Prime Labeling of Duplication of Some Star related Graphs

S. Meena[1], P. Kavitha[2]

[1]Department of Mathematics,
Government Arts College, C-Mutlur, Chidambaram– 608 102,
Tamil Nadu, India.
[2] Department of Mathematics, S.R.M University, Chennai– 603 203,
Tamil Nadu, India

Abstract:
A graph $G=(V,E)$ with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the label of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper we investigate prime labeling for some star related to graph. We also discuss prime labeling in the context of duplication of graph elements.

Keywords: *Graph Labeling, Prime Labeling, Prime Graph.*

1.Introduction:

We begin with finite, undirected and non trivial graph $G=(V(G),E(G))$ with vertex set V(G) and edge set E(G). The elements of V(G) and E(G) are commonly termed as graph elements. Throughout this work, the Subdivision of Star $S_{1,n}$ is obtained from $K_{1,n}$ by joining n pendant edges with $K_{1,n}$ that is a graph with a vertex of n degree called apex, n vertices of degree 2 and n vertices of degree one called pendent vertices. Throughout this paper $|V(G)|$ and $|E(G)|$ denote the cardinality of the vertex set and edge set respectively. For various graph theoretic notation and terminology we refer to Bondy and Murthy [1]. We give brief summary of definitions and other information which are useful for the present investigation.

Definition 1.1:
If the vertices of the graph are assigned values subject to certain condition(s) then it is known as graph labeling.

Definition 1.2:
A prime labeling of a graph G is an injective function $f: \mathrm{V(G)} \to \{1,2,...,|\mathrm{V(G)}|\}$ such that for every pair of adjacent vertices u and v, $\gcd(f(u), f(v)) = 1$. The graph which admits a prime labeling is called a prime graph.

The notion of a prime labeling was originated by Entringer and it was discussed by Tout et al [8]. Fu and Huang [3] proved that P_n and $K_{1,n}$ are prime graphs. Lee et al [5] proved that W_n is a prime graph if and only if n is even. Deretsky et al [2] proved that C_n is a prime graph. Vaidya and Prajapati [9] discussed prime labeling in the context of duplication of graph elements. Meena and Vaithilingam [7] have investigated existence of the prime labeling for some crown related graph. In [6] Meena and Kavitha proved the prime labeling for some butterfly related graphs. A variant of prime labeling known as vertex – edge prime labeling is also introduced by Venkatachalam and Antoni Raj in [10]. For latest Dynamic survey on graph labeling we refer to [4] (Gallian .J.A., 2009). Vast amount of literature is available on different types of graph labeling. More than 1000 research papers have been published so far in last four decades.

Definition 1.3:
Duplication of a vertex v of a graph G produces a new graph $G^{'}$ by adding a new vertex $v^{'}$ such that $N(v^{'}) = N(v)$. In other words a vertex $v^{'}$ is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to $v^{'}$ in $G^{'}$.

Definition 1.4:
Duplication of a vertex v_k by a new edge $e = v_k^{'}v_k^{''}$ in a graph G produces a new graph $G^{'}$ such that $N(v_k^{'}) = \{v_k, v_k^{''}\}$ and $N(v_k^{''}) = \{v_k, v_k^{'}\}$.

Definition 1.5:
Duplication of an edge $e = uv$ by a new vertex w in a graph G produces a new graph $G^{'}$ such that $N(w) = \{u,v\}$.

Definition 1.6:
Duplication of an edge $e = uv$ of a graph G produces a new graph G' by adding an edge $e' = u'v'$ such that $N(u') = N(u) \cup (v') - \{v\}$ and $N(v') = N(v) \cup (u') - \{u\}$.

In this paper, we investigate prime labeling for some graphs obtained by duplication of graph elements and also we derive some result for subdivision of star $S_{1,n}$ in this context.

2.Main Results:

Theorem 2.1:
The graph obtained by duplication of all vertices by an edges in subdivision of star $S_{1,n}$ is not a prime graph.

Proof:
Let v_0 be the apex vertex $v_1, v_2, \dots v_n$ be consecutive vertices of degree 2 and $u_1, u_2, \dots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplication of each and every vertex v_0, v_i, u_i by each of the edges $v_0'v_0'', u_i'u_i'', v_i'v_i''$ for $1 \le i \le n$. Then G is a graph with $6n+3$ vertices and having $2n+1$ vertex disjoint cycles each of length three. Any prime labeling of G must contains at the most one even label in each of these $2n+1$ cycles as it is not possible to assign even labels for two adjacent vertices. Consequently at the most $2n+1$ vertices will receive even labels out of $6n+3$ vertices. Hence the number of even integers which are left to be used as vertex labels is at least $\left\lceil \frac{6n+3}{2} \right\rceil - (2n+1) = \left\lceil \frac{6n+3}{2} - (2n+1) \right\rceil = \left\lceil \frac{2n+1}{2} \right\rceil \ge 1$. That means at least one even integer from $\{1, 2, \dots, 6n+3\}$ is left for label assignment. This labeling is not a prime labeling. Hence G is not a prime graph.

Theorem 2.2:
The graph obtained by duplication of a vertex in subdivision of star $S_{1,n}$ is a prime graph.

Proof:
Let v_0 be the apex vertex $v_1, v_2, \dots v_n$ be consecutive vertices of degree 2 and $u_1, u_2, \dots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplication of vertex v in a subdivision of star by a vertex v'. We consider the following cases depending on degree of v:

Case (i): If $\deg(v) = n$ then $v = v_0$. Let v_0' be the duplication of v_0 in G then define $f : V(G) \to \{1, 2, \dots 2n+1, 2n+2\}$ as

$$f(v) = \begin{cases} 2i+1 & if \;\; v = v_i \;\; for \;\; i = 0,1,2,3,\dots n; \\ 2i+2 & if \;\; v = u_i \;\; for \;\; i = 1,2,3,\dots n; \\ 2 & if \;\; v = v_0', \end{cases}$$

then f is an injection and it is a prime labeling for G.

Case (ii): If $\deg(v) = 2$ then $v = v_k$ and let v_k' be the duplication of v_k in G then define $f : V(G) \to \{1, 2, \dots 2n+1, 2n+2\}$ as

$$f(v) = \begin{cases} 1 & if \;\; v = v_0; \\ 2i & if \;\; v = v_i \;\; for \;\; 1 \le i \le k \\ 2i+1 & if \;\; v = u_i \;\; for \;\; 1 \le i \le n; \\ 2i+2 & if \;\; v = v_i \;\; for \;\; k+1 \le i \le n; \\ 2k+2 & if \;\; v = v_k', \end{cases}$$

then f is an injection and it is a prime labeling for G.

Case (iii): If $\deg(v) = 1$ then $v = u_k$ and let u_k' be the duplication of u_k in G where $k = 1, 2, \dots n$ then define $f : V(G) \to \{1, 2, \dots 2n+1, 2n+2\}$ as

$$f(v) = \begin{cases} 2i+1 & if \;\; v = v_i \;\; for \;\; i = 1,2,\dots k,\dots n; \\ 2i+2 & if \;\; v = u_i \;\; for \;\; i = 1,2,\dots k,\dots n; \\ 2 & if \;\; v = v_k', \end{cases}$$

Then f is an injection and it is a prime labeling for G.

Thus from all cases described above G is a prime graph.

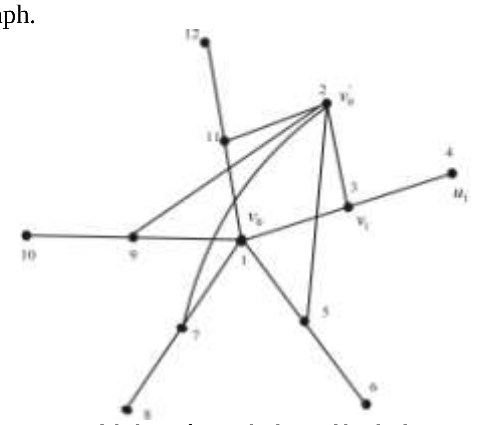

Fig. 1 A prime labeling of a graph obtained by duplication of the apex vertex in $S_{1,5}$

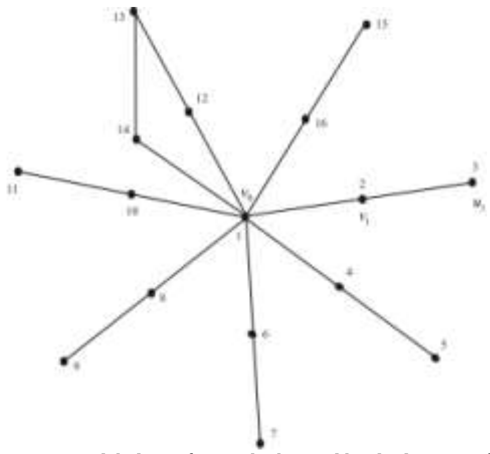

Fig. 2 A prime labeling of a graph obtained by duplication of the vertex of degree 2 in $S_{1,7}$

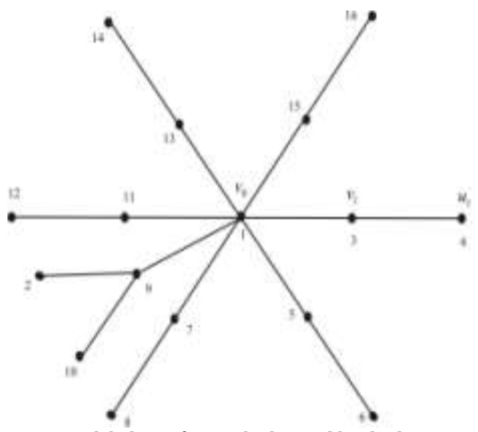

Fig. 3 A prime labeling of a graph obtained by duplication of the pendant vertex in $S_{1,7}$

Theorem 2.3:

The graph obtained by duplicating all the pendent vertices in subdivision of star $S_{1,n}$ is a prime graph.

Proof:

Let v_0 be the apex vertex $\mathrm{v}_1, \mathrm{v}_2, \dots \mathrm{v}_n$ be consecutive vertices of degree 2 and $u_1, u_2, \dots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplicating all the pendent vertices in subdivision of star and let the new vertices $u_1', u_2', \dots, u_n'$ then the vertex set $V(G) = \{\mathrm{v}_0, \mathrm{v}_1, \mathrm{v}_2, \dots \mathrm{v}_n, u_1, u_2, \dots u_n, u_1', u_2', \dots, u_n'\}$ and edge set $E(G) = \{v_0 v_i, v_i \mathrm{u}_i, v_i \mathrm{u}_i' / 1 \le i \le n\}$. Here $|\mathrm{V(G)}| = 3\mathrm{n}+1$, $|E(G)| = 3n$ then define a labeling $f : \mathrm{V(G)} \to \{1, 2, 3, \dots 3n+1\}$ as

$$f(v) = \begin{cases} 1 & if \;\; v = v_0; \\ 3i & if \;\; v = v_i \;\; for \;\; i = 1, 2, \dots, n; \\ 3i-1 & if \;\; v = u_i \;\; for \;\; i = 1, 2, \dots, n; \\ 3i+1 & if \;\; v = u_i' \;\; for \;\; i = 1, 2, \dots, n, \end{cases}$$

then f is an injection and it is a prime labeling for G. Hence G is a prime graph.

Theorem 2.4:

The graph obtained by duplicating all the vertices of degree 2 in subdivision of star $S_{1,n}$ is a prime graph.

Proof:

Let v_0 be the apex vertex $\mathrm{v}_1, \mathrm{v}_2, \dots \mathrm{v}_n$ be consecutive vertices of degree 2 and $u_1, u_2, \dots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplicating all the vertices of degree 2 in subdivision of star and let the new vertices $v_1', \mathrm{v}_2', \dots, \mathrm{v}_n'$ .Then the vertex set $V(G) = \{\mathrm{v}_0, \mathrm{v}_1, \mathrm{v}_2, \dots \mathrm{v}_n, u_1, u_2, \dots u_n, \mathrm{v}_1', \mathrm{v}_2', \dots, \mathrm{v}_n'\}$ and edge set $E(G) = \{v_0 v_i, v_0 v_i', \mathrm{v}_i \mathrm{u}_i, v_i' \mathrm{u}_i / 1 \le i \le n\}$.

Here $|\mathrm{V(G)}| = 3\mathrm{n}+1, |E(G)| = 4n$, then define a labeling $f : \mathrm{V(G)} \to \{1, 2, 3, \dots 3n+1\}$ as

$$f(v) = \begin{cases} 1 & if \;\; v = v_0; \\ 3i+1 & if \;\; v = v_i \;\; for \;\; i = 1, 2, \dots, n; \\ 3i & if \;\; v = u_i \;\; for \;\; i = 1, 2, \dots, n; \\ 3i-1 & if \;\; v = v_i' \;\; for \;\; i = 1, 2, \dots, n, \end{cases}$$

then f is an injection and it is a prime labeling for G. Hence G is a prime graph.

Theorem 2.5:

The graph obtained by duplicating all the vertices of the subdivision of star $S_{1,n}$, except the apex vertex is a prime graph.

Proof:

Let v_0 be the apex vertex $\mathrm{v}_1, \mathrm{v}_2, \dots \mathrm{v}_n$ be the consecutive vertices of degree 2 and $u_1, u_2, \dots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplicating all the vertices in subdivision of star, except the apex vertex v_0. Now let $u_1', u_2', \dots, u_n'$ and $v_1', \mathrm{v}_2', \dots, \mathrm{v}_n'$ be the new vertices of G by duplicating $u_1, u_2, \dots u_n$ and $\mathrm{v}_1, \mathrm{v}_2, \dots \mathrm{v}_n$ then the vertex set $\mathrm{V}(G) = \{v_0, u_i, \mathrm{u}_i', v_i v_i' / 1 \le i \le n\}$ and edge set $E(G) = \{v_0 v_i, v_0 v_i', \mathrm{v}_i \mathrm{u}_i, v_i' \mathrm{u}_i, v_i' u_i' / 1 \le i \le n\}$.

Here $|\mathrm{V(G)}| = 4\mathrm{n}+1, |E(G)| = 5n$, then define a labeling $f : \mathrm{V(G)} \to \{1, 2, 3, \dots 4n+1\}$ as

$$f(v) = \begin{cases} 1 & if \;\; v = v_0; \\ 4i-2 & if \;\; v = v_i' \;\; for \;\; 1 \le i \le n; \\ 4i-1 & if \;\; v = u_i \;\; for \;\; 1 \le i \le n; \\ 4i+1 & if \;\; v = u_i' \;\; for \;\; 1 \le i \le n; \\ 4i & if \;\; v = v_i \;\; for \;\; 1 \le i \le n \end{cases}$$

then f is an injection and it is a prime labeling for G. Hence G is a prime graph.

Theorem 2.6:
The graph obtained by duplicating all the vertices of the subdivision of star $S_{1,n}$ is a prime graph.

Proof:
Let v_0 be the apex vertex $v_1, v_2, \ldots v_n$ be consecutive vertices of degree 2 and $u_1, u_2, \ldots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplicating all the vertices of the subdivision of star. Now let the new vertices $v_0^{'}, v_1^{'}, v_2^{'}, \ldots, v_n^{'}$ and $u_1^{'}, u_2^{'}, \ldots, u_n^{'}$ be the new vertices of G by duplicating $v_0, v_1, v_2, \ldots v_n$ and $u_1, u_2, \ldots u_n$ respectively, then define a labeling $f : V(G) \to \{1, 2, 3, \ldots 4n+2\}$ as

$$f(v) = \begin{cases} 1 & if \;\; v = v_0; \\ 2 & if \;\; v = v_0^{'}; \\ 4i+1 & if \;\; v = v_i \;\; for \;\; 1 \le i \le n; \\ 4i-1 & if \;\; v = v_i^{'} \;\; for \;\; 1 \le i \le n; \\ 4i & if \;\; v = u_i \;\; for \;\; 1 \le i \le n; \\ 4i+2 & if \;\; v = u_i^{'} \;\; for \;\; 1 \le i \le n, \end{cases}$$

then f is an injection and it is a prime labeling for G. Hence G is a prime graph.

Theorem 2.7:
The graph obtained by duplication of the vertex by an edge in subdivision of star $S_{1,n}$ is a prime graph.

Proof:
Let v_0 be the apex vertex $v_1, v_2, \ldots v_n$ be consecutive vertices of degree 2 and $u_1, u_2, \ldots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplication of a vertex v_k by an edge $v_k^{'} v_k^{"}$. Here $|V(G)| = 2n+3, |E(G)| = 2n+3$. We consider the following cases depending on degree of v:

Case (i): If $\deg(v_k) = n$ in subdivision of star then $v_k = v_0$. Now let $v_0^{'} v_0^{"}$ be the new vertices of G, then define a labeling $f : V(G) \to \{1, 2, 3, \ldots 2n+3\}$ as

$$f(v) = \begin{cases} 1 & if \;\; v = v_0; \\ 2i+2 & if \;\; v = v_i \;\; for \;\; i = 1, 2, \ldots, n; \\ 2i+3 & if \;\; v = u_i \;\; for \;\; i = 1, 2, \ldots, n; \\ 2 & if \;\; v = v_0^{'}; \\ 3 & if \;\; v = v_0^{"}, \end{cases}$$

then f is an injection and it is a prime labeling for G.

Case (ii): If $\deg(v_k) = 2$ in subdivision of star then $v_k = v_j$. Let $v_j^{'} v_j^{"}$ be the new vertices of G, then define a labeling $f : V(G) \to \{1, 2, 3, \ldots 2n+3\}$ as

$$f(v) = \begin{cases} 1 & if \;\; v = v_0; \\ 2i+4 & if \;\; v = v_i \;\; for \;\; i = 1, 2, \ldots, k-1; \\ 2i+2 & if \;\; v = v_i \;\; for \;\; i = k+1, k+2, \ldots, n; \\ 3 & if \;\; v = v_k; \\ 2 & if \;\; v = v_k^{'}; \\ 5 & if \;\; v = v_k^{"}; \\ 4 & if \;\; v = u_k; \\ 2i+5 & if \;\; v = u_i \;\; for \;\; i = 1, 2, \ldots, k-1; \\ 2i+3 & if \;\; v = u_i \;\; for \;\; i = k+1, k+2, \ldots, n, \end{cases}$$

then f is an injection and it is a prime labeling for G.

Case (iii): If $\deg(v_k) = 1$ in subdivision of star then $v_k = u_j$. Let $u_j^{'} u_j^{"}$ be the new vertices of G, then define a labeling f_1 using the labeling f defined in case (i) as follows: $f_1(v_k) = 2$, $f_1(u_k) = 3$, $f_1(u_k^{'}) = 4$, $f_1(u_k^{"}) = 5$ for $k = 1, 2, \ldots, n$ and $f_1(v) = f(v)$ for all the remaining vertices. Then the resulting labeling f is a prime labeling. Thus from all the cases described above G is a prime graph.

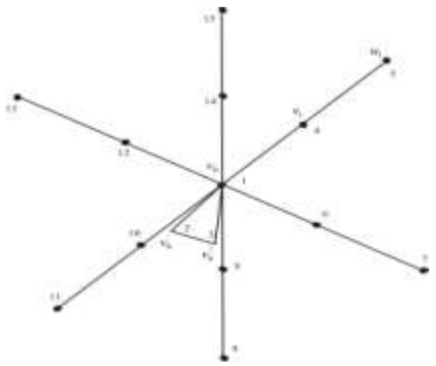

Fig. 4 A prime labeling of a graph obtained by duplication of a apex vertex by an edge in $S_{1,6}$

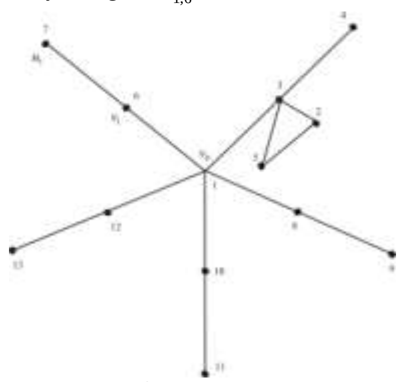

Fig. 5 A prime labeling of a graph obtained by duplication of a vertex of degree 2 by an edge in $S_{1,5}$

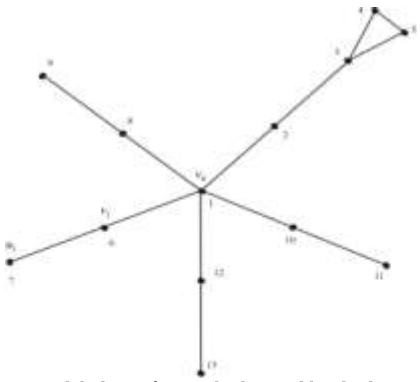

Fig. 6 A prime labeling of a graph obtained by duplication of a pendant vertex by an edge in $S_{1,5}$

Theorem 2.8:

The graph obtained by duplication of an edge by a vertex in subdivision of star $S_{1,n}$ is a prime graph.

Proof:

Let v_0 be the apex vertex $\mathrm{v}_1, \mathrm{v}_2, \ldots \mathrm{v}_n$ be consecutive vertices of degree 2 and $u_1, u_2, \ldots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplication of an edge by a vertex. Here $|\mathrm{V(G)}| = 2\mathrm{n}+2, |E(G)| = 2n+2$ and we consider two cases.

Case (i): The duplication of an edge $v_0 v_k$ by a vertex $v_k^{'}$ for $1 \le k \le n$ in subdivision of star, then define

$f : \mathrm{V(G)} \to \{1, 2, 3, \ldots 2n+2\}$ as

$$f(v) = \begin{cases} 1 & if \;\; v = v_0; \\ 2 & if \;\; v = v_k^{'} \; ; \\ 2i+1 & if \;\; v = v_i \;\; for \; i = 1, 2, \ldots, \mathrm{n}; \\ 2i+2 & if \;\; v = u_i \;\; for \; i = 1, 2, \ldots, \mathrm{n}, \end{cases}$$

then f is an injection and it is a prime labeling for G.

Case (ii): Here the duplication of an edge $v_0 u_k$ in subdivision of star by a vertex $u_k^{'}$ for $1 \le k \le n$ then define a labeling f_1 using the labeling f defined in case (i) as follows: $f_1(v_0) = f(v_k^{'})$ and $f_1(v_k^{'}) = f(v_0)$ for $1 \le k \le n$ and $f_1(v) = f(v)$ for all the remaining vertices. Thus f_1 is a prime labeling. Hence from the above cases described above G is a prime graph.

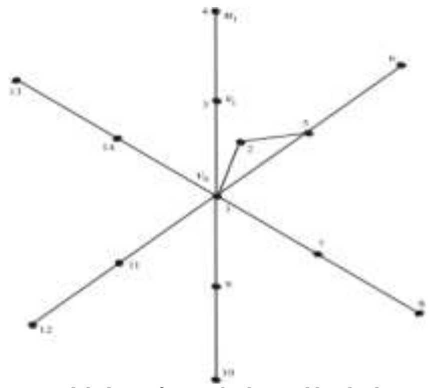

Fig. 7 A prime labeling of a graph obtained by duplication of an edge by a vertex in $S_{1,6}$

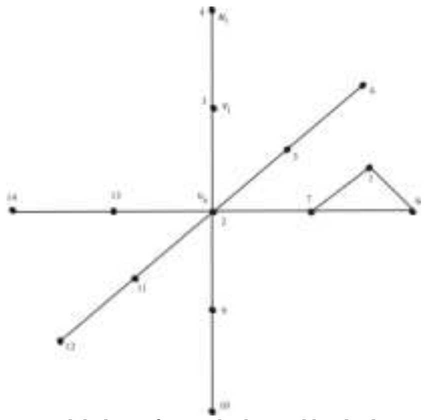

Fig. 8 A prime labeling of a graph obtained by duplication of an edge by a vertex in $S_{1,6}$

Theorem 2.9:

The graph obtained by duplication of an edge in subdivision of star $S_{1,n}$ is a prime graph.

Proof:

Let v_0 be the apex vertex $\mathrm{v}_1, \mathrm{v}_2, \ldots \mathrm{v}_n$ be consecutive vertices of degree 2 and $u_1, u_2, \ldots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplication of an edge e by a new edge $e^{'}$. Here $|\mathrm{V(G)}| = 2\mathrm{n}+3$ then define a labeling $f : \mathrm{V(G)} \to \{1, 2, 3, \ldots 2n+3\}$ by considering the following cases:

Case (i): If the edge e is one of the pendent edges of subdivision of star say, $e = v_i u_i$ then G can be thought as a graph with new $e^{'} = v_i^{'} u_i^{'}$ incident with $v_0 v_i^{'}$ and $v_i^{'} u_i^{'}$, which is again a subdivision of star. Hence it is a prime graph as discussed [3].

Case (ii): Let G be the graph obtained by duplication of an edge $e = v_0 v_i$ by a new edge $e^{'} = v_0^{'} v_i^{'}$ which vertex $v_0^{'}$ is incident with

$v_1, v_2, \dots v_{i-1}, v_{i+1}, \dots, v_n$ and the vertex $v_i^{'}$ is incident with $v_i^{'}$ only.

$$\text{Now } f(v) = \begin{cases} 1 & \text{if } v = v_0; \\ 2 & \text{if } v = v_0^{'}; \\ 3 & \text{if } v = v_i^{'} \text{ for } i = 1,2,\dots,n; \\ 2i+3 & \text{if } v = v_i \text{ for } i = 1,2,\dots,n; \\ 2i+2 & \text{if } v = u_i \text{ for } i = 1,2,\dots,n \end{cases}$$

Then f is an injection and it is a prime labeling for G. Thus from all the cases described above G is a prime graph.

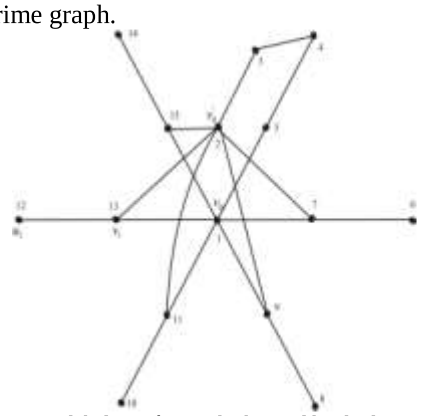

Fig. 9 A prime labeling of a graph obtained by duplication of an edge by a vertex in $S_{1,6}$

Theorem 2.10:

The graph obtained by duplication of every edge by a vertex in subdivision of star $S_{1,n}$ is a prime graph.

Proof:

Let v_0 be the apex vertex $v_1, v_2, \dots v_n$ be consecutive vertices of degree 2 and $u_1, u_2, \dots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplication of every edges $v_0 v_i$ by a vertex $v_i^{'}$ and $v_i u_i$ by a vertex $u_i^{'}$ then the vertex set

$V(G) = \{v_0, v_i, u_i, v_i^{'}, u_i^{'} / 1 \le i \le n\}$, and edge set

$E(G) = \{v_0 v_i, v_0 v_i^{'} / 1 \le i \le n\} \cup \{v_i u_i, v_i^{'} u_i^{'}, u_i^{'} u_i / 1 \le i \le n\}$

.

Here $|V(G)| = 4n+1$ and $|E(G)| = 6n$, then define a labeling $f : V(G) \to \{1, 2, 3, \dots 4n+1\}$ as

$$f(v) = \begin{cases} 1 & \text{if } v = v_0; \\ 4i-1 & \text{if } v = v_i \text{ for } i = 1,2,\dots,n; \\ 4i+1 & \text{if } v = u_i \text{ for } i = 1,2,\dots,n; \\ 4i-2 & \text{if } v = v_i^{'} \text{ for } i = 1,2,\dots,n; \\ 4i & \text{if } v = u_i^{'} \text{ for } i = 1,2,\dots,n, \end{cases}$$

then f is an injection and it is a prime labeling for G. Thus from all the cases described above G is a prime graph.

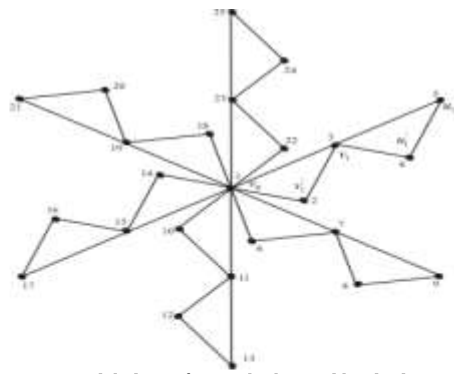

Fig. 10 A prime labeling of a graph obtained by duplication of each edge by a vertex in $S_{1,6}$

Theorem 2.11:

The graph obtained by duplication of the edges $v_0 v_i$ by a vertex $v_i^{'}$ in subdivision of star $S_{1,n}$ is a prime graph.

Proof:

Let v_0 be the apex vertex $v_1, v_2, \dots v_n$ be consecutive vertices of degree 2 and $u_1, u_2, \dots u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplication of each of the edges $v_0 v_i$ by a vertex $v_i^{'}$. Now the vertex set $V(G) = \{v_0, v_i, u_i, v_i^{'} / 1 \le i \le n\}$ and the edge set $E(G) = \{v_0 v_i, v_0 v_i^{'}, v_i v_i^{'}, v_i u_i / 1 \le i \le n\}$.

Here $|V(G)| = 3n+1$ and $|E(G)| = 4n$ then define a labeling $f : V(G) \to \{1, 2, 3, \dots 3n+1\}$ as

$$f(v) = \begin{cases} 1 & \text{if } v = v_0; \\ 3i & \text{if } v = v_i \text{ for } i = 1,2,\dots,n; \\ 3i-1 & \text{if } v = v_i^{'} \text{ for } i = 1,2,\dots,n; \\ 3i+1 & \text{if } v = u_i \text{ for } i = 1,2,\dots,n, \end{cases}$$

then f is an injection and it is a prime labeling for G. Thus from all the cases described above G is a prime graph.

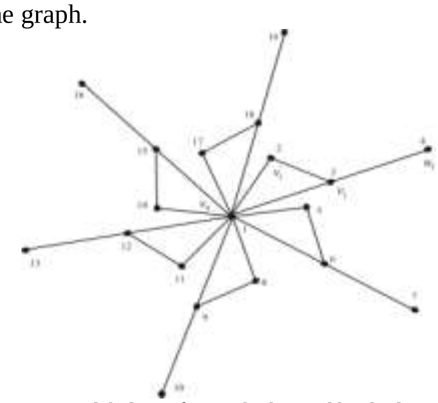

Fig. 11 A prime labeling of a graph obtained by duplication of the edges $v_0 v_i$ by vertices $v_i^{'}$ in $S_{1,6}$

Theorem 2.12:

The graph obtained by duplication of the edges by a vertex which are incident with pendent vertices in subdivision of star $S_{1,n}$ is not a prime graph.

Proof:

Let v_0 be the apex vertex $\mathrm{v}_1, \mathrm{v}_2, ... \mathrm{v}_n$ be consecutive vertices of degree 2 and $u_1, u_2, ... u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplication of the edges $v_i u_i$ by a vertex $u_i^{'}$ in subdivision of star. Then G is a graph with $3n+1$ vertices and having n vertex disjoint cycles each of length three. Any prime labeling of G most contains at the most one even label in each of these n cycles as it is not possible to assign even labels for two adjacent vertices. Consequently at the most n vertices will receive even label out of $3n+1$ vertices. Hence the number of even integer which are left to be used as vertex labels are atleast

$\left[\frac{3n+1}{2}\right] - n = \left[\frac{3n+1}{2} - n\right] = \frac{n+1}{2} \geq 1$. That means atleast one even integer from $\{1, 2, 3, ..., 3\mathrm{n}+1\}$ is left for label assignment. This not possible as prime labeling is bijective. Hence G is not a prime graph.

Theorem 2.13:

The graph obtained by duplication of every pendent vertex by an edge in subdivision of star $S_{1,n}$ is a prime graph.

Proof:

Let v_0 be the apex vertex $\mathrm{v}_1, \mathrm{v}_2, ... \mathrm{v}_n$ be consecutive vertices of degree 2 and $u_1, u_2, ... u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplication of every pendent vertex u_i by an edges $u_i^{'} u_i^{''}$ for $1 \leq i \leq n$, then the vertex set $\mathrm{V}(G) = \{v_0, v_i, \mathrm{u}_i, \mathrm{u}_i^{'}, \mathrm{u}_i^{''} / 1 \leq i \leq n\}$ and the edge set

$E(G) = \{v_0 v_i \,/ 1 \leq i \leq n\} \cup \{\mathrm{v}_i\, u_i, \mathrm{u}_i\, u_i^{'}, u_i u_i^{''}, u_i^{'} u_i^{''} \,/ 1 \leq i \leq n\}$

.

Here $|\mathrm{V(G)}| = 4\mathrm{n}+1$ and $|E(G)| = 5n$, then define a labeling $f : \mathrm{V(G)} \rightarrow \{1, 2, 3, ... 4n+1\}$ as

$$f(v) = \begin{cases} 1 & \text{if } v = v_0; \\ 4i-2 & \text{if } v = v_i \text{ for } i = 1,2,...,n; \\ 4i-1 & \text{if } v = u_i \text{ for } i = 1,2,...,n; \\ 4i & \text{if } v = u_i^{'} \text{ for } i = 1,2,...,n; \\ 4i+1 & \text{if } v = u_i^{''} \text{ for } i = 1,2,...,n, \end{cases}$$

then f is an injection and it is a prime labeling for G. Hence G is a prime graph.

Theorem 2.14:

The graph obtained by duplication of every vertex of degree 2 by an edge in subdivision of star $S_{1,n}$ is a prime graph.

Proof:

Let v_0 be the apex vertex $\mathrm{v}_1, \mathrm{v}_2, ... \mathrm{v}_n$ be consecutive vertices of degree 2 and $u_1, u_2, ... u_n$ be the consecutive pendent vertices of subdivision of star. Let G be the graph obtained by duplication of every vertex of degree 2 by an edge $v_i^{'} v_i^{''}$, then the vertex set $\mathrm{V}(G) = \{v_0, v_i, \mathrm{u}_i, \mathrm{v}_i^{'}, \mathrm{v}_i^{''} / 1 \leq i \leq n\}$ and edge set

$E(G) = \{v_0 v_i \,/ 1 \leq i \leq n\} \cup \{\mathrm{v}_i\, u_i, \mathrm{v}_i\, v_i^{'}, \mathrm{v}_i\, v_i^{''}, \mathrm{v}_i^{'}\, \mathrm{v}_i^{''} \,/ 1 \leq i \leq n\}$

.

Here $|\mathrm{V(G)}| = 4\mathrm{n}+1$ and $|E(G)| = 5n$ then define a labeling f_1 using the labeling f defined in above theorem as follows: $f_1(u_i) = f(\mathrm{v}_i)$, $f_1(\mathrm{v}_i) = f(\mathrm{u}_i)$, $f_1(\mathrm{v}_i^{'}) = f(\mathrm{u}_i^{'})$, $f_1(\mathrm{v}_i^{''}) = f(\mathrm{u}_i^{''})$ *for* $i = 1, 2, ..., n,$ and $f_1(v) = f(v)$ for the remaining vertices. Then the resulting labeling f_1 is a prime labeling. Hence G is a prime graph.

References

[1] J.A.Bondy and U.S.R.Murthy, *Graph Theory and Applications* (North-Holland), Newyork, 1976.

[2] T. Deretsky, S. M. Lee and J. Mitchem, “On Vertex Prime Labelings of Graphs in Graph Theory, *Combinatorics and Applications*, Wiley, New York, 1 (1991), 359-369.

[3] H.L.Fu, K.C.Huang, *On Prime Labellings*, Discrete Mathematics, 127(1994), 181-186,1994.

[4] J.A.Gallian, *A Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinatorics, 18(2011), 147, #DS6.

[5] S.M.Lee, I.Wui, J.Yeh, *On the amalgamation of prime graphs*, Bull. Malaysian Math. Soc. (Second Series), 11(1988), 59-67.

[6] S. Meena and P. Kavitha, *Prime labeling for some butterfly related graphs*, International Journal of Mathematical Archive, 5(10), 2014, 15-25.

[7] S. Meena and K. Vaithilingam, *Prime labeling for some crown related graphs*, International Journal of Scientific & Technology Research Vol.2, Issue 3, (2013) 92-95.

[8] A.Tout, A.N.Dabboucy, K.Howalla, *Prime labeling of Graphs*, Nat. Acad. Sci. Letters, 11(1982), 365-368.

[9] S.K.Vaidya and U.M.Prajapati, *Prime labeling in the context of duplication of graph elements* , International Journal of Mathematics and Soft Computing, *Vol.3, No.1 (2013), 13 - 20.*

[10] M.Venkatachalam and J.Arockia Antoni Raj, *A Note on Vertex-Edge Prime Labeling for Graphs*, Antarctica J. Math., 1,(2009), 79-84