Geometric mean labeling of some more Disconnected Graphs

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Abstract

A Graph G = (V,E) with p vertices and q edges is said to be a Geometric mean graph if is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2…..q+1 is such a way that when each edge e=uv is labeled with f(e=uv) = $\lceil \sqrt{f(u)f(v)} \rceil$ (or) $\lfloor \sqrt{f(u)f(v)} \rfloor$, then the resulting edge labels are distinct. In this case f is called Geometric mean labeling of G. In this paper we prove that some disconnected graphs are Geometric Mean graphs.

Key Words: *Graph, Geometric Mean labeling, Path, Cycle, Comb, Ladder etc.*

1.Introduction

The graph considered here are finite and undirected graph

$G = (V, E)$with$p$ vertices and q edges. For a detailed survey of graph labeling we refer to Gallain [1]. For all other standard terminology and notations we follow Harary [2]. S. Somasundaram and P. Vidhyarani introduced the concept of Geometric Mean labeling of graphs in [3] and studied their behavior in [4], [5], [6] and [7]. In this paper we investigate the Geometric mean labeling behavior of some disconnected graphs. The following definitions are useful for our present study.

Definition1.1: A Graph G = (V,E) with p vertices and q edges is said to be a Geometric mean graph if is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2…..q+1 is such a way that when each edge e=uv is labeled with f(e=uv) = $\lceil \sqrt{f(u)f(v)} \rceil$ (or) $\lfloor \sqrt{f(u)f(v)} \rfloor$, then the resulting edge labels are distinct. In this case f is called *Geometric mean labeling* of G.

Definition1.2: The *union* of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $E = E_1 \cup E_2$

Definition1.3: The *Corona* of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i[th]vertex of G_1 is adjacent to every vertex in the i[th] copy of G_2.

2.Main Results

Theorem2.1:$C_m \cup (P_n \odot K_1)$ is a Geometric Mean graph.

Proof: Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m . Let $v_1 v_2 \cdots v_n$ be the path P_n and let w_i be the vertex which is joined to the vertexv_i , $1 \le i \le n$of the path P_n. Let$G = C_m \cup (P_n \odot K_1)$.

Define a function $f: V(G) \to \{1,2, \dots, q+1\}$ by

$f(u_i) = i , 1 \le i \le m$

$f(v_i) = m + 2i - 1 , 1 \le i \le n$

$f(w_i) = m + 2i , 1 \le i \le n$

Then the edge labels are distinct.

Hence f is a Geometric Mean Labeling of G.

Example2.2: The Geometric Mean labeling of $C_8 \cup (P_5 \odot K_1)$ is given below.

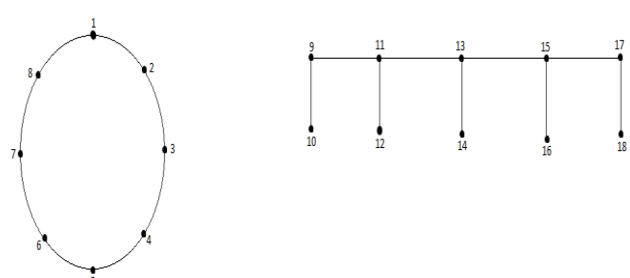

Figure1

Theorem2.3: $(C_m \odot K_1) \cup (P_n \odot K_1)$ is a Geometric Mean graph.

Proof: Let C_m be the cycle $u_1 u_2 \cdots u_m u_1$ and let v_i be the pendent vertex attached to u_i, $1 \le i \le m$. Let $P_n = w_1 w_2 \cdots w_n$ be the path on n vertices. Join a vertex t_i to w_i, $1 \le i \le n$. Let $G = (C_m \odot K_1) \cup (P_n \odot K_1)$.

Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$f(u_i) = 2i - 1$, $1 \le i \le 2$

$f(u_i) = 2i$, $3 \le i \le m$

$f(v_i) = 2i$, $1 \le i \le 2$

$f(v_i) = 2i - 1$, $3 \le i \le m$

$f(w_i) = 2m + 2i - 1$, $1 \le i \le n$

$f(t_i) = 2m + 2i$, $1 \le i \le n$

Then the edge labels are distinct. Hence f is a Geometric Mean Labeling of G.

Example2.4: The Geometric Mean labeling of $(C_6 \odot K_1) \cup (P_5 \odot K_1)$ is given below.

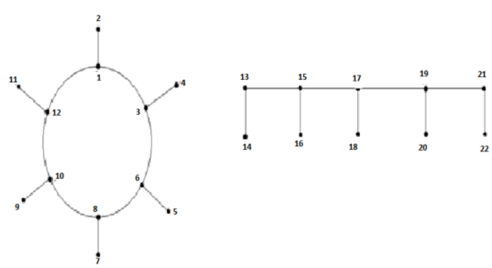

Figure2

Theorem2.5: $(C_m \odot \overline{K_2}) \cup (P_n \odot K_1)$ is a Geometric Mean graph.

Proof: Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m and let v_i, w_i be the vertices which are joined to the vertex u_i, $1 \le i \le m$ of the cycle C_m. Let $s_1 s_2 \cdots s_n$ be the path P_n and let t_i be the vertex which is joined to the vertex s_i $(1 \le i \le n)$ of the path P_n. Let $G = (C_m \odot \overline{K_2}) \cup (P_n \odot K_1)$.

Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$f(u_i) = 3i - 1$, $1 \le i \le m$

$f(v_i) = 3i - 2$, $1 \le i \le m$

$f(w_i) = 3i$, $1 \le i \le m$

$f(s_i) = 3m + 2i - 1$, $1 \le i \le n$

$f(t_i) = 3m + 2i$, $1 \le i \le n$

From the above labeling pattern, we get distinct edge labels. Hence f is a Geometric Mean Labeling of G.

Example2.6: The Geometric Mean labeling of $(C_6 \odot \overline{K_2}) \cup (P_6 \odot K_1)$ is given below.

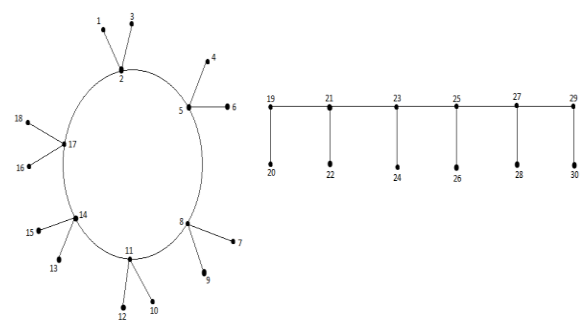

Figure3

Theorem2.7: $C_m \cup L_n$ is a Geometric Mean graph.

Proof: Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m. Let L_n be the Ladder graph with vertices v_i and w_i, $1 \le i \le n$. Let $G = C_m \cup L_n$.

Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$f(u_i) = i$, $1 \le i \le m$

$f(v_i) = m + 3i - 2$, $1 \le i \le n$

$f(w_i) = m + 3i - 1$, $1 \le i \le n$

Then the edge labels are distinct.

Hence f is a Geometric Mean Labeling of G.

Example2.8: The Geometric Mean labeling of $C_7 \cup L_6$ is given below.

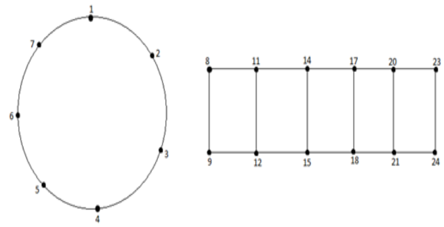

Figure4

Theorem2.9:$(C_m \odot K_1) \cup L_n$ is a Geometric Mean graph.

Proof:Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m and letv_i be the vertex which is joined to the vertex u_i , $1 \leq i \leq m$ of the cycle C_m . Letx_i andy_i , $1 \leq i \leq n$be the vertices ofL_n. Let $G = (C_m \odot K_1) \cup L_n$.

Define a function $f: V(G) \rightarrow \{1,2, \dots, q+1\}$ by

$f(u_i) = 2i - 1 , 1 \leq i \leq 2$

$f(u_i) = 2i , \quad 3 \leq i \leq m$

$f(v_i) = 2i , \quad 1 \leq i \leq 2$

$f(v_i) = 2i - 1 , 3 \leq i \leq m$

$f(x_i) = 2m + 3i - 1 , 1 \leq i \leq n$

$$f(y_i) = 2m + 3i - 2 , 1 \leq i \leq n$$

Then the edge labels are distinct.

Hence f is a Geometric Mean Labeling of G.

Example2.10: The Geometric Mean labeling of$(C_9 \odot K_1) \cup L_6$ is given below.

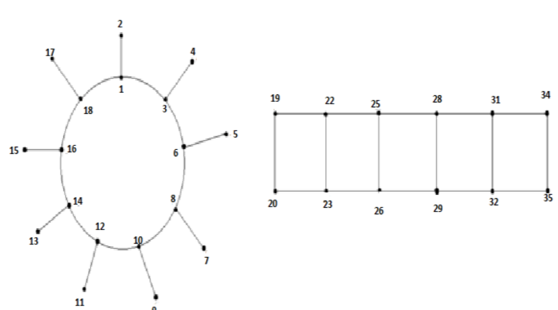

Figure5

Theorem2.11:$C_m \cup (P_n \odot K_3)$ is a Geometric Mean graph.

Proof: Let C_m be the cycle $u_1 u_2 \cdots u_m u_1$. Let $v_1 v_2 \cdots v_n$be the path P_n .

Let x_i , y_i , $1 \leq i \leq n$ be the vertices of K_3 which are attached to the vertices of P_n . Let $G = C_m \cup (P_n \odot K_3)$.

Define a function $f: V(G) \rightarrow \{1,2, \dots, q+1\}$ by

$f(u_i) = i , 1 \leq i \leq m$

$f(v_i) = m + 4i - 2 , 1 \leq i \leq n$

$f(x_i) = m + 4i - 3 , 1 \leq i \leq n$

$$f(y_i) = m + 4i - 1 , 1 \leq i \leq n$$

Then the edge labels are distinct.

Hence f is a Geometric Mean Labeling of G.

Example2.12: The Geometric Mean labeling of$C_9 \cup (P_5 \odot K_3)$ is given below.

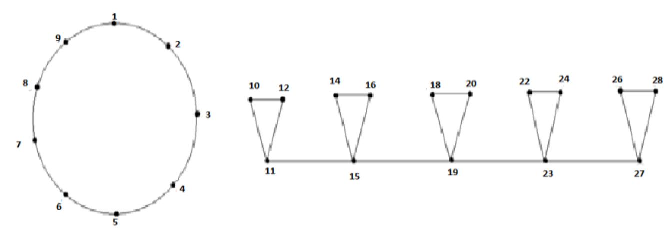

Figure6

Theorem2.13:$(C_m \odot K_1) \cup (P_n \odot K_3)$ is a Geometric Mean graph.

Proof: Let C_m be the cycle$u_1 u_2 \cdots u_m u_1$and let v_i be the vertex which is joined to the vertex u_i , $1 \leq i \leq m$ of the cycle C_m . Let $w_1 w_2 \cdots w_n$ be the path P_n . Let x_i , y_i be the vertices of K_3 which are attached to eachvertex of P_m .Let $= (C_n \odot K_1) \cup (P_m \odot K_3)$.

Define a function $f: V(G) \rightarrow \{1,2, \dots, q+1\}$ by

$f(u_i) = 2i - 1 , 1 \leq i \leq 2$

$f(u_i) = 2i , 3 \leq i \leq m$

$f(v_i) = 2i\,, 1 \leq i \leq 2$

$f(v_i) = 2i - 1\,, 3 \leq i \leq m$

$f(w_i) = 2m + 4i - 2\,, 1 \leq i \leq n$

$f(x_i) = 2m + 4i - 3\,, 1 \leq i \leq n$

$f(y_i) = 2m + 4i - 1\,, 1 \leq i \leq n$

Then the edge labels are distinct.

Hence f is a Geometric Mean Labeling of G.

Example2.14: The Geometric Mean labeling of $(C_8 \odot K_1) \cup (P_4 \odot K_3)$ is given below.

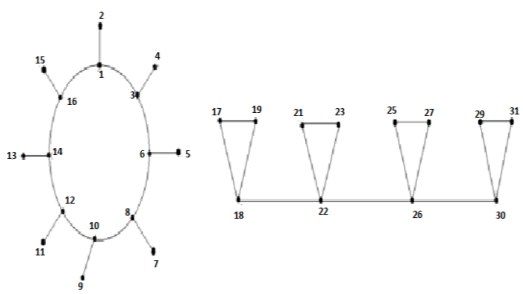

Figure7

Theorem2.15: $(C_m \odot K_3) \cup P_n$ is a Geometric Mean graph.

Proof: Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m . Let $v_i, w_i, 1 \leq i \leq m$ be the vertices of K_3 which are attached to the vertices of C_n .Let $t_i, 1 \leq i \leq n$ be the vertices of the path P_n. Let $G = (C_m \odot K_3) \cup P_n$.

Define a function $f: V(G) \to \{1,2, \dots, q+1\}$ by

$f(u_i) = 4i - 1\,, 1 \leq i \leq m$

$f(v_i) = 4i - 3, 1 \leq i \leq m$

$f(w_i) = 4i\ ,\ 1 \leq i \leq m$

$f(t_i) = 4m + i, 1 \leq i \leq n$

Then the edge labels are distinct.

Hence f is a Geometric Mean Labeling of G.

Example2.16: The labeling pattern of $(C_6 \odot K_3) \cup P_5$ is given below.

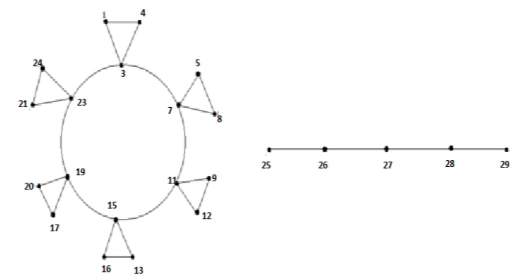

Figure8

Theorem2.17: $(C_m \odot K_3) \cup (P_n \odot K_3)$ is a Geometric Mean graph.

Proof: Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m . Let $v_i, w_i, 1 \leq i \leq m$ be the vertices of K_3 which are attached to the vertices of C_m . Let $t_1 t_2 \cdots t_n$ be the path P_n .Let x_i, y_i be the vertices of K_3 which are attachedto $t_i, 1 \leq i \leq n$ .Let $G = (C_m \odot K_3) \cup (P_n \odot K_3)$.

Define a function $f: V(G) \to \{1,2, \dots, q+1\}$ by

$f(u_i) = 4i - 1\,, \quad 1 \leq i \leq m$

$f(v_i) = 4i - 3, \quad 1 \leq i \leq m$

$f(w_i) = 4i\,, \quad 1 \leq i \leq m$

$f(t_i) = 4m + 4i - 2, 1 \leq i \leq n$

$f(x_i) = 4m + 4i - 3, 1 \leq i \leq n$

$f(y_i) = 4m + 4i - 1, 1 \leq i \leq n$

Then the edge labels are distinct.

Hence f is a Geometric Mean Labeling of G.

Example2.18: The labeling pattern of $(C_6 \odot K_3) \cup (P_5 \odot K_3)$ is given below.

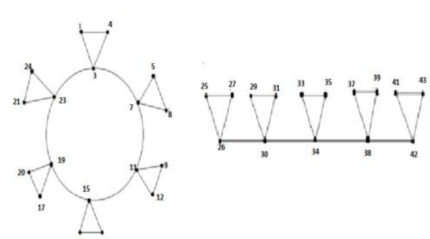

Figure9

REFERENCES:

[**1**] Gallian. J.A, 2012, A dynamic Survey of graph labeling. The electronic Journal of Combinatories $17\#DS6$.

[**2**] Harary.F, 1988, Graph Theory, Narosa Publishing House Reading, New Delhi.

[**3**] S. Somasundaram, P. Vidyarani and R. Ponraj **"**Geometric Mean Labeling of Graphs", Bullettin of Pure and Applied Sciences. 30E(2)(2011), page 153-160.

[**4**]Sandhya.S.S , Somasundaram.S , "Geometric Mean Labeling of Disconnected Graphs" Future Prospects in Multi Disciplinary Research. ISBN 978-81-910747-7-2 page no:134 to 136.

[**5**] S. Somasundaram. S.S. Sandhya, S.P Viji, "Some New Results in Geometric Mean Graphs" Kanyakumari Academy of Arts and Science. ISBN 978-93-81658-10-9 Vol. 3 page 13-17.

[**6**] S. Somasundaram. S.S. Sandhya, S.P Viji, "A Note on Geometric Mean Graphs", International Journal of Mathematical Archive- 5(10)(2014), 1-8 ISSN 2229-5046.

[**7**] S. Somasundaram. S.S. Sandhya, S.P Viji, "Few Results on Geometric Meam Graphs", International Journal of Mathematical Trends & Technology. ISSN 2331-5373 Vol. 16 No. 1 (2014).

[**8**] S. Somasundaram. S.S. Sandhya, S.P Viji, "On Geometric Mean Graphs", communicated to International Journal of Mathematical Forum.