i e_i; 1 \le i \le n-1,\ e_i e_{i+1}; 1 \le i \le n-2,\ u_i e_{i-1}; 2 \le i \le n\}$

Here |V(G)|= 2n-1 and |E(G)| = 4n-5

Define f:V(G) $\rightarrow$ {1,2,….q+1} as follows

$f(u_1) = 1$

$f(u_i) = 4i, \quad 2 \le i \le n$

$f(e_1) = 3$

$f(e_i) = 4i-2, \quad 2 \le i \le n-1$

Edges are labeled with

$f(u_i u_{i+1}) = 4i-2, \quad 1 \le i \le n-1$

$f(u_ie_i) = 4i-3, \quad 1 \le i \le n-1$

$f(e_ie_{i+1}) = 4i, \quad 1 \le i \le n-2$

$f(u_i e_{i-1}) = 4i-1, \quad 2 \le i \le n$

The above defined function f provides an Harmonic mean labeling for $T(P_n)$. Hence $T(P_n)$ is a Harmonic mean graph.

Example: 2.6 $T(P_6)$ and its Harmonic mean labeling is shown below.

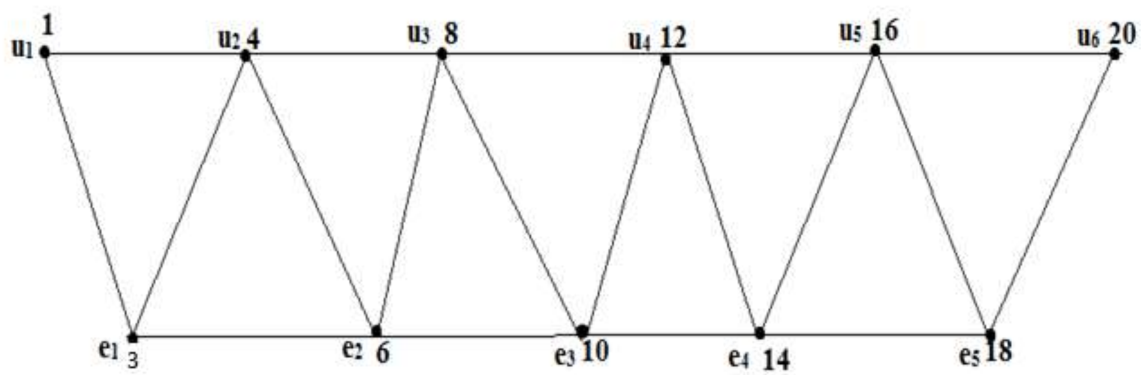

Figure: 3

Theorem 2.7: Splitting graph of path P_n is a Harmonic mean graph

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices and $e_1, e_2, \ldots, e_{n-1}$ be the edges of path P_n. Let $v_1, v_2, \ldots, v_n$ be the newly added vertices to form the splitting graph of path P_n. Let $G = S'(P_n)$ be the splitting graph of path P_n. $V(S'(P_n)) = \{u_i\} \cup \{v_i\}$, $1 \le i \le n$ and

$E(S'(P_n)) = \{ v_iu_{i+1} ; 1 \le i \le n-1 ,\ v_iu_{i-1} ; 2 \le i \le n ,\ v_i v_{i+1} ; 1 \le i \le n-1\}$

Here $|V(G)| = 2n$ and $|E(G)| = 3n-3$

we define $f: V(G) \to \{1,2,\ldots,q+1\}$ as follows

$f(u_1) = 2$

$f(u_i) = 3i, \quad 2 \le i \le n$

$f(v_1) = 1$

$f(v_i) = 3i-2, \quad 2 \le i \le n$

Edges are labeled with

$f(v_1u_2) = 1, \quad f(v_iu_{i+1}) = 3i-1, \quad 2 \le i \le n-1$

$f(v_2u_1) = 3, \quad f(v_i u_{i-1}) = 3i-2, \quad 3 \le i \le n-1$

$f(v_1v_2) = 3, \quad f(v_iv_{i+1}) = 3i, \quad 2 \le i \le n-1$

The above defined function f provides an Harmonic mean labeling for $S'(P_n)$. Hence $S'(P_n)$ is a Geometric mean graph.

Example 2.8: $S'(P_6)$ and its Harmonic mean labeling is shown below.

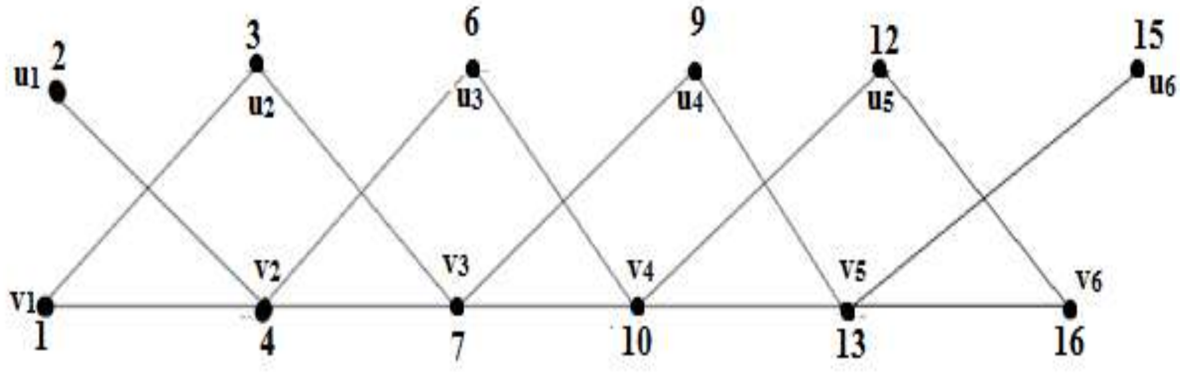

Figure: 4

Theorem 2.9: Duplicating each vertex by an edge in path P_n is a Harmonic mean graph.

Proof: Let $u_1, u_2, u_3 \ldots, u_n$ be the vertices of path P_n Let G be the graph obtained by duplicating each vertex v_i of P_n by an edge v_i', v_i'' at a time, where $1 \le i \le n$. Note that $|V(G)| = 3n$ and $|E(G)| = 4n-1$

We define $f: V(G) \rightarrow \{1,2,\ldots,q+1\}$ as follows

$f(u_1) = 3$

$f(u_i) = 4i-2$, $2 \le i \le n$

$f(v_i') = 4i-3$, $1 \le i \le n$

$f(v_1'') = 2$

$f(v_i'') = 4i-1$, $2 \le i \le n$

Edges are labeled with

$f(u_i u_{i+1}) = 4i$, $1 \le i \le n-1$

$f(u_1 v_1') = 2$, $f(u_i v_i') = 4i-3$, $2 \le i \le n$

$f(u_i v_i'') = 4i-1$, $1 \le i \le n$

$f(v_1' v_1'') = 1$ $f(v_i' v_i'') = 4i-2$, $2 \le i \le n$

The above labeling pattern we get distinct edge labels. Thus f provides is a Harmonic mean labeling for graph G. Hence, duplicating each vertex by edge in Path P_n is a Harmonic mean graph.

Example 2.10: Duplicating each vertex by edge in path P_7 and its Harmonic mean labeling is shown below.

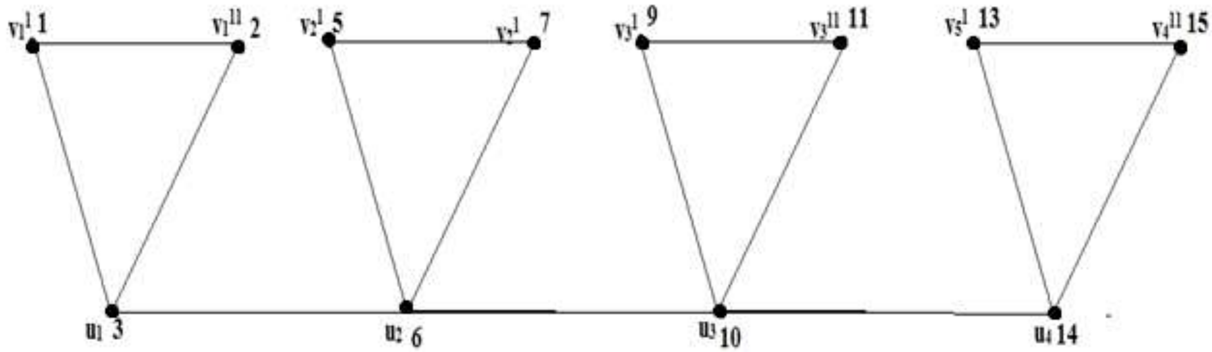

Figure: 5

Theorem 2.11: A Graph is obtained by attaching triangle at each pendent vertex of a crown is a Harmonic mean graph.

Proof: Let $u_1 u_2 \ldots u_n u_1$ be the cycle C_n. Let v_i be the vertex which is adjacent to u_i, where $1 \le i \le n$.The resultant graph is $C_n A K_1$

Let x_i, y_i and z_i be the vertices of i^{th} copy of C_3 and identify z_i with v_i . For$1 \le i \le n$, join v_i with x_i and y_i and then join x_i and y_i the required graph of G whose edge set is $E = \{u_iu_{i+1}, u_nu_1 \mid 1 \le i \le n-1\} \cup \{u_iv_i, v_ix_i, v_iy_i, x_iy_i \mid 1 \le i \le n\}$

Define a function $f: V(G) \rightarrow \{1,2,\ldots,q+1\}$ by

$f(u_1) = 3$

$f(u_i) = 5i-1, \quad 2 \le i \le n-1$

$f(v_1) = 4$

$f(v_i) = 5i, \quad 2 \le i \le n$

$f(x_i) = 5i-4, \quad 1 \le i \le n$

$f(y_i) = 5i-3, \quad 1 \le i \le n$

From the above labeling pattern we get distinct edge labels. Thus f provides an Harmonic mean labeling for graph G and its labeling pattern is shown in the following figure.

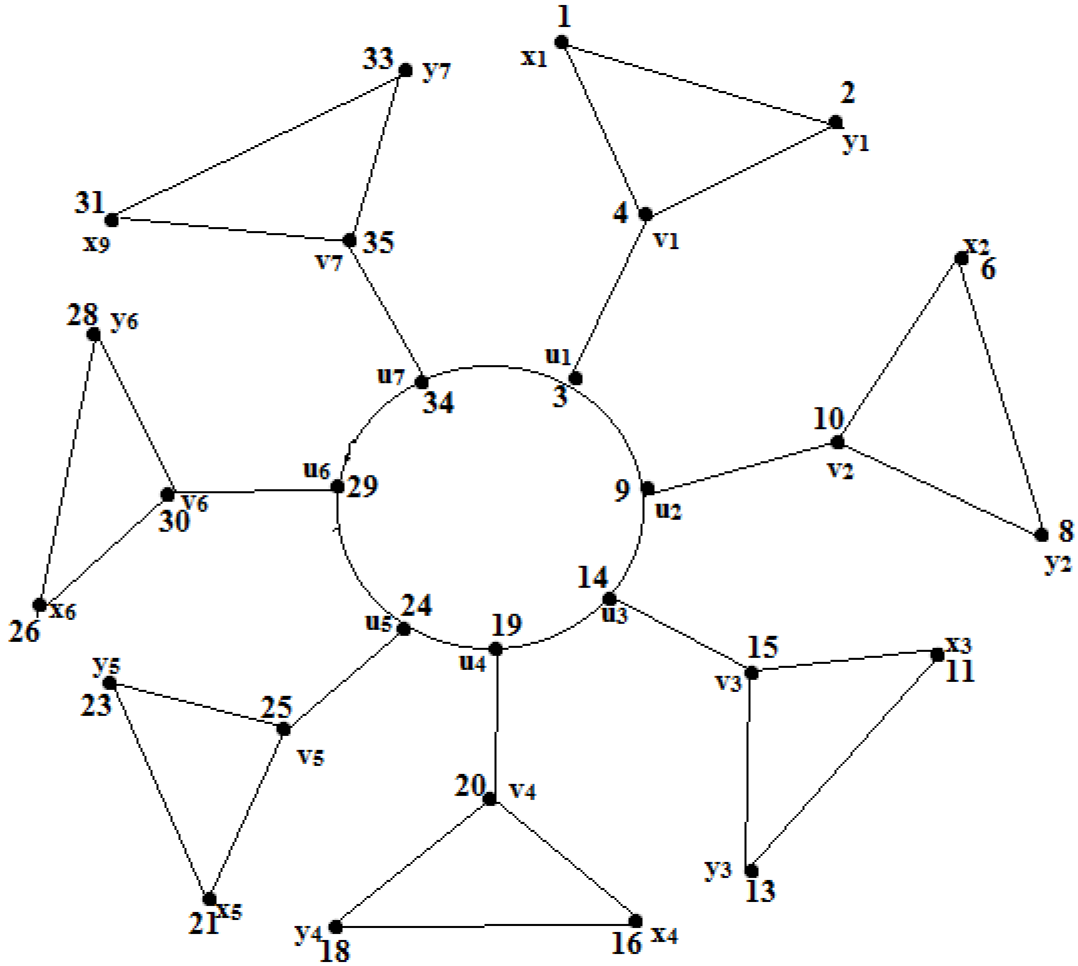

Figure: 6

Theorem 2.11: $D_n A\overline{K_2}$ is a Geometric mean graph.

Proof: Let $u_1 u_2 \ldots u_n u_1$ and $v_1 v_2 \ldots v_n v_1$ be the two cycles of length n. Join u_i and v_i where $1 \leq i \leq n$. The resultant graph is D_n .Let x_i, y_i be the vertices of i^{th} copy of $\overline{K_2}$ which are adjacent to the vertex v_i of D_n . The resultant graph is $D_n A\overline{K_2}$ whose edge set is $E = \{u_i u_{i+1}, v_i v_{i+1} \mid 1 \leq i \leq n-1\} \cup \{v_i x_i, v_i y_i \mid 1 \leq i \leq n\}$

Define a function $f: (D_n A\overline{K_2}) \rightarrow \{1,2,\ldots,q+1\}$ by,

$f(x_i) = 8i-7, \quad 1 \leq i \leq 2$

$f(x_i) = 5i-3, \quad 3 \leq i \leq n$

$f(y_i) = 8i-6, \quad 1 \leq i \leq 2$

$f(y_i) = 5i-2, \quad 3 \leq i \leq n$

$f(v_i) = 5i-2, \quad 1 \leq i \leq 2$

$f(v_i) = 5i-1, \quad 3 \leq i \leq n$

$f(u_i) = 5i-4, \quad 1 \leq i \leq 2$

$f(w_i) = 5i$, $3 \leq i \leq n$

From the above labeling pattern we get distinct edge labels. Thus f provides an Harmonic mean labeling for G. Hence $D_n A \overline{K_2}$ is a Harmonic mean graph and its labeling pattern is shown below

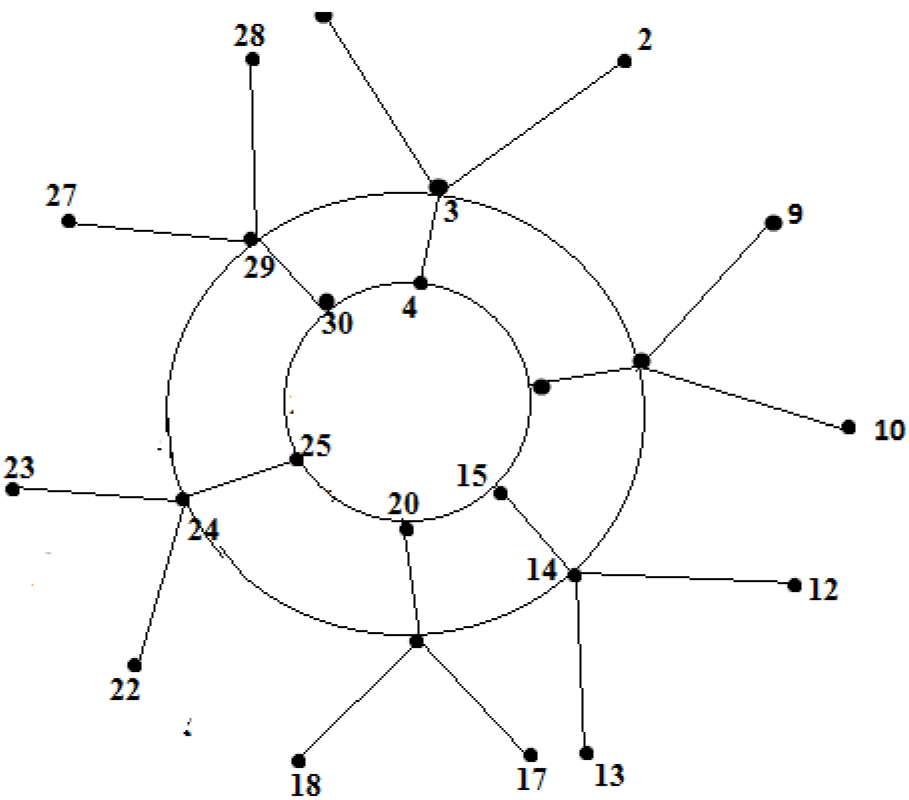

Figure : 7

References:

[1] J.A.Gallian, A dynamic survey of graph labeling. The Electronic Journal of combinators 17#DS6.

[2] F.Harary, Graph theory, Narosa publishing House New Delhi.

[3] S. Somasundram and R.Ponraj, Mean labeling of graphs, National Academy of Science letters vol.26, p210-2013

[4] S.S.Sandhya,and S. Somasundaram, Harmonic mean labeling of graphs International Journal of Mathematics Research vol.6, No.2(2014) pp179-182.