On the Monophonic Number of Line Graph

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Abstract.

For a connected graph G = (V, E) of order at least 2, a subset S of V is said to be a monophonic set of G if each vertex V of G lies on an x-y monophonic path for some elements x and y in S. The minimum cardinality of a monophonic set of G is the monophonic number of G. In this paper, we obtain the monophonic number of line graph.

Keywords: Monophonic set, monophonic number, monophonic distance, line graph.

1. Introduction

By a graph G = (V, E) we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For any graph G = (V, E), the *Line graph* L(G) whose vertices correspond to the edges of G and two vertices in L(G) are adjacent if and only if the corresponding edges in G are adjacent. The vertex set and edge set of L(G) is denoted by V' and E', where as the order and size of L(G) is denoted p' and q' respectively. A *chord* of a path P is an edge joining two non-adjacent vertices of P. A path P is called *monophonic* if it is a chordless path. A vertex v is an *extreme vertex* if the subgraph induced by its neighbours is complete. For any two vertices u and v in a connected graph G, the *monophonic distance* $d_m(u, v)$ from u to v is defined as the length of a longest u-v monophonic path in G.A set S of vertices of a graph G is *a monophonic set* of G if each vertex v of G lies on an x − y monophonic path in G for some x, y ∈ S. The minimum cardinality of a monophonic set of G is the *monophonic number* of G and is denoted by m(G). The *monophonic eccentricity* $e_m(v)$ of a vertex v in G is $e_m(v) = \max\{d_m(v, u) : u \in V(G)\}$. The *monophonic radius*, $rad_m G$ of G is $rad_m G = \min\{e_m(v) : v \in V(G)\}$ and the *monophonic diameter*, $diam_m G$ of G is $diam_m G = \max\{e_m(v) : v \in V(G)\}$. A vertex u in G is *monophonic eccentric vertex* of a vertex v in G if $e_m(u) = d_m(u, v)$. The *monophonic closure* $J_G[S]$ is the set formed by the union of all monophonic closed intervals $J_G[u, v]$ with u, v ∈ S. If e = {u, v} is an edge of a graph G with d(u) = 1 and d(v) > 1, then we call 'e' a *pendant edge*.

In this paper, we find the monophonic number line graph of a graph G. A set S' ⊆ V' is called a monophonic set of L(G) if $J_G[S'] = V'$ for some vertices in S' and the minimal cardinality of S' is the monophonic number of L(G) and is denoted by m(L(G)) = p'.

2. Preliminary Results

Theorem 2.1:*[2]* Each extreme vertex of a connected graph G belongs to every monophonic set of G. Moreover, if the set S of all extreme vertices of G is a monophonic set, then S is the unique minimum monophonic set of G.

Corollary 2.2*:[2]* If T is a tree with k end vertices, then m(T) = k.

Theorem 2.3*:[2]* For any connected graph G, $2 \leq m(G) \leq p$.

Corollary 2.4*:[2]* For the path P_p, $m(P_p) = 2$.

Corollary 2.5:*[2]* For the path C_p, $m(C_p) = 2$.

Theorem 2.6:*[3]* For integers m, n ≥ 2, $m(K_m \times K_n) = 2$.

3. Monophonic number of line graph

Theorem 3.1. Each vertices in L(G) corresponds to the pendant edges e_i $(1 \leq i \leq r)$ in G must belongs to S' of L(G).

Proof. Let $e = vv_i$ be the pendant edge in G and Let S and S' be the monophonic set of G and L(G) respectively. By the definition of pendant edge, consider $d(v_i) = 1$ and $d(v) \geq 2$, this shows that v_i is pendant vertex in G for all i, which is an extreme vertex of G. by Theorem 2.1 all v_i ∈ S. Since $d(v) \geq 2$, let $v_1, v_2, \ldots, v_i$ be the vertices adjacent with v then the edge $vv_1, vv_2, \ldots, vv_i$ has a common vertex v in G, then vertices in L(G) corresponds to the pendant edges forms a complete graph K_i in L(G). Then by Theorem 2.1 the result follows.

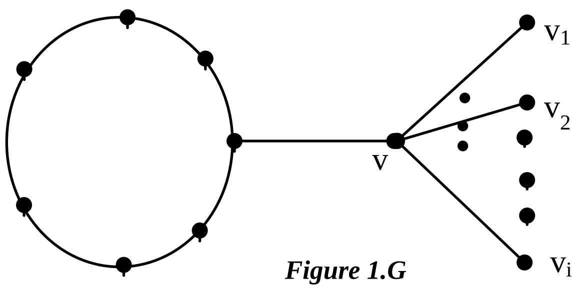

Figure 1.G

Example 3.2 For the graph G given in Figure 2, a monophonic set S = {v_1, v_5, v_6} and the line graph L(G) in Figure 3, S' = { E_5, E_6, E_4}

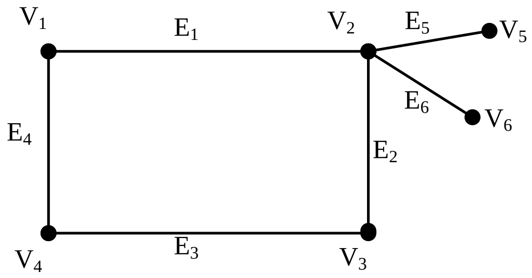

Figure 2. G

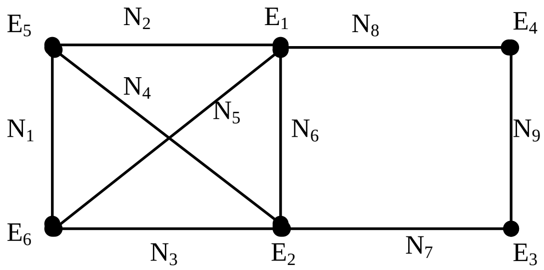

Figure 3. L(G)

Theorem 3.2*:* For any tree T with k pendant edges, m(L(T)) = k.

Proof*:* Let S be the set of all end vertices of T. By Corollary 2.2 |S| = m(T) = k. Each vertex in S has a pendant edge in T. By Theorem 3.1 the corresponding pendant edges forms a unique extreme vertex set S' in L(T), which is minimal, then by Theorem 2.1 |S'| = m(L(T)) = k.

Corollary 3.3*:* For the path $P_p(p \geq 2)$, $m(L(P_p)) = 2$.

Corollary 3.4: For the cycle $C_p(p \geq 2)$, $m(L(C_p)) = 2$.

Proof: Since the line graph of C_p is again C_p, then by Corollary 2.5 $m(L(C_p)) = 2$.

Theorem 3.5: Let G be a connected graph of order p and q, m(L(G)) = p' if and only if G is star graph.

Proof*:* Let G be a graph with m(L(G)) = p'. Since m(L(G)) = p' then each vertex in L(G) must be a extreme vertex. Then the edges in G corresponds to the vertices in V' are pendent edges. So that it must be star graph. Conversely let G be a star graph, then all the edges in G are pendant edge in G then by Theorem 3.1. that correspond edges forms a unique minimal monophonic set S' of L(G), so that m(L(G)) = p'

4. Bounds on monophonic number of a line graph

Theorem 4.1. For any connected graph G, $2 \leq m(L(G)) \leq p'$.

Remark 4.2. The bound in the above theorem are sharp. For the star graph $K_{1,p-1}(p \geq 2)$. The set of two end edges of path $P_p(p \geq 2)$ is its unique minimal monophonic set of $L(P_p)$, so that $m(L(P_p)) = 2$.

Theorem 4.3. For any integer k such that $2 \leq k \leq p'$ there is a connected graph of order p such that m(L(G)) = k.
Proof. For k = p-1, the theorem follows from theorem 3.5 by taking $G = K_{1,p-1}$. For $2 \leq m(L(G)) \leq p-2$, the theorem follows from theorem 3.2 by taking $G = T \neq K_{1,p-1}$.

Theorem 4.4. For any non-complete connected graph G of order p, $2 \leq m(L(G)) \leq m(G)$.
Proof. Since by Theorem 3.5 and by Theorem 2.3, $m(L(G)) \leq m(G)$. The other inequality is trivial.

Corollary 4.5. For the complete bipartite graph $K_{m,n}$, then $m(L(K_{m,n})) = 2$.
Proof. By result 7.1.8.in [4], $L(K_{m,n}) = K_m \times K_n$, then $m(L(K_{m,n})) = m(K_m \times K_n)$ and result follows by Theorem 2.6

Remark 4.6. The bounds in Theorem 4.4 are sharp. For the star graph $K_{1,p-1}$, $m(K_{1,p-1}) = m(L(K_{1,p-1}))$. For the non-trivial path P_p, $m(P_p) = m(L(P_p)) = 2$. For the complete bipartite graph $K_{m,n}$, $m(L(K_{m,n})) < m(K_{m,n})$.

Conclusion

In this paper we define the monophonic number of line graph of G. We also determine Lower and upper bound of the monophonic number line graph L(G). Also we establish a relation between monophonic number of graph and line graph

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