Surface area mensuration relation of two Cuboids (Relation All Mathematics)

Deshmukh Sachin Sandipan
Corps of Signal (Indian Army), India

Abstract

In this research paper, the relation between Cube, Cuboid and two cuboids surface area is explained with the help of formula. We can also understand difference/relation between their vertical surface area and total surface area with this formula. This surface area relation is considered in three parts as (i) Vertical surface area (ii) Total surface area and (iii)Vertical - total surface area. Based on height again this relation is divided in two parts (i) when height is same and (ii) when height is un-equal. We are trying to give a new concept "Relation All Mathematics" to the world. I am sure that this concept will be helpful in Agricultural, Engineering, Mathematical world etc.

Keywords: *Surface area, Sidemeasurement, Relation, Cube, Cuboid.*

I. Introduction

Relation All Mathematics is a new field and the various relations shown in this research, “Surface area mensuration relation of two cuboids” is the 4^{th} research paper of Relation All Mathematics and in future, any research related to this concept, must be part of “ Relation Mathematics ” subject. Here, we have studied and shown new variables, letters, concepts, relations and theorems. Inside the research paper, relation between two cuboids explained in three parts i.e. as (i) vertical surface area (ii) total surface area and (iii) vertical- total surface area. Based on height this relations is again divided in two parts (i) when height is same and (ii) when height is un-equal. We have explained a new concept i.e. Side measurement, which is very important related to ‘Relation Mathematics’ subject.

In this “Relation All Mathematics” we have Proved the relation between cube –cuboid and two cuboids with the help of formula. This “Relation All Mathematics” research work is near by 300 pages. This research is prepared considering the Agricultural sector mainly, but I am sure that it will also be helpful in other sectors.

II. Basic concept of Cube and Cuboids

2.1. Side measurement (B):- If sides of any geometrical figure are in right angle with each other, then those sides or considering one of the parallel and equal sides after adding them, the addition is called sidemeasurement. Sidemeasurement is indicated with letter ‘B’.

Sidemeasurement is a one of the most important concept and maximum base of the Relation All Mathematics depend upon this concept.

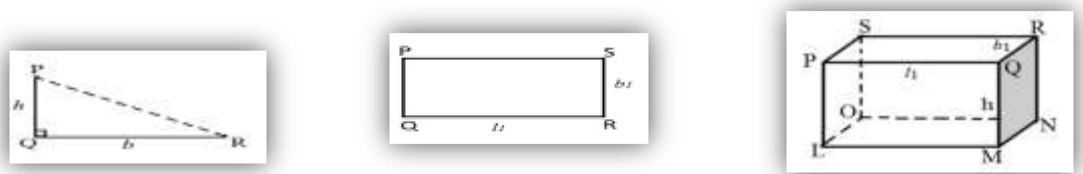

Figure I : Concept of Sidemeasurement

Sidemeasurement of right angled triangle - B (ΔPQR) = b+h

In ΔPQR, sides PQ and QR are right angle, performed to each other.

Sidemeasurement of rectangle-B (□PQRS) = l_1+ b_1

In □PQRS, opposite sides PQ and RS are similar to each other and m<Q = 90^o. Here side PQ and QR are right angle performed to each other.

Sidemeasurement of cuboid–E_B (□PQRS) = l_1+ b_1+ h_1

In E (□PQRS),opposite sides are parallel to each other and QM are right angle performed to each other. Sidemeasurement of cuboid written as = E_B (□PQRS)

2.2) Important points of square-rectangle relation :-

I) For explanation of square and rectangle relation following variables are used

a) Area – A
b) Perimeter – P
c) Side measurement – B

II) For explanation of square and rectangle relation following letters are used

a) Area of square ABCD – A (□ABCD)
b) Perimeter of square ABCD – P (□ABCD)
c) Sidemeasurement of square ABCD – B (□ABCD)
d) Area of rectangle PQRS – A (□PQRS)
e) Perimeter of rectangle PQRS – P (□PQRS)
f) Sidemeasurement of rectangle PQRS – B (□PQRS)

2.3) Important points of cube-cuboid relation:-

I) For explanation of Cube-Cuboid relation following variables are used

a) cuboid - E
b) Cube - G
c) Volume - V
d) Vertical surface area - U
e) Total surface area - A
f) Side measurement - B

II) Concept of explanation of Cube-Cuboid

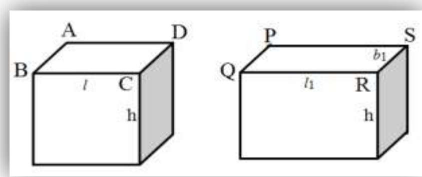

Figure II : Concept of cube and cuboid

Explanation of cube and cuboid is given with the reference of its upper side .
In Fig.I, cuboid is explained with the reference of rectangle. i.e. E (□PQRS) and cube is explained with reference of square. i.e. G(□ABCD).

III) For explanation of Cube-Cuboid relation, following letters are used

a) Volume of cube ABCD – G_V (□ABCD)
b) Volume of cuboid PQRS – E_V (□PQRS)
c) Vertical surface area of the cube ABCD – G_U (□ABCD)
d) Total surface area of the cube ABCD – G_A (□ABCD)
e) Vertical surface area of the cuboid PQRS – E_U (□PQRS)
f) Total surface area of the cuboid PQRS – E_A (□PQRS)
g) Sidemeasurement of cube ABCD – G_B (□ABCD)
h) Sidemeasurement of cuboid PQRS – E_B (□PQRS)

2.4) Un-equal height Surface Area Relation formula of cube and cuboid (Y)

In cube and cuboid when Area of square and rectangle is same but height of both are un-equal then difference between surface area of both are maintained with the help of 'Unequal height surface area Relation formula of cube and cuboid (Y)' and both side surface area relation of cube and cuboid become equal.

Unequal height Surface Area Relation formula of cube and cuboid indicated with letter 'Y'

Y= 4[$l_1.b_1$- $h_1\sqrt{l_1.b_1}$]

2.5) Important Reference theorem of previous paper which used in this paper:-

Theorem : Basic theorem of area relation of square and rectangle

Perimeter of square and rectangle is same then area of square is more than area of rectangle, at that time area of square is equal to sum of the, area of rectangle and Relation area formula of square-rectangle(K) .

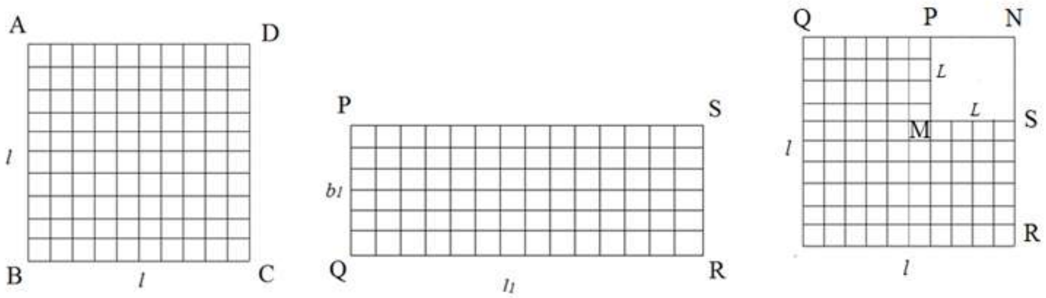

Figure III : Area relation of square and rectangle

Proof formula :- A(□ABCD) = A(□PQRS) + $\left[\frac{(l_1+b_1)}{2} - b_1\right]^2$

[Note :- The proof of this formula given in previous paper and that available in reference**]**

Theorem :- Basic theorem of perimeter relation of square-rectangle

Area of square and rectangle is same then perimeter of rectangle is more than perimeter of square , at that time perimeter of rectangle is equal to product of the, perimeter of square and Relation perimeter formula of square-rectangle(V).

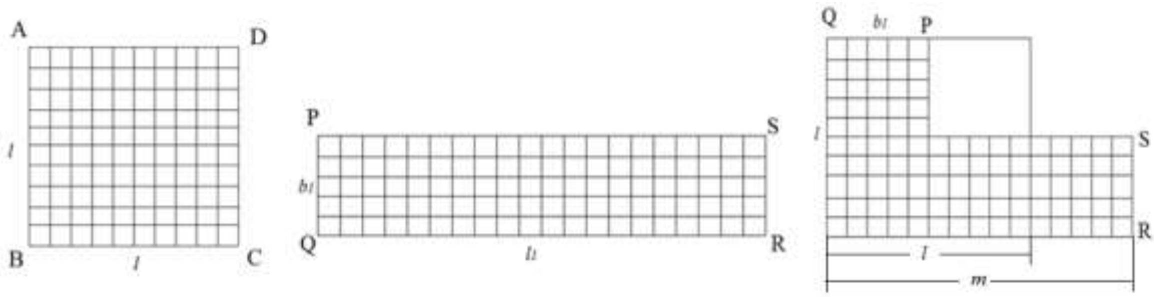

Figure IV : Perimeter relation of square-rectangle

Proof formula :- P(□ PQRS)= P(□ABCD) x $\frac{1}{2}\left[\frac{(n^2+1)}{n}\right]$

[Note :- The proof of this formula given in previous paper and that available in reference**]**

III. Relation between cube and cuboids.

Relation –I: Surface area relation of Cube – Cuboid :-

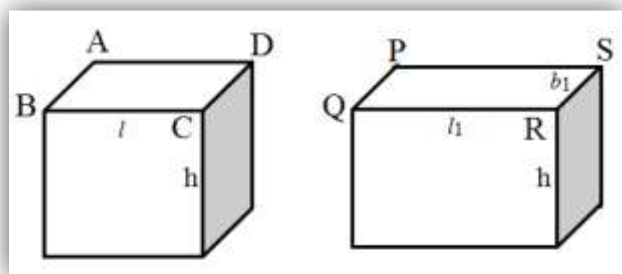

Figure V : Surface area relation of Cube – Cuboid

Side of cube is 'l' and length, width and height of cuboid is l_1, b_1 and h_1 respectively.We know that, concept of vertical and total surface area of cube and cuboid which explain as below,

Vertical surface area of the cuboid $= E_U$ (□PQRS) $= 2(l_1 + b_1) \times h$

Total surface area of the cuboid $= E_A$ (□PQRS) $= 2(l_1.b_1+ l_1.h_1+b_1.h_1)$

Vertical surface area of the cube $= G_U$ (□ABCD) $= 4l^2$

Total surface area of the cube $= G_A$ (□ABCD) $= 6l^2$

Surface area relation of Cube – Cuboid is important relation for this research paper. This relation is used to convert Total surface area into vertical surface area and vise varsa of cube /cuboids.

Cuboid /Cube Total surface area = Cuboid /Cube Vertical surface area+$2l_1.b_1$

. ... $(l_1.b_1 = 2l)$

Relation –II: Vertical surface area relation of cube and cuboid, when height is same

Known information: Side of cube G(□ABCD) is 'l' and length, width and height of cuboid E(□PQRS) is l1,b1 and h1respectively.

height of G(□ABCD) = height of E (□PQRS)

A(□ABCD) = A (□PQRS)

G_V(□ABCD)= E_V (□PQRS) ... $l_1 > l$.

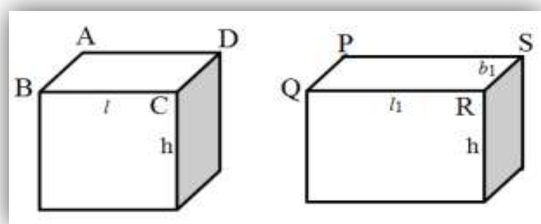

Figure –VI : Equal height Vertical surface area relation of cube and cuboid

To Prove : E_U(□PQRS) = $\frac{1}{2}$ G_U (□ABCD) . $\left[\frac{(n^2+1)}{n}\right]$

Proof: In G(□ABCD) and E(□PQRS) ,

P(□PQRS) = $\frac{1}{2}$ P(□ABCD) . $\left[\frac{(n^2+1)}{n}\right]$ ... V=$\left[\frac{(n^2+1)}{n}\right]$

... (Basic theorem of perimeter relation of square and rectangle)

But vertical surface area of cuboid is $= 2(l_1+b_1) \times h$

Multiply height 'h' to both side.

P(□PQRS) x h = $\frac{1}{2}$ P(□ABCD) x h . $\left[\frac{(n^2+1)}{n}\right]$

here, $\frac{1}{2}$. $\left[\frac{(n^2+1)}{n}\right]$ is a Surface area relation formala of cube and cuboid and is explained with letter V\`

...V=V\`

E_U (□PQRS) = G_U(□ABCD) $\frac{1}{2}$. $\left[\frac{(n^2+1)}{n}\right]$

Hence , we have Prove that Vertical surface area relation of cube and cuboid, when height is same

. This Relation cleared the following points-

1) Volume of cube and cuboid are equal.

2) But vertical surface area of cuboid is greater than vertical surface area of cube and this relation is explained with the help of formula and both sides of equation become equal.

Relation –III: Vertical surface -total surface area relation of cube and cuboid, when height is same

Known information: Side of cube G(□ABCD) is 'l' and length, width and height of cuboid E(□PQRS) is l1, b1 and h1 respectively.

height of G(□ABCD) = height of E (□PQRS)

A(□ABCD) = A (□PQRS)
G_V(□ABCD)= E_V (□PQRS) … $l_1 > l$.

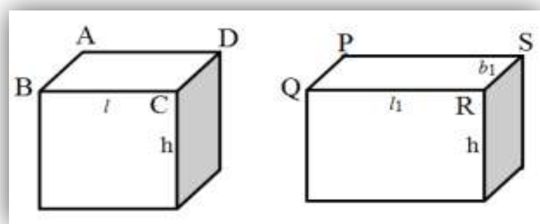

Figure –VII : Equal height Vertical surface -total surface area relation of cube and cuboid

To Prove : E_A(□PQRS) = $\frac{1}{2}$ G_U (□ABCD) . $\left[\frac{(n^2+1)}{n}\right]$ + $2l_1\, b_1$

Proof:

In G(□ABCD) ,

G_A(□ABCD) = G_U (□ABCD) + $2l^2$ … (i)

(Surface area relation of Cube – Cuboid) here , $l_1 b_1 = l^2$

but,in In G(□ABCD) and E(□PQRS) ,

G_U (□ABCD) =2E_U(□PQRS) x $\left[\frac{n}{(n^2+1)}\right]$ … (ii)

…[Vertical surface area relation of cube and cuboid when height is same]

put the value of equation no (ii) in equation no (i)

G_A(□ABCD) = 2E_U(□PQRS) x $\left[\frac{n}{(n^2+1)}\right]$ + $2l^2$ … ($l^2 = l_1$ x b_1)

G_A(□ABCD) = 2E_U(□PQRS) x $\left[\frac{n}{(n^2+1)}\right]$ + 2A(□PQRS)

here equation no (i) written as

In E(□PQRS) ,

G_A(□PQRS) = G_U (□PQRS) + $2l_1\, b_1$ … (iii)

(Surface area relation of Cube – Cuboid)

also, E_U (□PQRS) = ½ G_U(□ABCD) . $\left[\frac{(n^2+1)}{n}\right]$ …(iv)

…[Vertical surface area relation of cube and cuboid, when height is same] …[from eq^n (ii)]

put the value of equation no (iv) in equation no (iii)

E_A(□PQRS) = $\frac{1}{2}$ G_U (□ABCD) . $\left[\frac{(n^2+1)}{n}\right]$ + $2l_1\, b_1$ … [$l^2 = l_1$ x b_1]

Hence , we have Prove that Vertical surface -total surface area relation of cube and cuboid, when height is same
This Relation cleared the following points-

1) Volume of cube and cuboid are equal.

2) Here Vertical surface -total surface area relation of cube and cuboid explained with the help of formula and both sides of equation become equal.

Relation –IV: Total surface area relation of cube and cuboid when height is same

Known information: Side of cube G(□ABCD) is ‘l’ and length, width and height of cuboid E(□PQRS) is l1, b1 and h1 respectively.

height of G(□ABCD) = height of E (□PQRS)
A(□ABCD) = A (□PQRS)
G_V(□ABCD)= E_V (□PQRS) … $l_1 > l$.

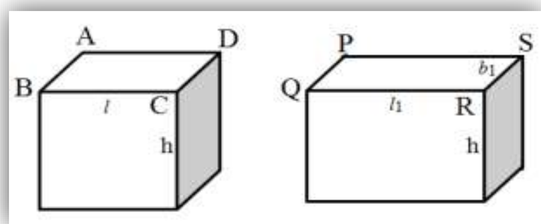

Figure –VIII : Equal height Total surface area relation of cube and cuboid

To Prove : $\mathbf{E_A(\square PQRS) = \frac{1}{2}\,[G_A(\square ABCD).\left[\frac{(n^2+1)}{n}\right] + l^2\,[2 - \frac{(n^2+1)}{n}\,]}$

Proof:

In G(□ABCD) ,

$G_U(\square ABCD) = G_A\,(\square ABCD) - 2l^2$ … (i)

In E(□PQRS) ,

$E_U(\square PQRS) = E_A\,(\square PQRS) - 2\,l_1.b_1$ … (ii),

[$l_1.b_1 = l^2$] (Surface area relation of Cube – Cuboid)

Now , In G(□ABCD) and E(□PQRS) ,

$E_U\,(\square PQRS) = \frac{1}{2}\,G_U(\square ABCD)\ x\left[\frac{(n^2+1)}{n}\right]$ …(iii)

…[Vertical surface area relation of cube and cuboid, when height is same]

put the value of equation no (i) and (ii) in equation no (iii)

$E_A(\square PQRS)-2\,l_1\,.b_1 = \frac{1}{2}\,[G_A(\square ABCD) - 2l^2\,]\ x\left[\frac{(n^2+1)}{n}\right]$

$= \frac{1}{2}\,[G_A(\square ABCD)\ x\left[\frac{(n^2+1)}{n}\right] - l^2\,x\left[\frac{(n^2+1)}{n}\right]$

$E_A(\square PQRS) = \frac{1}{2}\,[G_A(\square ABCD).\left[\frac{(n^2+1)}{n}\right] - l^2 .\left[\frac{(n^2+1)}{n}\right] +2\,l_1.b_1$

$\mathbf{E_A(\square PQRS) = \frac{1}{2}\,[G_A(\square ABCD).\left[\frac{(n^2+1)}{n}\right] + l^2\,[2 - \frac{(n^2+1)}{n}\,]}$ … $l_1.b_1 = l^2$

Hence , we have Proved that Total surface area relation of cube and cuboid, when height is same.

This Relation cleared the following points-

1) Volume of cube and cuboid are equal .

2) But Total surface area of cuboid is greater than Total surface area of cube and that relation explained with the help of formula and both sides of equation become equal.

Example :-

	WATER TANK	□PQRS (CUBOID)	□ABCD (CUBE)	.V`	REMARK
GIVEN	LENGTH	20	10		
	WIDTH	5	10		
	HEIGHT	10	10		EQUAL
EXPLANATION	Volume	1000	1000		EQUAL
	WATER CAPACITY ON TANK	27,300 ltr (27.3 ltr/sq ft)	27,300 ltr (27.3 ltr/sq ft)		Water storage capacity of both water tank is equal i.e. 27,300 ltr/sq.ft

BENEFIT/LOSS		100 sq.ft bilding cost is greater	Minimum bilding cost		
		LHS	**RHS**	**RHS**	
i.e.	Vertical surface area	500	400	x 1.25	100 sq.ft difference in vertical surface area
ANS		500	500		
i.e.	Total surface area	700	600	x 1.16667	100 sq.ft difference in Total surface area
ANS		700	700		
Result in real life		Less volume more surface area	Less surface area more volume		
		LHS	**RHS**		

Relation –V : Vertical surface area relation of Cube and Cuboid, when height is un-equal.
Known information: Side of cube G(□ABCD) is 'l' and length, width and height of cuboid E(□PQRS) is l1, b1 and h1respectively.
A(□ABCD)=A (□PQRS) … $l_1 > l$.
But side of cube (h) $\neq$ height of cuboid (h1)

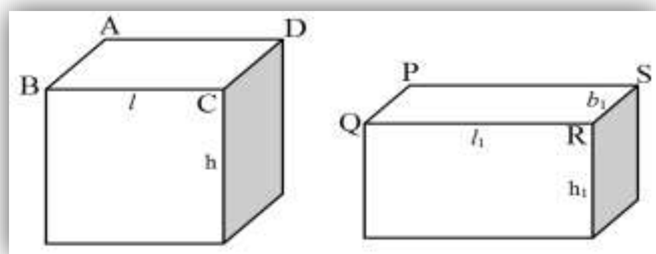

Figure –IX :Un-equal height Vertical surface area relation of Cube and Cuboid

To Prove : $\mathbf{G_U(\square ABCD) = 2 \,.\, E_U(\square PQRS) \,.\, \left[\frac{n_1}{(n_1{}^2+1)}\right] + 4(l_1b_1 - h_1\sqrt{l_1.b_1})}$

Proof: In G(□ABCD) and E(□PQRS) ,

$G_U(\square ABCD) = 2 \,.\, E_U(\square PQRS) \,.\, \left[\frac{n_1}{(n_1{}^2+1)}\right]$ ….(i)

…[Vertical surface area relation of cube and cuboid, when height is same]
but height of cuboid (□PQRS) and cube (□ABCD) is un-equal.($l \neq h_1$)
therefore,

$G_U(\square ABCD) \neq 2 \,.\, E_U(\square PQRS) \,.\, \left[\frac{n_1}{(n_1{}^2+1)}\right]$

here height of cube and cuboid is unequal, so add value of 'Y in RHS, now the equation is,

G_U (□ABCD) =2 . E_U (□PQRS) . $\left[\frac{n_1}{(n_1{}^2+1)}\right]$ +4(l_1b_1-$h_1\sqrt{l_1.b_1}$) … Y=4(l_1b_1-$h_1\sqrt{l_1.b_1}$)

E_U (□PQRS)= $\frac{1}{2}$ G_U (□ABCD) $\left[\frac{(n_1{}^2+1)}{n_1}\right]$ +2(l_1b_1-$h_1\sqrt{l_1.b_1}$) x $\left[\frac{(n_1{}^2+1)}{n_1}\right]$

Hence, we have Proved that Vertical surface area relation of Cube and Cuboid when height is un-equal.
This relation cleared the following points,
1) In cube and cuboid, area of square and rectangle are same but height is unequal.
2) Remember related to cube, length and width of cube should be equal but height is not necessary to be equal with its side.
3) Height of cube –cuboid is un-equal at that time vertical surface area relation between them explained with the help of formula and both sides of equation become equal.

Relation –VI : Vertical surface -total surface area relation of Cube and Cuboid, when height is un-equal.
Known information: Side of cube G(□ABCD) is 'l'.and length ,width and height of cuboid E(□PQRS) is l1,b1 and h1respectively.
A(□ABCD)=A (□PQRS) … $l_1 > l$.
But , side of cube (h) ≠ height of cuboid (h1)

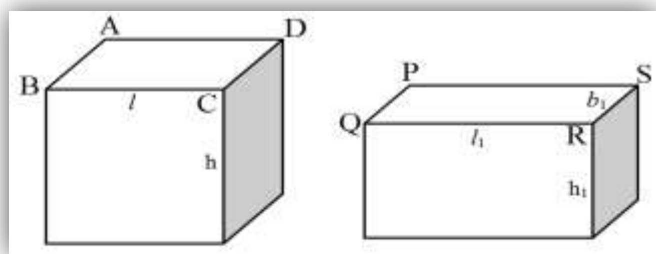

Figure –X :Un-equal height Vertical surface -total surface area relation of Cube and Cuboid

To Prove : E_A(□PQRS) = $\frac{1}{2}$ G_U (□ABCD) . $\left[\frac{(n_1{}^2+1)}{n_1}\right]$ - 2(l_1b_1-$h_1\sqrt{l_1.b_1}$. $\left[\frac{(n_1{}^2+1)}{n_1}\right]$ + 2l_1 b_1

Proof: In G(□ABCD) ,
G_A(□ABCD) = G_U (□ABCD) + $2l^2$ … (i),
here $l_1 b_1 = l^2$ (Surface area relation of Cube – Cuboid)
But, In G(□ABCD) and E (□PQRS) ,
G_U (□ABCD) =2 . E_U (□PQRS) . $\left[\frac{n_1}{(n_1{}^2+1)}\right]$ + 4(l_1b_1-$h_1\sqrt{l_1.b_1}$) … (ii)
…(Vertical surface area relation of Cube and Cuboid, when height is un-equal)
Now ,Value of eq^n no. (ii) put in eq^n no (i)
G_A(□ABCD) = 2E_U(□PQRS) x $\left[\frac{n_1}{(n_1{}^2+1)}\right]$ + 4(l_1b_1-$h_1\sqrt{l_1.b_1}$) +$2l^2$ … ($l^2 = l_1$ x b_1)
G_A(□ABCD) = 2E_U(□PQRS) x $\left[\frac{n_1}{(n_1{}^2+1)}\right]$ + 4(l_1b_1-$h_1\sqrt{l_1.b_1}$) +2A(□PQRS)
. … ($l^2 = l_1$ x b_1)
This equation is explained with a different method which is given below,
E_A(□PQRS) = E_U (□PQRS) + 2l_1 b_1
… (iii)(Surface area relation of Cube – Cuboid)
Now , E_U (□PQRS) = $\frac{1}{2}$ G_U(□ABCD) . $\left[\frac{(n_1{}^2+1)}{n_1}\right]$ - 2(l_1b_1-$h_1\sqrt{l_1.b_1}$). $\left[\frac{(n_1{}^2+1)}{n_1}\right]$.
…(iv)
…(Vertical surface area relation of Cube and Cuboid, when height is un-equal)
put the value of eq^n (iv) in eq^n no (iii) ,
E_A(□PQRS) = $\frac{1}{2}$ G_U (□ABCD) . $\left[\frac{(n_1{}^2+1)}{n_1}\right]$ - 2(l_1b_1-$h_1\sqrt{l_1.b_1}$. $\left[\frac{(n_1{}^2+1)}{n_1}\right]$ + 2l_1 b_1 .
…[$l^2 = l_1 .b_1$]

Hence, we have Proved that Vertical surface -total surface area relation of Cube and Cuboid, when height is un-equal.
This relation cleared the following points,
1) In cube and cuboid , area of square and rectangle are same but height is unequal .
2) Remember related to cube, length and width of cube should be equal but height is not necessary to be equal with its side
3) Height of cube –cuboid is unequal at that time Vertical surface -total surface area relation between them explained with the help of formula and both sides of equation become equal.

Relation –VII : Total surface area relation of Cube and Cuboid when height is un-equal.
Known information: Side of cube G(□ABCD) is 'l' and length, width and height of cuboid E(□PQRS) is l1, b1 and h1 respectively.
A(□ABCD)=A (□PQRS) … $l_1 > l$.
But, side of cube (h) ≠ height of cuboid (h1)

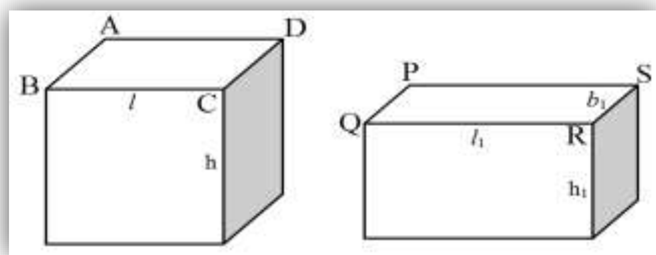

Figure –XI :Un-equal height total surface area relation of cube and cuboid

To Prove : G_A (□ABCD) =2 . [E_A (□PQRS) . $\left[\frac{n_1}{(n_1{}^2+1)}\right]$ + $2l^2$. $\left[1-2\left(\frac{n_1}{(n_1{}^2+1)}\right)\right]$ +4(l_1b_1-$h_1\sqrt{l_1.b_1}$)

Proof: In G(□ABCD) ,
G_U(□ABCD) = G_A (□ABCD) - $2l^2$
… (i) here , $l_1.b_1 = l^2$ (Surface area relation of Cube – Cuboid)
In E(□PQRS) ,
E_U(□PQRS) = E_A (□PQRS) - $2l^2$
… (ii) here , $l_1.b_1 = l^2$ (Surface area relation of Cube – Cuboid)
But,In G(□ABCD) and E(□PQRS) ,
G_U (□ABCD) =2 . E_U (□PQRS). $\left[\frac{n_1}{(n_1{}^2+1)}\right]$ +4(l_1b_1-$h_1\sqrt{l_1.b_1}$)] … (iii)
…(Vertical surface area relation of Cube and Cuboid, when height is un-equal)
put the value of eq^n no.(i) and (ii) in eq^n no. (iii) ,
G_A (□ABCD) =2 . E_A(□PQRS) . $\left(\frac{n_1}{(n_1{}^2+1)}\right)$] - $4l^2$. $\left[\frac{n_1}{(n_1{}^2+1)}\right]$ + $2l^2$ +4(l_1b_1-$h_1\sqrt{l_1.b_1}$)
G_A (□ABCD) =2 . E_A (□PQRS) . $\left[\frac{n_1}{(n_1{}^2+1)}\right]$ + $2l^2$. $\left[1-2\left(\frac{n_1}{(n_1{}^2+1)}\right)\right]$ +4(l_1b_1-$h_1\sqrt{l_1.b_1}$)
G_A (□ABCD) =2 . E_A (□PQRS) . $\left[\frac{n_1}{(n_1{}^2+1)}\right]$ + $2l^2$. $\left[1-2\left(\frac{n_1}{(n_1{}^2+1)}\right)\right]$ +4(l_1b_1-$h_1\sqrt{l_1.b_1}$)
Hence, we have Proved that Total surface area relation of Cube and Cuboid, when height is un-equal.
This relation cleared the following points,
1) In cube and cuboid ,area of square and rectangle are same but height is unequal .2) Remember related to cube, length and width of cube should be equal but height is not necessary to be equal with its side.
3) Height of cube –cuboid is unequal at that time Total surface area relation between them is explained with the help of formula and both sides of equation become equal.

Relation –VIII : Vertical surface area relation of two cuboids when height is same
Known information: The length, width and height of cuboid E(□PQRS) is l_1,b_1 and h_1 and cuboid E(□LMNO) is l_2,b_2 and h_2 respectively. As well as side of cube G(□ABCD) is l .

height of G(□ABCD) = height of E (□PQRS) = height of E (□LMNO)
A(□ABCD) = A (□PQRS) = A (□LMNO)
G_V (□ABCD) =E_V(□PQRS)= E_V (□LMNO) … $l_2 > l_1 > h$ & $h=h_1=h_2$

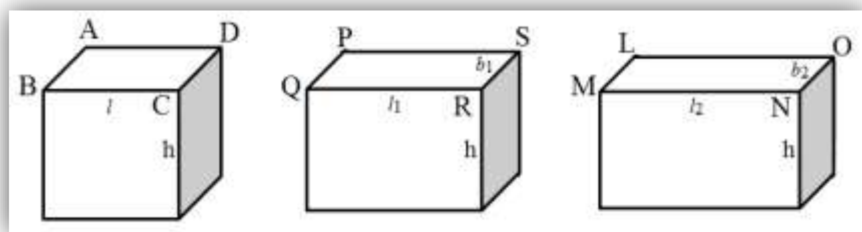

Figure –XII : Equal height Vertical surface area relation of two cuboids

To Prove : E_U (□PQRS)= E_U (□LMNO) . $\left[\frac{n_2}{n_1} \cdot \frac{(n_1^2+1)}{(n_2^2+1)}\right]$

Proof: In G(□ABCD) and E(□PQRS) ,
G_U(□ABCD) = 2 E_U (□PQRS) . $\left[\frac{n_1}{(n_1^2+1)}\right]$ …(i)
…(Vertical surface area relation of cube and cuboids, when height is same)
In G(□ABCD) and E(□LMNO) ,
G_U(□ABCD) = 2E_U(□LMNO). $\left[\frac{n_2}{(n_2^2+1)}\right]$ …(ii)
…(Vertical surface area relation of cube and cuboids when height is same)
2 E_U (□PQRS) . $\left[\frac{n_1}{(n_1^2+1)}\right]$ = 2 E_U (□LMNO) . $\left[\frac{n_2}{(n_2^2+1)}\right]$
E_U (□PQRS)= E_U (□LMNO) . $\left[\frac{n_2}{n_1} \cdot \frac{(n_1^2+1)}{(n_2^2+1)}\right]$ …From equatin no. (i) and (ii)
Hence, we have Proved that Vertical surface area relation of two cuboids, when height is same
This relation cleared the following points,
1) Volume of two cuboid are equal.
2) But among the both cuboid, if the length of one cuboid campaired with another cuboid is greater than the vertical surface area of that cuboid will be greater than the vertical surface area of another cuboid, this relation is explained with the help of formula and both sides of equation become equal.

Relation –IX : Vertical surface -total surface area relation of two cuboids when height is same
Known information: The length, width and height of cuboid E(□PQRS) is l_1,b_1 and h_1 and cuboid E(□LMNO) is l_2,b_2 and h_2 respectively, as well as side of cube G(□ABCD) is l .
height of G(□ABCD) = height of E (□PQRS) = height of E (□LMNO)
A(□ABCD) = A (□PQRS) = A (□LMNO)
G_V (□ABCD) =E_V(□PQRS)= E_V (□LMNO) … $l_2 > l_1 > h$ & $h=h_1=h_2$

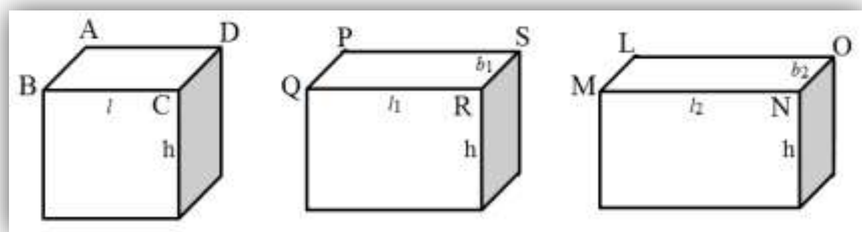

Figure –XIII : Equal height Vertical surface -total surface area relation of two cuboids

To Prove : $E_A(\square PQRS) = E_A(\square LMNO).\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ $+2l_1.b_1$.

Proof: In E(□PQRS) ,

$E_A(\square PQRS) = E_U(\square PQRS) + 2l_1.b_1$ …(i) here , $l_1.b_1 = l^2$ (Surface area relation of Cube – Cuboid)

but, In G(□PQRS) and E(□LMNO) ,

$E_U(\square PQRS) = E_U(\square LMNO).\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ … (ii)

…(Vertical surface area relation of two cuboids, when height is same)

value of eq^n no. (ii) put in eq^n no. (i) ,

$$\boldsymbol{E_A(\square PQRS) = E_U(\square LMNO).\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right] + 2l_1.b_1} \quad \ldots\ l_1.b_1=l_2.b_2$$

Hence, we have Proved the Vertical surface -total surface area relation of two cuboids, when height is same. This relation cleared the following points,

1) Volume of two cuboid are equal

2) Here Vertical surface -total surface area relation of two cuboids is explained with the help of formula and both sides of equation become equal.

Relation –X : Total surface area relation of two cuboids, when height is same

Known information: The length,width and height of cuboid E(□PQRS) is l_1,b_1 and h_1 and cuboid E(□LMNO) is l_2,b_2 and h_2 respectively as well as side of cube G(□ABCD) is l .

height of G(□ABCD) = height of E (□PQRS) = height of E (□LMNO)

A(□ABCD) = A (□PQRS) = A (□LMNO)

$G_V(\square ABCD) = E_V(\square PQRS) = E_V(\square LMNO)$ … $l_2 > l_1 > h$ & $h=h_1=h_2$

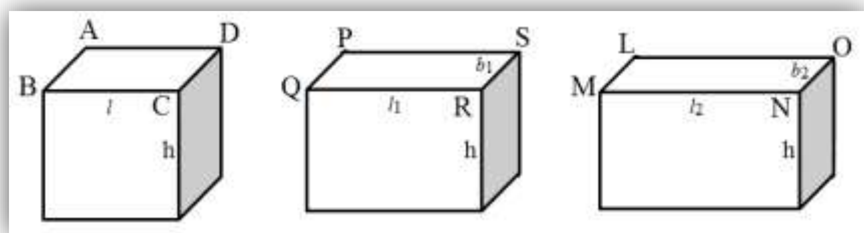

Figure –XIV : Equal height Total surface area relation of two cuboids

To Prove : $E_A(\square PQRS) = E_A(\square LMNO).\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ $+2l_1.b_1.\left[1-\left(\frac{n_2}{n_1}.\frac{n_1{}^2+1}{n_2{}^2+1}\right)\right]$

Proof: In E(□PQRS) ,

$E_U(\square PQRS) = E_A(\square PQRS) - 2\,l_1.b_1$ … (i) here , $l_1\,b_1 = l^2$ (Surface area relation of Cube – Cuboid)

In E(□LMNO) ,

$E_U(\square LMNO) = E_A(\square LMNO) - 2\,l_2.b_2$ … (ii) here , $l_2\,b_2 = l^2$ (Surface area relation of Cube – Cuboid)

but, In G(□ABCD) and E(□PQRS) ,

$E_U(\square PQRS)=E_U(\square LMNO).\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ … (iii)

…(Vertical surface area relation of two cuboids when height is same)

put the value of eq^n no. (i) and (ii) in eq^n no. (iii) ,

$E_A(\square PQRS)- 2l_1.b_1 = [E_A(\square LMNO) -2l_2.b_2]$ x $\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$

$E_A(\square PQRS) = E_A(\square LMNO) .\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ $-2l_1.b_1.\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ $+2\,l_1.b_1$ …$l_1b_1=l_2b_2$

$$\boldsymbol{E_A(\square PQRS) = E_A(\square LMNO).\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right] +2l_1.b_1.\left[1-\left(\frac{n_2}{n_1}.\frac{n_1{}^2+1}{n_2{}^2+1}\right)\right]}$$

Hence, we have Proved that Total surface area relation of two cuboids, when height is same

This relation cleared the following points,

1) Volume of two cuboid are equal

2) But among the both cuboid, if the length of one cuboid campaired with another cuboid is greater than the total surface area of that cuboid will be greater than the total surface area of another cuboid, this relation is explained with the help of formula and both sides of equation become equal.

Relation -XI: Vertical surface area relation of two cuboids when height is un-equal.
Known information: The length, width and height of cuboid E(□PQRS) is l_1, b_1 and h_1 and cuboid E(□LMNO) is l_2,b_2 and h_2 respectively as well as side of cube G (□ABCD) is l .
A (□ABCD) = A(□PQRS)= A (□LMNO) … $l_2 > l_1 > l$ & $h \neq h_1 \neq h_2$
But , side of cube □ABCD (l) ≠ height of cuboid □PQRS (h1) ≠ height of cuboid □LMNO (h2)

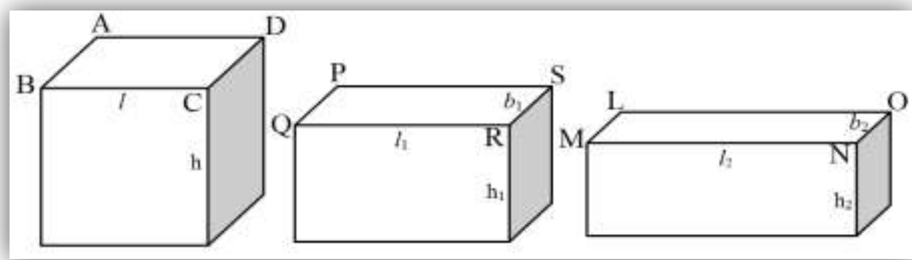

Figure –XV: Un-equal height Vertical surface area relation of two cuboids

To Prove : E_U (□PQRS) . = E_U (□LMNO) . $\left[\frac{n_2}{n_1} \cdot \frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right] + 2\sqrt{l_1 . b_1}\,(h_1 - h_2).\ \left[\frac{(n_1{}^2+1)}{n_1}\right]$

Proof: In G(□ABCD) and E(□PQRS) ,

G_U(□ABCD)= $2E_U$(□PQRS). $\left[\frac{n_1}{(n_1{}^2+1)}\right] + 4(l_1 . b_1 - h_1\sqrt{l_1 . b_1}) + 2l_1 . b_1$ …(i)

…(Vertical surface area relation of cube and cuboids when height is un-equal.)

In G(□ABCD) and E(□LMNO) ,

G_U(□ABCD)= $2E_U$(□LMNO $\left[\frac{n_2}{(n_2{}^2+1)}\right] + 4(l_2 . b_2 - h_2\sqrt{l_2 . b_2}) + 2l_2 . b_2$ …(ii)

Vertical surface area relation of cube and cuboids when height is un-equal.

2 E_U (□PQRS) . $\left[\frac{n_1}{(n_1{}^2+1)}\right] + 4(l_1 . b_1 - h_1\sqrt{l_1 . b_1}) + l_1 . b_1$ = 2 E_U (□LMNO) . $\left[\frac{n_2}{(n_2{}^2+1)}\right] + 4(l_2 . b_2 - h_2\sqrt{l_2 . b_2}) + 2l_2 . b_2$

…From equation no.(i) and (ii)

2 E_U (□PQRS) . $\left[\frac{n_1}{(n_1{}^2+1)}\right]$= $2E_U$ (□LMNO) . $\left[\frac{n_2}{(n_2{}^2+1)}\right] + 4(l_2 . b_2 - h_2\sqrt{l_2 . b_2}) - 4(l_1 b_1 - h_1\sqrt{l_1 . b_1})$

2 E_U (□PQRS) . $\left[\frac{n_1}{(n_1{}^2+1)}\right]$= 2 E_U(□LMNO) . $\left[\frac{n_2}{(n_2{}^2+1)}\right] + 4(h_1\sqrt{l_1 . b_1} - h_2\sqrt{l_2 . b_2})$

E_U (□PQRS) . = E_U (□LMNO) . $\left[\frac{n_2}{n_1} \cdot \frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right] + 2\sqrt{l_1 . b_1}\,(h_1 - h_2).\ \left[\frac{(n_1{}^2+1)}{n_1}\right]$

Hence, we have Proved the Vertical surface area relation of two cuboids when height is un-equal.
This Relation cleared the following points-
1) Two cuboid inside area of two rectangle are same but height is un-equal.
2) In this relation two cuboids Vertical surface area relation is explained with the help of formula when height is un-equal and both sides of equation become equal.

Note : in above relation $l_1 . b_1 = l_2 . b_2$ and h is difind as $h = \sqrt{l_1 . b_1}$

Relation -XII: Vertical surface -total surface area relation of two cuboids, when height is un-equal.
Known information: The length, width and height of cuboid E(□PQRS) is l_1, b_1 and h_1 and cuboid E(□LMNO) is l_2,b_2 and h_2 respectively as well as side of cube G (□ABCD) is l.
A (□ABCD) = A(□PQRS)= A (□LMNO) … $l_2 > l_1 > l$ & $h \neq h_1 \neq h_2$
But , side of cube □ABCD (l) ≠ height of cuboid □PQRS (h1) ≠ height of cuboid □LMNO (h2)

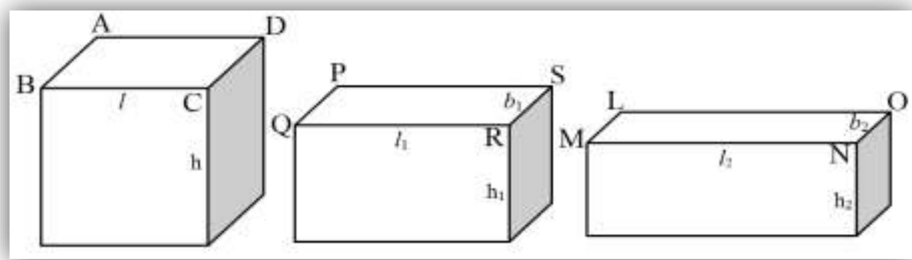

Figure –XVI : Un-equal height Vertical surface -total surface area relation of two cuboids

To Prove : E_A(□PQRS) = E_U (□LMNO) . $\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ +2$\sqrt{l_1.b_1}$ (h_1-h_2). $\left[\frac{(n_1{}^2+1)}{n_1}\right]$ +2l_1.b_1

Proof: In E(□PQRS) ,

E_A(□PQRS) = E_U (□PQRS) + 2 l_1 .b_1 … (i) here, $l_1 b_1 = l^2$ (Surface area relation of Cube – Cuboid)

BUT, In G(□PQRS) and E(□LMNO) ,

E_U (□PQRS)= E_U (□LMNO) . $\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ +2$\sqrt{l_1.b_1}$ (h_1-h_2). $\left[\frac{(n_1{}^2+1)}{n_1}\right]$ … (ii)

…(Vertical surface area relation of two cuboids when height is un-equal.)

value of eq^n no. (ii) put in eq^n no. (i)

E_A(□PQRS) = E_U (□LMNO) . $\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ +2$\sqrt{l_1.b_1}$ (h_1-h_2). $\left[\frac{(n_1{}^2+1)}{n_1}\right]$ +2l_1.b_1 …l_1b_1=l_2b_2

Hence, we have Proved the Vertical surface -total surface area relation of two cuboids, when height is un-equal.
This Relation cleared the following points-

1) Two cuboid inside area of two rectangle are same but height is un-equal.
2) In this relation two cuboids Vertical surface -total surface area relation is explained with the help of formula, when height is un-equal and both sides of equation become equal.

Note : in above relation $l_1.b_1$=$l_2.b_2$ and h is difind as h = $\sqrt{l_1.b_1}$

Relation -XIII: Total surface area relation of two cuboids when height is un-equal.

Known information: The length, width and height of cuboid E(□PQRS) is l_1,b_1 and h_1 and cuboid E(□LMNO) is l_2,b_2 and h_2 respectively as well as side of cube G (□ABCD) is l .

A (□ABCD) = A(□PQRS)= A (□LMNO) … $l_2 > l_1 > l$ & $h \neq h_1 \neq h_2$

But , side of cube □ABCD (l) ≠ height of cuboid □PQRS (h1) ≠ height of cuboid □LMNO (h2)

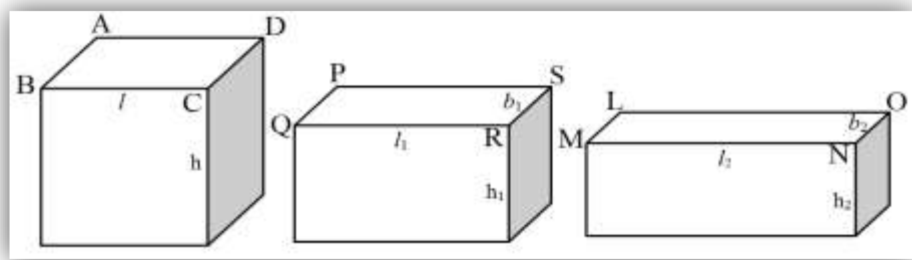

Figure –XVII : Un-equal height Total surface area relation of two cuboids

To Prove : E_A (□PQRS) = [E_A (□LMNO) $\left[\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ +2l_1.b_1. $\left[1-\left(\frac{n_2}{n_1}.\frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right)\right]$ + 2 $\sqrt{l_1.b_1}$ (h_1-h_2). $\left[\frac{(n_1{}^2+1)}{n_1}\right]$

Proof: In E(□PQRS) ,

E_U(□PQRS) = E_A (□PQRS) - 2l_1.b_1 … (i) here, $l_1 b_1 = l^2$ (Surface area relation of Cube – Cuboid)

In E(□LMNO) ,

E_U(□LMNO) = E_A (□LMNO) - $2l_1.b_1$ … (ii) here, $l_1.b_1 = l^2$ (Surface area relation of Cube – Cuboid)
But, In G(□ABCD) and E(□PQRS) ,

E_U (□PQRS)= E_U (□LMNO) . $\left[\frac{n_2}{n_1} \cdot \frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ + 2 $\sqrt{l_1.b_1}$ (h_1-h_2). $\left[\frac{(n_1{}^2+1)}{n_1}\right]$… …(iii) (Vertical surface area relation of two cuboids when height is un-equal.)

E_A(□PQRS)-2 $l_1.b_1$ = E_A(□LMNO) - $2l_1.b_1$.] x $\left[\frac{n_2}{n_1} \cdot \frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ +2$\sqrt{l_1.b_1}$ (h_1-h_2) x $\left[\frac{(n_1{}^2+1)}{n_1}\right]$

E_A (□ PQRS) = E_A (□LMNO). $\left[\frac{n_2}{n_1} \cdot \frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$- 2 $l_1.b_1$. $\left[\frac{n_2}{n_1} \cdot \frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ + $2l_1.b_1$ + 2 $\sqrt{l_1.b_1}$ (h_1-h_2). $\left[\frac{(n_1{}^2+1)}{n_1}\right]$

E_A (□PQRS) = E_A (□LMNO) $\left[\frac{n_2}{n_1} \cdot \frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$ **+$2l_1.b_1$.** $\left[1-\left(\frac{n_2}{n_1} \cdot \frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right)\right]$ **+2** $\sqrt{l_1.b_1}$ **(h_1-h_2).** $\left[\frac{(n_1{}^2+1)}{n_1}\right]$

Hence, we have Proved the Total surface area relation of two cuboids when height is un-equal.
This Relation cleared the following points-
1) Two cuboid inside area of two rectangle are same but height is un-equal.
2) Total surface area relation of two cuboids is explained with the help of formula, when height is un-equal and both sides of equation become equal.

Note : in above relation $l_1.b_1=l_2.b_2$ and h is defind as h = $\sqrt{l_1.b_1}$

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