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Inventory Model for Deteriorating Item with Exponential Demand Rate and Partial Backlogging

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Abstract:

We developed an inventory model for deteriorating item with exponential demand rate. Shortage is allowed in this model and is partially backlogged. The backlogged rate is time-dependent for the next replenishment. We have derived the most favorable order quantity model by minimizing the total inventory cost. To explain the model a numerical illustration and sensitivity analysis has been carried out to study the outcome of parameters on variables and the total inventory cost of this model.

Keywords:

Inventory Model**,** Deterioration, Exponential Demand Rate, Partial Backlogging

1. Introduction:

Inventory modeling is a mathematical approach to determine when to order and how much order to minimize the total inventory cost. The deterioration of items plays a significant role in the inventory of goods. The deterioration of goods is always happened in daily life. All types of consumable goods such as vegetables, fruits, milk products etc. are a few examples of objects in which deterioration can take place during the ordinary storage of goods and consequently this loss must be taken into account when analyzing the model. Shortages are permitted and so the customers will wait till the next replenishment. All demands are fulfilled instantly. An inventory model for fashion items deteriorating at the end of a recommended storage period was developed by Whitin [21]. Shah and Jaiswal [19] developed an order-level inventory model for consumable items with a constant deterioration rate. An inventory model for deteriorating items using price discount was established by Jeyaraman and Sugapriya [18]. Wee [20] established an inventory model for deteriorating items with shortages has revealed the economic order quantity with a branded market demand rate. Dave and Patel [17] established an inventory model in the field of deteriorating items with time dependent demand. Donaldson [16] developed the classical no shortage inventory strategy for linear, time-dependent demand. Goyal [13] has developed an EOQ model with constant demand rate under the conditions of permissible delay in payments.

Aggarwal and Jaggi [6] extended the inventory model with an exponential deterioration rate under the condition of permitted delay in payments. Sana [23] considered that the demand rate is price dependent; the deterioration rate is time-dependent and partially backlogged. Ghare and Schrader [3] was the pioneer in the study of inventory model considering the cause of deterioration. In this paper they considered constant rate of deterioration without shortage. Silver [10] proposed an inventory model for deteriorating items with time dependent linear demand. Shah and Shah [5], Goyal and Giri [15] extended review on inventory model with deteriorating items. Covert and Philip [11] developed an inventory model where the time to deterioration is described with two parameters Weibull distribution deterioration. Ghosh and Chaudhuri [22] developed an inventory model for two parameters Weibull deterioration, with shortages and quadratic demand rate. Sanni [14] developed an inventory model for three parameters Weibull deterioration, with shortages and quadratic demand rate.

The aim of the present paper is to give a new approach to the inventory system. Exponential demand rate is an increasing function of time and shortage is allowed. To explain the inventory model a numerical illustration and sensitivity analysis has been carried out to analyze the result of parameters on decision variables and the entire cost of this inventory model.

2. Assumptions

We need the following assumptions for developing mathematical model.

1. The Inventory system consider single item.

2. The demand rate $D(t)$ at any time t is given by

$$D(t)=\begin{cases} e^{\lambda t}, & i(t)>0 \\ \alpha, & i(t)\leq 0 \end{cases}$$

Where $\lambda>0$ and $\alpha>0$ and $i(t)$ is the inventory level at time t.

3. The lead time is zero.

4. Shortage is allowed and partially backlogged.

5. The rate of deterioration $\theta\ (0<\theta<1)$ is constant.

6. Replenishment rate is infinite.

7. The backlogging rate through the shortage phase is inconsistent and depends on the length of the waiting time till the next

replenishment. The partially backlogged rate will be $B(t)=\dfrac{1}{1+\delta(T-t)}$, where $\delta>0$ denote the backlogging

parameter during (t_1,T).

8. $I_a(t)$ is the inventory level at time $t\ (0\leq t\leq t_1)$ in which the product has no deterioration. $I_b(t)$ is the inventory level at

time $t\ (t_1\leq t\leq t_2)$ in which the product has deterioration. $I_c(t)$ is the inventory level at time $t\ (t_2\leq t\leq T)$ in which

the product has shortage.

3. Notations.

Let us consider the following notations for developing mathematical model.

A: Ordering cost per order

H_c : Holding cost per unit time

D_c : Deterioration cost per unit per item

S_c : Shortage cost per unit per item

O_c : Cost of lost sale per unit

$D(t)$: Demand rate at any time t

T: Cycle length considered as annually

t_2 : Inventory becomes zero at that time

$I_{\max}$: Size of initial inventory

Q: Total ordering quantity

$I_a(t)$: Positive inventory at time t during $0 \le t \le t_1$

$I_b(t)$: Positive inventory at time t during $t_1 \le t \le t_2$

$I_c(t)$: Decreasing inventory at time t during $t_2 \le t \le T$

TC: Total average cost per unit time

4. Mathematical Model:

At the beginning of the cycle, the inventory level reaches its maximum $I_{\max}$ units of item at time $t = 0$. The inventory depletes due to exponential demand rate during $(0, t_1)$ .The inventory depletes due to both exponential demand rate and deterioration till it becomes zero during the interval (t_1, t_2) .The inventory level reaches to zero at time $t = t_2$. The inventory $I_a(t)$ is defined by the following differential equation:

$$\frac{dI_a(t)}{dt} = -e^{\lambda t}, \qquad 0 \le t \le t_1 \qquad \dots (1)$$

With condition $I_a(0) = I_{\max}$

The solution of equation (1) is

$$I_a(t) = I_{\max} + \frac{1}{\lambda}\left[1 - e^{\lambda t}\right], \; 0 \le t \le t_1 \qquad \dots (2)$$

Inventory level decreases due to exponential demand rate and deterioration rate during the interval (t_1, t_2) and reaches zero at time t_2 and then shortage occurs. The inventory system $I_b(t)$ is defined by the following differential equation:

$$\frac{dI_b(t)}{dt} + \theta I_b(t) = -e^{\lambda t}, \qquad t_1 \le t \le t_2 \qquad \dots (3)$$

With condition $I_b(t_2) = 0, \; t_1 \le t \le t_2$

The solution of equation (3) is

$$I_b(t) = \frac{1}{\theta+\lambda}\left(e^{-\theta t}\cdot e^{(\theta+\lambda)t_2} - e^{\lambda t}\right), \qquad t_1 \le t \le t_2 \qquad \ldots (4)$$

Considering continuity of $I(t)$ at $t = t_1$, it follows that $I_a(t_1) = I_b(t_1)$

$$I_{\max} + \frac{1}{\lambda}\left[1-e^{\lambda t_1}\right] = \frac{1}{\theta+\lambda}\left[e^{-\theta t_1}\cdot e^{(\theta+\lambda)t_2} - e^{\lambda t_1}\right]$$

The maximum level of inventory per cycle is

$$I_{\max} = \frac{1}{\theta+\lambda}\left(e^{-\theta t_1}\cdot e^{(\theta+\lambda)t_2} - e^{\lambda t_1}\right) - \frac{1}{\lambda}\left(1-e^{\lambda t_1}\right) \qquad \ldots (5)$$

Substitute the value of $I_{\max}$ from equation (5) into equation (2), we have

$$I_a(t) = \frac{1}{\theta+\lambda}\left(e^{-\theta t_1}\cdot e^{(\theta+\lambda)t_2} - e^{\lambda t_1}\right) + \frac{1}{\lambda}\left(e^{\lambda t_1} - e^{\lambda t}\right), \quad 0 \le t \le t_1 \qquad \ldots (6)$$

The inventory level reaches to zero at time t_2 and then shortage occurs. The rate of demand is partially backlogged. The backlogged demand is given by the following differential equation:

$$\frac{dI_c(t)}{dt} = \frac{-\alpha}{1+\delta(T-t)}, \qquad t_2 \le t \le T \qquad \ldots (7)$$

With condition $I_c(t_2) = 0, \quad t_2 \le t \le T$

The solution of equation (7) is

$$I_c(t) = -\frac{\alpha}{\delta}\left[\ln\left(1+\delta(T-t_2)\right) - \ln\left(1+\delta(T-t)\right)\right], t_2 \le t \le T \qquad \ldots (8)$$

The maximum amount of demand backlogged per cycle by putting $t = T$ in equation (8), we have

$$S = -I_c(T) = \frac{\alpha}{\delta}\left[\ln\left(1+\delta(T-t_2)\right)\right], t_2 \le t \le T \qquad \ldots (9)$$

From equations (5) and (9), we obtained the order quantity Q per cycle is

$$Q = I_{\max} + S \quad Q = \frac{1}{\theta-\lambda}\left[e^{-\theta t_1}\cdot e^{(\theta+\lambda)t_2} - e^{\lambda t_1}\right] - \frac{1}{\lambda}\left[1-e^{\lambda t_1}\right] + \frac{\alpha}{\delta}\left[\ln\left(1+\delta(T-t_2)\right)\right], t_2 \le t \le T$$

Ordering cost per order is $\frac{A}{T}$ ... 10

The holding cost per cycle is

$$H_c=\int_0^{t_1} I_a(t)\,dt+\int_{t_1}^{t_2} I_b(t)\,dt$$

$$=\frac{e^{\lambda t_2}}{\theta+\lambda}\left\{e^{-\theta t_1}e^{\theta t_2}\left(t_1+\frac{1}{\theta}\right)-\left(\frac{1}{\theta}+\frac{1}{\lambda}\right)\right\}+e^{\lambda t_1}\left(\frac{t_1}{\lambda}-\frac{t_1}{\theta+\lambda}-\frac{1}{\lambda^2}+\frac{1}{\lambda(\theta+\lambda)}\right)+\frac{1}{\lambda^2} \quad \dots 11$$

The cost of deterioration per cycle is

$$D_c=I_b(t_1)-\int_{t_1}^{t_2} D(t)=e^{\lambda t_2}\left(\frac{e^{-\theta t_1}e^{\theta t_2}}{\theta+\lambda}-\frac{1}{\lambda}\right)+e^{\lambda t_1}\left(\frac{1}{\lambda}-\frac{1}{\theta+\lambda}\right) \quad \dots 12$$

The shortage cost per cycle is

$$S_c=\int_{t_2}^{T}\left[-I_c(t)\right]dt=\frac{\alpha}{\delta}\left[(T-t_2)-\frac{\ln(1+\delta(T-t_2))}{\delta}\right] \quad \dots 13$$

The opportunity cost per cycle is

$$O_c=\int_{t_2}^{T}\alpha\left[1-\beta(T-t)\right]dt=\alpha\left[(T-t_2)-\frac{\ln(1+\delta(T-t_2))}{\delta}\right] \quad \dots 14$$

Total relevant cost is

$$TC(t_2)=\frac{1}{T}(A+H_c+D_c+S_c+O_c)$$

$$TC(t_2)=\left[B+Ce^{Dt_2}-Ee^{\lambda t_2}-Ft_2-F\frac{\ln(G-\delta t_2)}{\delta}\right] \quad \dots 15$$

Where,

$$B=\frac{A}{T}+\frac{1}{\lambda^2 T}+\frac{e^{\lambda t_1}}{T}\left(\frac{t_1+1}{\lambda}-\frac{1}{\lambda^2}-\frac{t_1+1}{\theta+\lambda}+\frac{1}{\lambda(\theta+\lambda)}\right)+\alpha\left(\frac{1}{\delta}+1\right),$$

$$C=\frac{e^{-\theta t_1}}{(\theta+\lambda)T}\left(t_1+1+\frac{1}{\theta}\right),D=\theta+\lambda,E=\frac{1+\theta}{\theta\lambda T},F=\frac{\alpha}{T}\left(\frac{1}{\delta}+1\right),G=1+\delta T$$

5. Solution Procedure:

$$\frac{\partial TC}{\partial t_2}=-F+CDe^{Dt_2}-\lambda Ee^{\lambda t_2}+\frac{F}{G-\delta t_2} \quad \dots 16$$

$$\frac{\partial^2 TC}{\partial t_2^{\,2}}=CD^2e^{Dt_2}-\lambda^2Ee^{\lambda t_2}+\frac{\delta F}{(G-\delta t_2)^2} \quad \dots 17$$

Main objective is to minimize the total relevant cost of the inventory model. The necessary condition to minimize the total relevant cost is

$\frac{\partial TC}{\partial t_2}=0$, We have

$$\delta\lambda Et_2e^{\lambda t_2}-\delta CDt_2e^{Dt_2}+CDGe^{Dt_2}-\lambda EGe^{\lambda t_2}+\delta Ft_2+F-FG=0 \quad \dots 18$$

Using the software Mathematica, we can calculate the optimal value of t_2 by equation (18) and the optimal value $TC\ t_2$ of the total relevant cost is determined by equation (15). The optimal value of t_2 satisfy the sufficient condition for minimizing total relevant cost $TC\ t_2$ is

$$\frac{\partial^2 TC}{\partial t_2{}^2} > 0 \qquad \ldots 19$$

The sufficient condition is satisfied.

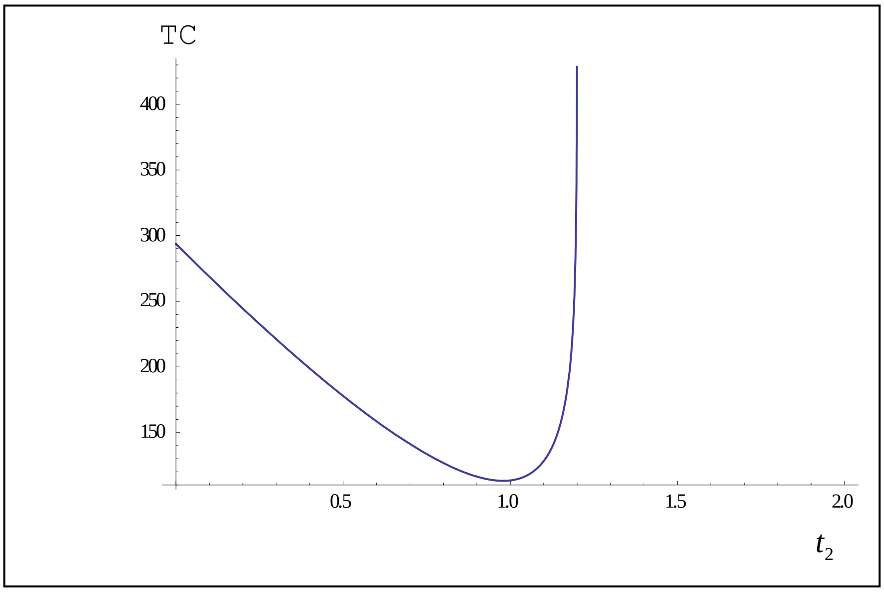

Fig. 1. Graphical representation of total relevant cost TC and time t_2 .

6. Numerical Example:

Let us consider

$$A=100, H_c=30, D_c=15, S_c=25, O_c=10, \alpha=20, \delta=5, t_1=0.5, \lambda=0.1, \theta=0.5, T=1 year.$$
$$B=146.1, C=4.5, D=0.6, E=30, F=24, G=6$$

Thus, the optimal value of t_2 is $t_2^* \to 0.98$. The optimal ordering quantity is $OQ=1.7$. The minimum relevant cost is $TC=113.17$.

7. Sensitivity Analysis:

To know, how the optimal solution is affected by the values of parameters, we derive the sensitivity analysis for some parameters. The particular values of some parameters are increased or decreased by $+10\%, -10\%$ and $+20\%, -20\%$. After that, we derive the value of t_2 and TC with the help of increased or decreased values of A, H_c, D_c, S_c and O_c. The result of the minimum relevant cost is existing in the following table 1.

Table1:

Parameters	Actual Values	+10% Increased	+20% Increased	–10% Decreased	–20% Decreased
A	100	110	120	90	80
H_c	30	33	36	27	24
D_c	15	16.5	18	13.5	12
S_c	25	27.5	30	22.5	20
O_c	10	11	12	9	8
t_2	0.98	0.98	0.98	0.98	0.98
TC	112.89	124.5	135.8	101.85	90.5

From the result of above table, we observe that total relevant cost and ordering quantity is affected by ordering cost, holding cost, deterioration cost, shortage cost and opportunity cost.

Convexity on the cost:

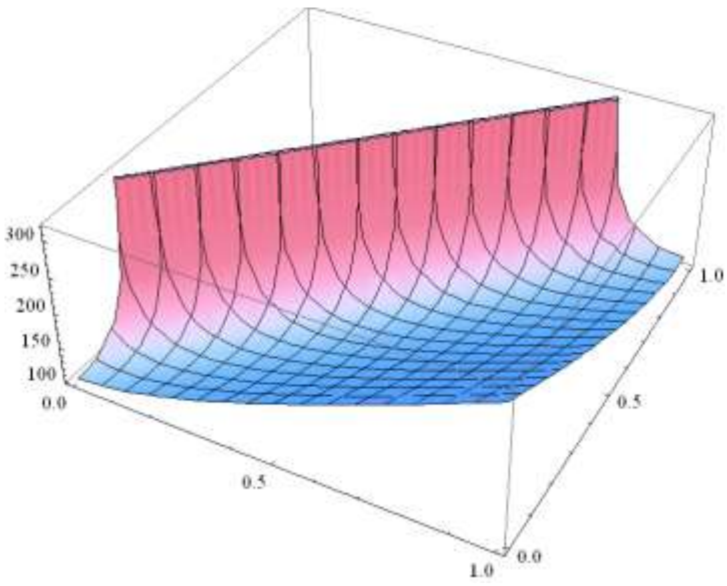

Fig.2. Effects of changes of the parameters on total relevant cost.

8. Conclusion:

In this paper, an optimal inventory model developed for deteriorating item with exponential rate of demand. Exponential rate of demand is an increasing function of time. Shortage is allowed in the inventory system and is partially backlogged. The backlogging rate is depends on the lead time for the next replenishment. We have derived the most favorable order quantity model by minimizing the total inventory cost. Advance research in this way can be carried out such as finite replenishment rate, ramp type demand, permissible delay in payments and quantity discounts.

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References:

[1] P.L. Abad, "Optimal pricing and lot-sizing under conditions of perish ability and partial backordering," Management science, 42, 1093-1104, 1996.

[2] P.L. Abad, "Optimal pricing and order-sizing for a reseller under partial backlogging," Computer and Operations Research, 28, 53-65, 2001.

[3] P.M. Ghare, and G.H. Schrader, "A model for exponentially decaying inventory system," Journal of Operational Research, 134, 1-16, 1963.

[4] M. Hariga, "An EOQ model for deteriorating items with shortages and time–varying demand," Journal of the Operational Research Society, 46, 398-404, 1996.

[5] N.H. Shah, and Y.K. Shah, "Literature survey on inventory model for deteriorating items," Economic Annals, 44, 221-237, 2000.

[6] S.P. Aggarwal, and C.K. Jaggi, "Ordering policies of deteriorating items under permissible delay in payments," Journal of Operational Research Society, 46, 658-662, 1995.

[7] T. Chakrabarti, and K.S. Chaudhary, "An EOQ model for deteriorating items with a linear trend in demand and shortages in all cycles," International Journal of Production Economics, 49, 205-213, 1997.

[8] R. Begum, S.K. Sahu, and R.R. Sahoo, "An EOQ model for deteriorating items with Weibull distribution deterioration, quadratic demand and shortages," Applied Mathematical Sciences, 4, 271-288, 2010.

[9] C.Y. Dye, "Joint pricing and ordering policy for a deteriorating inventory with partial backlogging," Omega, 35, 184-189, 2007.

[10] E.A. Silver, "A simple inventory replenishment decision rule for a linear trend in demand," Journal of Operational Research, 30, 71-75, 1979.

[11] R.P. Convert, and G.C. Philip, "An EOQ model for items with Weibull distribution deterioration," AIIE Trans., 5, 323-326, 1973.

[12] G.C. Philip, "A generalized EOQ model for items with Weibull distribution deterioration," AIIE Trans., 6, 159-162, 1974.

[13] S.K. Goyal, "Economic order quantity model under condition of permissible delay in payments," Journal of Operational Research Society, 36, 335-338, (1985).

[14] S.S. Sanni, "An EOQ model inventory model with time dependent Weibull deterioration and trend demand," M.Sc. Thesis, University of Nigeria, 2012.

[15] S.K. Goyal, and B.C. Giri, "Recent trends in modeling of deteriorating inventory," European Journal of Operational Research, 134, 1-16, 2001.

[16] W.A. Donaldson, "Inventory replenishment policy for a linear trend in demand: an analytical solution," Operational Research Q., 28, 663-670, 1977.

[17] U. Dave, and L.K. Patel, "On (T,Si) policy inventory model for deteriorating items with time proportional demand," Journal of Operational Research Society, 32,137-142, 1981.

[18] K. Jayaraman, and C. Sugapriya, "Determining a common production cycle time for an EPQ model with non-instantaneous deteriorating items allowing price discount using permissible delay in payments," ARPN Journal of Engineering and Applied Sciences, 3,26-30, 2008.

[19] Y.K. Shah, and M.C. Jaiswal, "An order-level inventory model for a system with constant rate of deterioration," Opsearch, 14,174-184, (1977).

[20] H.M. Wee, "A deterministic lot size inventory model for deteriorating items with shortages and a declining market," Computer and Operations Research, 22, 345-356, (1995).

[21] T.M. Whitin, "Theory of inventory management," Princeton University Press, Princeton, NJ, 62-72, (1957).

[22] S.K. Ghosh, and K.S.Chaudhuri, "An order-level inventory model for a deteriorating item with Weibull distribution deterioration, time quadratic demand and shortages," Adv. Model. Optim. 6(1), 21-35, 1979.

[23] S.S. Sana, "Optimal selling price and lot-size with time varying deterioration and partial backlogging," Applied Mathematics and Computation, 217, 185-194, 2010a.