Homomorphism and Anti Homomorphism on Multi L-Fuzzy Quotient Group of a Group

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Abstract

In this paper we introduce the notion of homomorphism and anti homomorphism of a multi L-fuzzy quotient subgroup of a group and some of its properties are investigated.

Keywords

Fuzzy set, multi-L-fuzzy subgroup, homomorphism of multi L-fuzzy group, anti homomorphism of multi L-fuzzy group, quotient subgroup, multi L-fuzzy quotient subgroup.

1.INTRODUCTION

L. A. Zadeh[19] introduced the notion of a fuzzy subset A of a set X as a function from X into I = [0, 1]. Rosenfeld [3] and Kuroki [12] applied this concept in group theory and semi group theory, and developed the theory of fuzzy subgroups and fuzzy sub semi groupoids respectively.The concept of anti – fuzzy subgroup was introduced by Biswas [5]. The Concept of multi fuzzy subgroups was introduced by Souriar Sebastian and S.Babu Sundar [17]. In all these studies, the closed unit interval [0, 1] is taken as the Membership lattice.

We introduce the notion of a multi L-fuzzy sub group G and discussed some of its properties. The characterizations of a Multi L-fuzzy subgroup under homomorphism and anti homomorphism on multi L-fuzzy quotient subgroup on a group is discussed.

2. Preliminaries

In this Section, we review some definitions and some results of Multi L-fuzzy subgroups which will be used in the later sections. Throughout this section we mean that $(G,*)$ is a group, e is the identity of G and xy as $x*y$.

2.1 Definition:

Let X be any nonempty set. A fuzzy set A of X is $A : X \to [0, 1]$.

2.2 Definition:

Let (G, .) be a group. A fuzzy subset A of G is said to be a fuzzy subgroup (FSG) of G if the following conditions are satisfied:

i. $A(xy) \geq \min\{A(x), A(y)\}$,

ii. $A(x^{-1}) = A(x)$, for all x and $y \in G$.

2.3 Definition:

Let (G, .) be a group. A fuzzy subgroup A of G is said to be a normal fuzzy subgroup of G if A(xy) = A (yx), for all x and $y \in G$.

2.4 Definition:

A fuzzy subset A of G is said to be a anti fuzzy group of G, if for all $x,y \in G$

i $A(xy) \leq \max\{A(x), A(y)\}$

ii $A(x^{-1}) = A(x)$.

2.5 Definition:

A anti fuzzy subgroup A of G is called a anti fuzzy normal subgroup (AFNS) of G if for every $x, y \in G$,

$A(xyx^{-1}) \leq A(y)$.

2.6 Definition:

Let X be a non – empty set. A multi L-fuzzy set A in X is defined as a set of ordered sequences, A = {(x, A1(x), A2(x), ..., Ai(x), ...) : $x \in X$}, where

Ai : X → L for all i.

2.7 Definition:

A multi L-fuzzy subset A of G is called a multi L-fuzzy subgroup (MLFS) of G if for every x,y∈G,

i $A(xy) \geq A(x) \wedge A(y)$

ii $A(x^{-1}) = A(x)$.

2.8 Definition:

A multi L-fuzzy subset A of G is called a multi anti L-fuzzy subgroup (MALFS) of G if for every x,y ∈G,

i $A(xy) \leq A(x) \vee A(y)$

ii A(x-1) = A (x).

2.9 Definition

The function f: G→G′ is said to be a homomorphism if f(xy)= f(x)f(y) ∀ x,y∈G.

2.10 Definition

The function f: G→ G′ (G and G′ are not necessarily commutative) is said to be an anti homomorphism if f(xy)= f(y)f(x) ∀ x,y∈G.

2.11 Definition

Let f be any function from a set X to a set Y, and let A be any L-fuzzy subset of X. Then A is called f-invariant if f(x) = f(y) implies A(x) = A(y), where x,y∈X.

2.12 Definition:

Let A be a multi L-fuzzy normal subgroup of G with identity e. Let

K = { x ∈ G / A(x) = A(e) }. Consider the map $\overline{A}$ = ($\tilde{A} = (\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_K)$) be a multi L-fuzzy group in G with

$A_i : G/K \to L$ defined by

$\overline{A}(xK) = \vee A(xk)$ for all k ∈K and x ∈ G. Then , the multi L-fuzzy subgroup $\overline{A}$ of G/K is called a multi L-fuzzy quotient group of A by K.

Remarks:

$\overline{A}$ is not a multi L-fuzzy normal quotient group of G/K.

Since, $\overline{A}(xKyK) \neq \overline{A}(yKxK)$.

Consider the map, $\overline{A} : G/K \to L$ defined by $\overline{A}(xK)$ = A(x) for all k ∈ K and x ∈ G. Then, $\overline{A}$ is a multi L-fuzzy normal quotient group of G/K .as a template and simply type your text into it.

3.Properties of multi L-fuzzy quotient group $\overline{A}$ determined by A and K under homomorphism and anti homomorphism:

In this section, we discuss some of the properties of multi L-fuzzy quotient group of a group G/K determined by A and K under homomorphism and anti homomorphism.

3.1 Theorem:

Let G and G′ be any two groups. Let f: G → G′ be a homomorphism and onto. Let $\overline{A}$ = ($\tilde{A} = (\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_K)$) be a multi L-fuzzy group in G with $A_i : G/K \to L$ be a multi L-fuzzy quotient group of G/K. Then $f(\overline{A})$ is a multi L-fuzzy quotient group of G'/K, if $\overline{A}$ has sup property and $\overline{A}$ is f-invariant and $f(\overline{A}) = \overline{f(A)}$.

Proof:

Let $\overline{A}$ be a multi L-fuzzy quotient group of G/K.

$f(\overline{A})(f(x)f(y)K)$

$= (f(\overline{A}))(f(xy)K)$

$= \overline{A}(xyK)$

$\geq \overline{A}(xK) \wedge \overline{A}(yK)$

$= (f(\overline{A}))(f(x)K) \wedge (f(\overline{A})(f(y)K)$

$f(\overline{A})(f(x)f(y)K) \geq (f(\overline{A}))(f(x)K) \wedge (f(\overline{A})(f(y)K).$

ii $f(\overline{A})([f(x)]-1K)$

$= f(\overline{A})[f(x-1)K] = \overline{A}(x-1K)$

$= \overline{A}(xK)$

$= f(\overline{A})[f(x)K]$

$= f(\overline{A})([f(x)]-1\,K)$

$= f(\overline{A})[f(x)K].$

Hence $f(\overline{A})$ is a multi L- fuzzy quotient group of G'/K.

Also , $\overline{f(A)}(yK) = \vee f(A)(yK)$,

for all $k \in K$ and $y \in G'$.

$= \vee f(A)(f(x)K)$, f is onto and $x \in G$.

$= \vee A(xK)$

$= \overline{A}(xK)$

$= f(\overline{A})(f(x)K)$

$= f(\overline{A})(yK).$

Hence $\overline{f(A)}(yK) = f(\overline{A})(yK).$

3.2 Theorem:

Let G and G′ be any two groups. Let f: G→ G′ be a homomorphism. Let $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, ..., \tilde{B}_K)$ be a multi L-fuzzy group in G with $B_i : G/K \to L$ be a multi L-fuzzy quotient group of G'/K. Then $f^{-1}(\overline{B})$ is a multi L-fuzzy quotient group of G/K and $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

Proof:

Let $\overline{B}$ be a multi L-fuzzy quotient group of G'/K.

$f^{-1}(\overline{B})(xyK) = \overline{B}(f(xy)K)$

$= \overline{B}(f(x)f(y)K)$

$\geq \overline{B}(f(x)K) \wedge \overline{B}(f(y)K)$

$\geq f^{-1}(\overline{B})(xK) \wedge f^{-1}(\overline{B})(yK)$

$f^{-1}(\overline{B})(xyK)$

$\geq f^{-1}(\overline{B})(xK) \wedge f^{-1}(\overline{B})(yK).$

$f^{-1}(\overline{B})(x-1K)$

$= \overline{B}(f(x-1)K)$

$= \overline{B}((f(x))-1K)$

$= \overline{B}(f(x)K)$

$= f^{-1}(\overline{B})(xK)$

$= f^{-1}(\overline{B})(x-1K)$

$= f^{-1}(\overline{B})(xK).$

Hence, $f^{-1}(\overline{B})$ is a multi L-fuzzy quotient group of G/K.

Also, $\overline{f^{-1}(B)}(xK) = \vee f\,{-1}(B)(xK)$,

for all $k \in K$ and $x \in G$.

$= \vee B(f(x)K)$

$= \overline{B}(f(x)K)$

$= f^{-1}(\overline{B})(xK).$

Hence, $\overline{f^{-1}(B)}(xK) = f^{-1}(\overline{B})(xK).$

3.3 Theorem:

Let G and G′ be any two groups. Let

f: G → G′ be an anti homomorphism and onto. Let

$\overline{A}$ = ($\tilde{A}=(\tilde{A}_1,\tilde{A}_2,....,\tilde{A}_K)$ be a multi L-fuzzy group in G with

$A_i : G/K \to L$ be a multi L-fuzzy quotient group of G/K. Then f($\overline{A}$) is a multi L-fuzzy quotient group of G'/K, if $\overline{A}$ has sup property and $\overline{A}$ is f- invariant and $f(\overline{A}) = \overline{f(A)}$.

Proof:

Let $\overline{A}$ be a multi L-fuzzy quotient group of G/K.

i. f ($\overline{A}$)(f(x)f(y)K) = (f ($\overline{A}$))(f(yx)K)

= $\overline{A}$ (yxK)

≥ $\overline{A}$ (yK) ∧ $\overline{A}$ (xK)

≥ $\overline{A}$ (xK) ∧ $\overline{A}$ (yK)

= (f ($\overline{A}$))(f(x)K) ∧ (f ($\overline{A}$)(f(y)K)

f ($\overline{A}$)(f(x)f(y)K)

≥ (f ($\overline{A}$))(f(x)K) ∧ (f ($\overline{A}$)(f(y)K).

ii f ($\overline{A}$)($[f(x)]^{-1}$K)

= f ($\overline{A}$)[f(x^{-1})K]

= $\overline{A}$ (x^{-1}K)

= $\overline{A}$ (xK)

= f ($\overline{A}$)[f(x)K]

= f ($\overline{A}$)($[f(x)]^{-1}$ K) = f ($\overline{A}$)[f(x)K].

Hence f ($\overline{A}$) is a multi L-fuzzy quotient group of G'/K.

Also, $\overline{f(A)}$ (yK) = ∨ f(A)(yK), for all k ∈K and y ∈ G′.

= ∨ f(A)(f(x)K), f is onto and x∈ G.

= ∨ A(xK)

= $\overline{A}$ (xK)

= f($\overline{A}$)(f(x)K)

= f($\overline{A}$)(yK).

Hence, $\overline{f(A)}$ (yK) = f($\overline{A}$)(yK).

3.4 Theorem:

Let G and G′ be any two groups. Let f: G→ G′ be an anti homomorphism.

Let $\tilde{B}=(\tilde{B}_1,\tilde{B}_2,....,\tilde{B}_K)$ be a multi L-fuzzy quotient group of G'/K. Then

f^{-1}($\overline{B}$) is a multi L-fuzzy quotient group of G/K and $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

Proof:

Let $\overline{B}$ be a multi L-fuzzy quotient group of G'/K.

f -1($\overline{B}$)(xyK) = $\overline{B}$ (f(xy)K)

= $\overline{B}$ (f(y)f(x)K)

≥ $\overline{B}$ (f(y)K) ∧ $\overline{B}$ (f(x)K)

≥ $\overline{B}$ (f(x)K) ∧ $\overline{B}$ (f(y)K)

≥ f^{-1}($\overline{B}$)(xK) ∧ f^{-1}($\overline{B}$)(yK)

f^{-1}($\overline{B}$)(xyK)

≥ f -1($\overline{B}$)(xK) ∧ f^{-1}($\overline{B}$)(yK).

f -1($\overline{B}$)(x^{-1}K) = $\overline{B}$ (f(x^{-1})K)

= $\overline{B}$ ($(f(x))^{-1}$K)

= $\overline{B}$ (f(x)K)

= f^{-1}($\overline{B}$)(xK)

f^{-1}($\overline{B}$)(x-1K) = f^{-1}($\overline{B}$)(xK).

Hence, $f^{-1}(\overline{B})$ is a multi L-fuzzy quotient subgroup of G/K.

Also,

$\overline{f^{-1}(B)}(xK) = \vee f^{-1}(B)(xK)$, for all $k \in K$ and $x \in G$.

$= \vee B(f(x)K)$

$= \overline{B}(f(x)K)$

$= f^{-1}(\overline{B})(xK)$.

Hence, $\overline{f^{-1}(B)}(xK) = f^{-1}(\overline{B})(xK)$.

3.5 Theorem:

Let G and G′ be any two groups. Let

$f: G \to G'$ be a homomorphism and onto. Let $\overline{A}$: $G/K \to Lk$ be a multi anti L-fuzzy quotient group of G/K. Then $f(\overline{A})$ is a multi anti L-fuzzy quotient group of G'/K, if $\overline{A}$ has sup property and $\overline{A}$ is f- invariant and $f(\overline{A}) = \overline{f(A)}$.

Proof:

Let $\overline{A}$ be a multi anti L-fuzzy quotient group of G/K.

i $f(\overline{A})(f(x)f(y)K)$

$= (f(\overline{A}))(f(xy)K)$

$= \overline{A}(xyK)$

$\leq \overline{A}(xK) \vee \overline{A}(yK)$

$= (f(\overline{A}))(f(x)K) \vee (f(\overline{A})(f(y)K)$

$f(\overline{A})(f(x)f(y)K) \leq (f(\overline{A}))(f(x)K) \vee (f(\overline{A})(f(y)K)$.

ii $f(\overline{A})([f(x)]^{-1}K)$

$= f(\overline{A})[f(x^{-1})K]$

$= \overline{A}(x^{-1}K)$

$= \overline{A}(xK)$

$= f(\overline{A})[f(x)K]$

$f(\overline{A})([f(x)]^{-1}K) = f(\overline{A})[f(x)K]$.

Hence $f(\overline{A})$ is a multi anti L- fuzzy quotient group of G'/K.

Also $\overline{f(A)}(yK)$

$= \wedge f(A)(yK)$, for all $k \in K$ and $y \in G'$.

$= \wedge f(A)(f(x)K)$,

f is onto and $= \wedge A(xK)$

$= \overline{A}(xK) = f(\overline{A})(f(x)K)$

$= f(\overline{A})(yK)$.

Hence, $\overline{f(A)}(yK) = f(\overline{A})(yK)$.

3.6 Theorem:

Let G and G′ be any two groups. Let f: G→ G′ be a homomorphism. Let $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, \ldots, \tilde{B}_K)$ be a multi anti L-fuzzy quotient group of G'/K. Then $f^{-1}(\overline{B})$ is a multi anti L-fuzzy quotient group of G/K and $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

Proof:

Let $\overline{B}$ be a multi anti L-fuzzy quotient group of G'/K.

i. $f^{-1}(\overline{B})(xyK)$

$= \overline{B}(f(xy)K)$

$= \overline{B}(f(x)f(y)K)$

$\leq \overline{B}(f(x)K) \vee \overline{B}(f(y)K)$

$\leq f^{-1}(\overline{B})(xK) \vee f^{-1}(\overline{B})(yK)$

$f^{-1}(\overline{B})(xyK)$

$\leq f^{-1}(\overline{B})(xK) \vee f^{-1}(\overline{B})(yK)$.

ii $f^{-1}(\overline{B})(x^{-1}K) = \overline{B}(f(x^{-1})K)$

$= \overline{B}((f(x))^{-1}K)$

$= \overline{B}(f(x)K)$

$= f^{-1}(\overline{B})(xK)$

$f^{-1}(\overline{B})(x^{-1}K) = f^{-1}(\overline{B})(xK)$.

Hence, $f^{-1}(\overline{B})$ is a multi anti L-fuzzy quotient group of G/K.

Also, $\overline{f^{-1}(B)}(xK)$

$= \wedge f^{-1}(B)(xK)$, for all $k \in K$ and $x \in G$.

$= \wedge B(f(x)K)$

$= \overline{B}(f(x)K)$

$= f^{-1}(\overline{B})(xK)$.

Hence, $\overline{f^{-1}(B)}(xK) = f^{-1}(\overline{B})(xK)$.

3.7 Theorem:

Let G and G′ be any two groups. Let $f: G \to G'$ be an anti homomorphism and onto. Let $\overline{A} = (\tilde{A} = (\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_K))$ be a multi anti L-fuzzy quotient group of G/K. Then $f(\overline{A})$ is a multi anti L-fuzzy quotient group of G'/K, if $\overline{A}$ has sup property and $\overline{A}$ is f- invariant and $\overline{f(A)} = f(\overline{A})$.

Proof:

Let $\overline{A}$ be a multi anti L-fuzzy quotient group of G/K.

i $f(\overline{A})(f(x)f(y)K)$

$= (f(\overline{A}))(f(yx)K)$

$= \overline{A}(yxK)$

$\leq \overline{A}(yK) \vee \overline{A}(xK)$

$\leq \overline{A}(xK) \vee \overline{A}(yK)$

$= (f(\overline{A}))(f(x)K) \vee (f(\overline{A})(f(y)K)$

$f(\overline{A})(f(x)f(y)K)$

$\leq (f(\overline{A}))(f(x)K) \vee (f(\overline{A})(f(y)K)$.

ii $f(\overline{A})([f(x)]^{-1}K)$

$= f(\overline{A})[f(x^{-1})K]$

$= \overline{A}(x^{-1}K)$

$= \overline{A}(xK)$

$= f(\overline{A})[f(x)K]$

$f(\overline{A})([f(x)]^{-1}K) = f(\overline{A})[f(x)K]$.

Hence $f(\overline{A})$ is a multi anti L- fuzzy quotient group of G'/K.

Also,

$\overline{f(A)}(yK) = \wedge f(A)(yK)$, for all $k \in K$ and $y \in G'$.

$= \wedge f(A)(f(x)K)$, f is onto and $x \in G$.

$= \wedge A(xK)$

$= \overline{A}(xK)$

$= f(\overline{A})(f(x)K)$

$= f(\overline{A})(yK)$.

Hence, $\overline{f(A)}(yK) = f(\overline{A})(yK)$.

3.8 Theorem:

Let G and G′ be any two groups. Let $f: G \to G'$ be an anti homomorphism. Let $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, ..., \tilde{B}_K)$ be a multi anti L-fuzzy quotient group of G'/K Then $f^{-1}(\overline{B})$ is a multi anti L-fuzzy quotient group of G/K and $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$

Proof:

Let $\overline{B}$ be a multi anti L-fuzzy quotient group of G'/K.

i $f^{-1}(\overline{B})(xyK) = \overline{B}(f(xy)K)$

$= \overline{B}(f(y)f(x)K)$

$\leq \overline{B}(f(y)K) \vee \overline{B}(f(x)K)$

$\leq \overline{B}(f(x)K) \vee \overline{B}(f(y)K)$

$\leq f^{-1}(\overline{B})(xK) \vee f^{-1}(\overline{B})(yK)$

$f^{-1}(\overline{B})(xyK)$

$\leq f^{-1}(\overline{B})(xK) \vee f^{-1}(\overline{B})(yK)$.

ii $f^{-1}(\overline{B})(x^{-1}K) = \overline{B}(f(x^{-1})K)$

$= \overline{B}((f(x))^{-1}K)$

$= \overline{B}(f(x)K)$

$= f^{-1}(\overline{B})(xK)$

$f^{-1}(\overline{B})(x^{-1}K) = f^{-1}(\overline{B})(xK)$.

Hence, $f^{-1}(\overline{B})$ is a multi anti L-fuzzy quotient subgroup of G/K.

Also,

$\overline{f^{-1}(B)}(xK) = \wedge f^{-1}(B)(xK)$, for all $k \in K$ and $x \in G$.

$= \wedge B(f(x)K)$

$= \overline{B}(f(x)K)$

$= f^{-1}(\overline{B})(xK)$.

Hence, $\overline{f^{-1}(B)}(xK) = f^{-1}(\overline{B})(xK)$.

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