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Computational Analysis of Compound Redundant System with Respect to Reliability Analysis

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Abstract — *This paper presents an overview of results related to the computational analysis of compound redundant system under the head-of-line repair policy. Suppose a compound system consisting of two subsystems designated as 'ψ_1' and 'ψ_2' connected in series. Subsystem 'ψ_1' consists of N non-identical units in series, while the subsystem 'ψ_2' consists of three identical components in parallel redundancy.*

Keywords — *Reliability theory, stochastic processes, Laplace transforms and Cost profit function*.

I. Introduction

In reliability analysis, it has been mostly assumed that the system has an immediate repair facility and after detection of failure, the unit goes under repair. But in many cases it is not advisable to always have a repair facility. In this paper the authors have considered a compound system consisting of two subsystems designated as 'ψ_1' and 'ψ_2' connected in series. Subsystem 'ψ_1' consists of *N* non-identical units in series, while the subsystem 'ψ_2' consists of three identical components in parallel redundancy. In this model it is considered that the system goes to complete breakdown state if any unit of subsystem '*P*' fails or more than 1 unit of subsystem 'ψ_2' is in the failed condition. Also, the system works with reduced efficiency if one unit of subsystem 'ψ_2' failed. The system as a whole can also fail from normal efficiency state if there is any failure due to environmental reasons. *Supplementary variable technique and Laplace transforms have been utilized to obtain various state probabilities and the cost incurred for the system is obtained*. Failure and repair times of the units follow exponential and general time distributions respectively.

Their many research [1, 2], different techniques have been applied to evaluate the reliability of distribution system, including distributed generation such as an analytical technique using the load duration curve, distributed processing technique, Characteristic function based approach for computing the probability distributers of reliability indices, probabilistic method for assessing the reliability and quantity of power supply to a customer, composite load point model, practical reliability assessment algorithm, validation method and impact of substation on distribution reliability respectively.

II. ASSUMPTIONS

(i) initially, all units are good
(ii) A failed unit is repaired at a single service channel.
(iii) The parallel subsystem is composed of three identical units, while series subsystem is composed of *N* non-identical units.
(iv) Failures are statistically independent.
(v) Environmental failure rates are constant.
(vi) after repair, units work like new.
(viii) First come first served (Head-of-line) repair policy is being adopted.

III. Mathematical Symbol

(i) $f' / f_i / f_E$: Constant failure rates of any unit of subsystem ψ_2^th^ unit of subsystem ψ_1/environmental failure.

(ii) $r_1(x)/r_2(y)/r_3(z)/r_4(\alpha)$: Repair rates with general time distribution from states S_4 to S_0, S_1 to S_0, or S_3 to S_4, S_2 to S_0, S_5 to S_0.

(iii) $P_N^2(y,t)\,\Delta$: The probability that at time '*t*', the system is in degraded state due to the failure of one unit of subsystem ψ_2. The elapsed repair time lies in the interval (y, y + Δ).

(iv) $P_N^F(z,t)\,\Delta$: The probability that at time '*t*', the system is in failed state due to the failure of more than one unit of subsystem ψ_2, the elapsed repair time lies in the interval (z, z + Δ).

(v) $P_i^3(x,t)\,\Delta$: The probability that at time 't', the system is in failed state due to the failure of i^{th} unit of subsystem ψ_1. The elapsed repair time lies in the interval (x, $x + \Delta$).

(vi) $P_i^2(y,t)\,\Delta$: The probability that at time 't' the repair time lies in the interval (y, y + Δ).

(vii) $P_E(\alpha,t)\,\Delta$: The probability that at time 't', the system is in failed state, due to the environmental failure, the elapsed repair time lies in the interval (α, $\alpha + \Delta$).

IV. TRANSITION STATE DIAGRAM

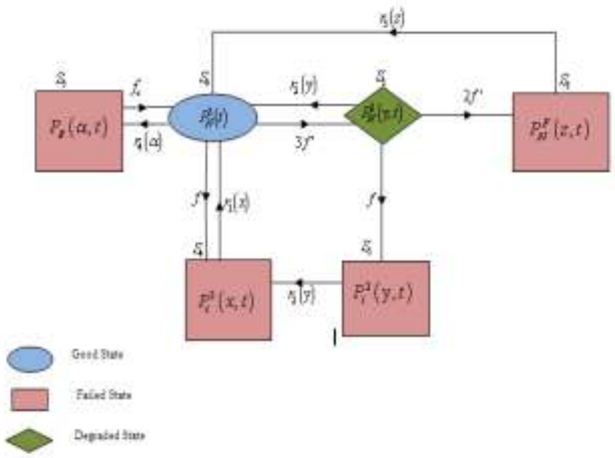


Figure 1. represents the state transition diagram of the system.

V. FORMULATION OF MATHEMATICAL MODEL

The analysis crucially depends on the method of supplementary variables technique and the supplementary variable x denotes the time that a unit has been elapsed undergoing repair. Viewing the nature of the problem, we obtain the following set of difference-differential equations:

$$[D_t + 3f' + f + f_e]\, P_N^3(t) = \int_0^\infty P_i^2(y,t) r_2(y)\, dy + \int_0^\infty P_N^F(z,t) r_3(z)\, dz$$

$$+ \int_0^\infty P_i^3(x,t) r_1(x)\, dx + \int_0^\infty P_E(\alpha,t) r_4(\alpha)\, d\alpha \quad \dots (1)$$

$$\left[D_t + D_y + 2f' + f + r_2(y)\right] P_N^2(y,t) = 0 \quad \dots (2)$$

$$[D_t + D_z + r_3(z)]\, P_N^F(z,t) = 0 \quad \dots (3)$$

$$[D_t + D_x + r_1(z)]\, P_i^3(x,t) = 0 \quad \dots (4)$$

$$\left[D_t + D_y + r_2(y)\right] P_i^2(y,t) = f\, P_N^2(y,t) \quad \dots (5)$$

$$[D_t + D_\alpha + r_4(\alpha)]\, P_E(\alpha,t) = 0 \quad \dots (6)$$

A. *Boundary Conditions:*

$$P_N^2(0,t) = 3\, f' P_N^3(t) \quad \dots (7)$$

$$P_N^F(0,t) = 2\, f' P_N^2(t) \quad \dots (8)$$

$$P_i^3(0,t) = f\, P_N^3(t) + \int_0^\infty P_i^2(y,t) r_2(y)\, dy \quad \dots (9)$$

$$P_i^2(0,t) = 0 \quad \dots (10)$$

$$P_i(0,t) = f_e\, P_N^3(t) \quad \dots (11)$$

B. *Initial Conditions:*

$$P_N^3(0) = 1 \text{ Otherwise zero} \quad \dots (12)$$

Where $\frac{\partial}{\partial t} \equiv D_t, \frac{\partial}{\partial x} \equiv D_x, \frac{\partial}{\partial y} \equiv D_y, \frac{\partial}{\partial z} \equiv D_z$ and $\frac{\partial}{\partial \alpha} \equiv D_\alpha$

VI. SOLUTION OF MATHEMATICAL MODEL

Taking Laplace transforms of equations (1) through (11) and using initial conditions one may obtain:

$$[s + 3f' + f + f_e]\, \bar{P}_N^3(s) = 1 + \int_0^\infty \bar{P}_N^2(y,s) r_2(y)\, dy + \int_0^\infty \bar{P}_N^F(z,s) r_3(z)\, dz$$

$$+ \int_0^\infty \bar{P}_N^3(x,s) r_1(x)\, dx$$

$$+ \int_0^\infty \bar{P}_E(\alpha,s) r_4(\alpha)\, d\alpha \quad \dots (13)$$

$$\left[D_y + s + 2f' + r_2(y)\right] \bar{P}_N^2(y,s) = 0 \quad \dots (14)$$

$$[D_z + s + r_3(z)]\, \bar{P}_N^F(z,s) = 0 \quad \dots (15)$$

$$[D_x + s + r_1(x)]\, \bar{P}_i^3(x,s) = 0 \quad \dots (16)$$

$$\left[D_y + s + r_2(y)\right] \bar{P}_i^2(y,s) = f \bar{P}_N^2(y,s) \quad \dots (17)$$

$$[D_\alpha + s + r_4(\alpha)]\, \bar{P}_E(\alpha,s) = 0 \quad \dots (18)$$

$$\bar{P}_N^2(0,s) = 3f' \bar{P}_N^3(s) \quad \dots (19)$$

$$\bar{P}_N^F(0,s) = 2f' \bar{P}_N^2(s) \quad \dots (20)$$

$$\bar{P}_i^3(0,s) = f\, \bar{P}_N^3(s) + \int_0^\infty \bar{P}_i^2(y,s)\, r_2(y)\, dy \quad \dots (21)$$

$$\bar{P}_i^2(0,s) = 0 \quad \dots (22)$$

$$\bar{P}_E(0,s) = f_e\, \bar{P}_N^3(s) \quad \dots (23)$$

After solving the above equations, we get finally

$$\bar{P}_N^3(s) = \frac{1}{A(s)} \quad \dots (24)$$

$$\bar{P}_N^2(s) = \frac{3f'}{A(s)} D_{r_2}(s + 2f' + f) \quad \dots (25)$$

$$\bar{P}_N^F(s) = \frac{6f'^2}{A(s)} D_{r_2}(s + 2f' + f) D_{r_3}(s) \quad \dots (26)$$

$$\bar{P}_i^2(s) = \frac{3f\, f'}{(2f' + f)\, A(s)} \left[D_{r_2}(s) - D_{r_2}(s + 2f' + f)\right] \quad \dots (27)$$

$$\bar{P}_i^3(s) = \frac{f}{A(s)} \left[1 + \frac{3f'}{2f' + f} \left\{\bar{S}_{r_2}(s) \bar{S}_{r_2}(s + 2f' + f)\right\}\right] D_\eta(s)$$

$$\dots (28)$$

$$\bar{P}_E(s) = \frac{f_e}{A(s)} D_{r_4}(s) \quad \dots (29)$$

Where

$$A(s) = s + 3f' + f + f_e - 3f' \bar{S}_{r_2}(s + 2f' + f)$$

$$-6f' D_{r_2}(s + 2f' + f) \bar{S}_{r_3}(s)$$

$$-f\left[1 + \frac{3f'}{2f' + f}\left\{\bar{S}_{r_2}(s) - \bar{S}_{r_2}(s + 2f' + f)\right\}\right] \bar{S}_\eta(s) - f_e \bar{S}_{r_4}(s)$$

$$\dots (30)$$

It is interesting to note that sum of relation (24) through (29) $= \frac{1}{s}$

VII. ERGODIC BEHAVIOUR OF SYSTEM

Using Abel's Lemma $\lim_{s\to 0} s\,\bar{F}(s) = \lim_{t\to\infty} F(t) = F$ (say), provided the limit on the R.H.S. exists, the time independent probabilities are obtained as follows by making use above lemma in the relations (24) through (29)

$$P_N^3 = \frac{1}{A'(0)} \qquad \text{... (31)}$$

$$P_N^2 = \frac{3f'}{A'(0)} D_{r_2}(2f' + f) \qquad \text{... (32)}$$

$$P_N^F = \frac{6f'}{A'(0)} D_{r_2}(2f' + f) M_{r_3} \qquad \text{... (33)}$$

$$P_i^2 = \frac{3ff'}{(2f' + f)A'(0)}\left[M_{r_3} - D_{r_2}(2f' + f)\right] \qquad \text{... (34)}$$

$$P_i^3 = \frac{f}{A'(0)} M_{r_1} \qquad \text{... (35)}$$

$$P_E = \frac{f_e}{A'(0)} M_{r_4} \qquad \text{... (36)}$$

Where, $A'(0) = \left[\frac{d}{ds}A(s)\right]_{s=0}$ and M_k = Mean time to repair k^{th} unit

VIII. EVALUATION OF UP AND DOWN STATE PROBABILITIES

We have,

$$\bar{P}_{up}(s) = \frac{1}{s + 3f' + f + f_e}\left[1 + \frac{3f'}{s + 2f' + f}\right] \qquad \text{... (37)}$$

On both sides taking inverse Laplace transform, we get

$$\bar{P}_{up}(t) = \left[1 - \frac{3f'}{f' + f_e}\right]\exp\{-(3f' + f_e + f)t\} + \frac{3f'}{f' + f_e}\exp\{-(2f' + f)t\} \qquad \text{... (38)}$$

and $P_{down}(t) = 1 - P_{up}(t)$ ... (39)

IX. COST PROFIT ANALYSIS FUNCTION

The cost profit function is defined as,

$$G(t) = C_1\int_0^t P_{up}(t)\,dt - C_2 t \qquad \text{... (40)}$$

Where,
G(*t*) = Expected cost for total time,
C_1 = Revenue cost per unit up time and
C_2 = Service cost per unit time
Putting the value of $P_{up}(t)$ in equation (40), we get

$$G(t) = C_1\left(1 - \frac{3f'}{f' + f_e}\right)\left[\frac{1 - \exp\{-(3f' + f_e + f)t\}}{3f' + f_e + f}\right] + C_1\left(\frac{3f'}{f' + f_e}\right)\left[\frac{1 - \exp\{-(2f' + f)t\}}{2f' + f}\right] - C_2 t \qquad \text{... (41)}$$

X. RELIABILITY OF THE SYSTEM

The Reliability of the system is

$$\bar{R}(s) = \frac{1}{s + 3f' + f + f_e}$$

On both sides taking inverse Laplace transform, we get

$$R(t) = \exp\left[-(3f' + f + f_e)t\right] \qquad \text{... (42)}$$

XI. NUMERICAL COMPUTATION

Substituting $f = 0.001$, $f' = 0.002$, $f_e = 0.003$ and $c_1 = 2$, $c_2 = 1$and all repair rates are zero, then from equations (38), (41) and (42), we get

A. *Availability of system*

$$P_{up}(t) = -0.2\exp(-0.010t) + 1.2\exp(-0.005t)$$

B. *Cost profit function of system*

$$G(t) = -0.4\left[\frac{1 - \exp(-0.010t)}{0.010}\right] + 2.4\left[\frac{1 - \exp(-0.005t)}{0.005}\right] - t$$

C. *Reliability of system*

$$R(t) = \exp(-0.010t)$$

XII. TABLE I AND FIGURE 2

COMPUTATION OF AVAILABILITY OF SYSTEM WITH RESPECT TO TIME

S.No.	t	Pup(t)
1	0	1
2	1	0.996005
3	2	0.9920201
4	3	0.9880452
5	4	0.9840805
6	5	0.980126
7	6	0.9761817
8	7	0.9722477
9	8	0.9683241
10	9	0.9644107
11	10	0.9605078
12	11	0.9566154
13	12	0.9527334
14	13	0.9488619
15	14	0.9450009
16	15	0.9411506

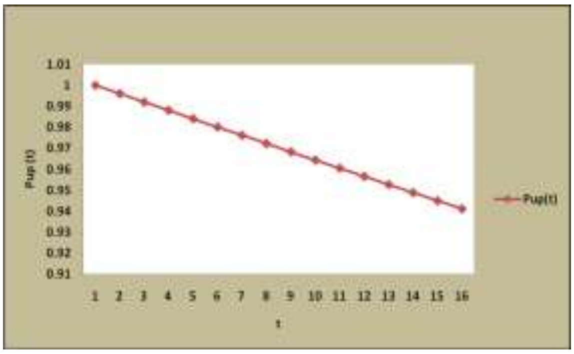


TABLE III AND FIGURE 3
COMPUTATION OF COST FUNCTION WITH RESPECT TO TIME

S.No.	*t*	*G(t)*
1	0	0
2	1	0.9960033
3	2	1.9840267
4	3	2.9640903
5	4	3.9362144
6	5	4.9004192
7	6	5.8567252
8	7	6.805153
9	8	7.7457231
10	9	8.6784561
11	10	9.603373
12	11	10.520494
13	12	11.429841
14	13	12.331435
15	14	13.225296
16	15	14.111446

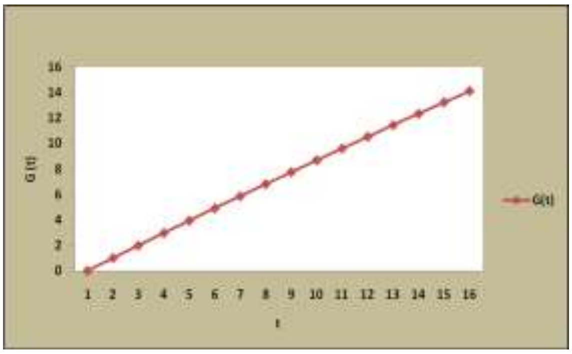
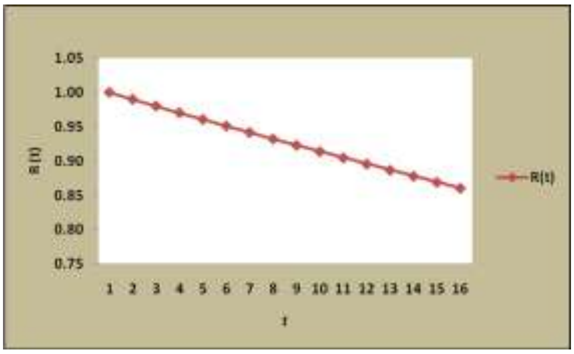


TABLE IIII AND FIGURE 4
COMPUTATION OF RELIABILITY OF SYSTEM WITH RESPECT TO TIME

S.No.	*t*	*R(t)*
1	0	1.00
2	1	0.99
3	2	0.98
4	3	0.97
5	4	0.96
6	5	0.95
7	6	0.94
8	7	0.93
9	8	0.92
10	9	0.91
11	10	0.90
12	11	0.90
13	12	0.89
14	13	0.88
15	14	0.87
16	15	0.86



XIII. CONCLUSIONS

The *Table-I* and *Figure 2* provide information how availability of the complex engineering repairable system change with respect to time when failure rate increases, then availability of system is decreases.

The *Table-II* & *Figure 3* when revenue cost per unit time C_1 and C_2 are fixed, then one can conclude by observing this graph that as cost increase with respect to time *t*.

The *Table-III* and *Figure 4* provide information how reliability of the complex engineering repairable system changes with respect to the time when failure rate increases reliability of the system decreases.

Hence the present study clearly proves the importance of head-of line repair policy in comparison of [5-6] which seem to be possible in many engineering systems when it is analysed with the help of the copula.

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