Subdivision of Super Geometric Mean Labeling for Quadrilateral Snake Graphs

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ABSTRACT

Let f:V(G)→{1,2,...,p+q} be an injective function.For a vertex labeling "f ", the induced edge labeling f* (e=uv) is defined by, f*(e)= $\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$.Then "f " is called a **"Super Geometric mean labeling"** if {f(V(G))}∪{f(e):e∈E(G)}= {1,2,...,p+q}. A graph which admits Super Geometric mean labeling is called **"Super Geometric mean graph"**

In this paper we prove that $S[A(Q_n)]$,$S[D(Q_n)]$, $S[A(D(Q_n))]$, Subdivision of triple Quadrilateral snake $S[T(Q_n)]$ and Subdivision of alternate triple Quadrilateral snake graph $S[A(T(Q_n))]$ are Super Geometric mean graphs.

Key words: Graph, Geometric mean graph, Super Geometric mean graph, Quadrilateral snake, Double Quadrilateral snake and Triple Qadrilateral snake

1. Introduction

Throughout this paper we consider only finite undirected and simple graphs. Let G be a graph with p vertices and q edges . There are several types of labeling and a detailed survey can be found in Gallian [1].For all terminologies and notations we follow Harary[2]

The concept of **"Geometric mean labeling"** has been introduced by S. Somasundaram, R .Ponraj and P. Vidhyarani in[6].

In this paper we investigate super Geometric mean labeling behavior of $S[A(Q_n)]$, $S[D(Q_n)]$, $S[A(D(Q_n))]$, Subdivision of triple Quadrilateral snake $S[(T(Q_n)]$ and Subdivision of alternate triple Quadrilateral snake $S[A(T(Q_n))]$.

We will provide a brief summary of definitions and other informations which are necessary for our present investigation.

Definition : 1.1

A graph G=(V,E) with p vertices and q edges is called a **"Geometric mean graph"** if it is possible to label the vertices $x\in$V with distinct labels f(x) from 1,2,…,q+1 in such a way that when each edge e=uv is labeled with, f(e=uv) = $\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$ then the edge labels are distinct. In this case, "f " is called a **"Geometric mean labeling"** of G.

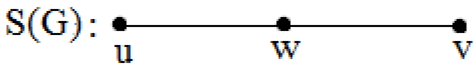
Definition : 1.2

Let f: V(G) → {1,2,…,p+q} be an injective function. For a vertex labeling " f ", the induced edge labeling f*(e=uv) is defined by, f*(e) = $\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$. Then " f " is called a **"Super Geometric mean labeling"** if {f(V(G))}∪{f(e):e∈E(G)={1,2,…,p+q}. A graph which admits Super Geometric mean labeling is called **"Super Geometric mean graph".**

Definition : 1.3

If e=uv is an edge of G and w is not a vertex of G, then e is said to be subdivided when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the **Subdivision** of G and it is denoted by S(G).

For example



Definition : 1.4

A **Quadrilateral snake** Q_n is obtained from a path $u_1u_2\ldots u_n$ by joining u_i and u_{i+1} with two new vertices v_i and w_i respectively and then joining v_i and w_i, $1\leq i\leq n-1$. That is every edge of a path is replaced by a cycle C_4.

Definition : 1.5

An **Alternate Quadrilateral snake** $A(Q_n)$ is obtained from a path $u_1u_2\ldots u_n$ by joining u_i and u_{i+1} alternatively with two new vertices v_i and w_i respectively and then joining v_i and w_i. That is every alternate edge of a path is replaced by a cycle C_4.

Definition : 1.6

A **Double Quadrilateral snake** $D(Q_n)$ consists of two Quadrilateral snakes that have a common path.

Definition : 1.7

An **Alternate Double Quadrilateral snake** $A[D(Q_n)]$ consists of two Alternate Quadrilateral snakes that have a common path.

Definition : 1.8

A **Triple Quadrilateral snake** $T(Q_n)$ consists of three Quadrilateral snakes that have a common path.

Definition : 1.9

An **Alternate Triple Quadrilateral snake** $A[T(Q_n)]$ consists of three Alternate Quadrilateral snakes that have a common path.

Theorem 1.10: Q_n, $A(Q_n)$, $D(Q_n)$ and $A[D(Q_n)]$ are mean graphs.

Theorem 1.11: Q_n, $A(Q_n)$, $D(Q_n)$ and $A[D(Q_n)]$ are Harmonic mean graphs.

Theorem 1.12: Q_n, $A(Q_n)$, $D(Q_n)$, $A[D(Q_n)]$, $T(Q_n)$ and $A[T(Q_n]$ are Geometric mean graphs.

Theorem 1.13: Q_n, $A(Q_n)$, $D(Q_n)$, $A[D(Q_n)]$, $T(Q_n)$ and $A[T(Q_n)]$ are Super Geometric mean graphs.

2. Main Results

Theorem : 2.1

Subdivision of Alternate Quadrilateral snake $S[A(Q_n)]$ is a Super Geometric mean graph.

Proof:

Let $A(Q_n)$ be an Alternate Quadrilateral snake which is obtained from a path $P_n = u_1u_2\ldots u_n$ by joining u_i and u_{i+1} alternatively with two new vertices v_i and w_i respectively and then joining v_i and w_i.

Let $S[A(Q_n)] = A(Q_N) = G$ be a graph obtained by subdividing all the edges of $A(Q_n)$. Here we consider the following cases.

Case 1: If Q_n starts from u_1,

Let t_i, $1\leq i\leq n-1$ be the vertices which subdivide the edges u_iu_{i+1}.

Let x_i, y_i and z_i be the vertices which subdivide the edges $u_{2i-1}v_i$, $u_{2i}w_i$ and v_iw_i respectively.

We have to consider two subcases.

Subcase (1) (a): If 'n' is odd, then.

Define a function $f: V(G) \to \{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 19i - 18$, $1\leq i\leq\left(\frac{n-1}{2}\right) + 1$

$f(u_{2i}) = 19i-3$, $1\leq i\leq\left(\frac{n-1}{2}\right)$

$f(t_{2i-1}) = 19i-5$, $1\leq i\leq\left(\frac{n-1}{2}\right)$

$f(t_{2i}) = 19i-1$, $1\leq i\leq\left(\frac{n-1}{2}\right)$

$f(x_1) = 4$

$f(x_i) = 19i\text{-}16,\ 2 \le i \le \left(\frac{n-1}{2}\right)$
$f(y_i) = 19i\text{-}7,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(v_1) = 6$
$f(v_i) = 19i\text{-}14,\ 2 \le i \le \left(\frac{n-1}{2}\right)$
$f(z_i) = 19i\text{-}11,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(w_i) = 19i\text{-}9,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

The labeling pattern of $S[A(Q_7)]$ is shown in the following figure.

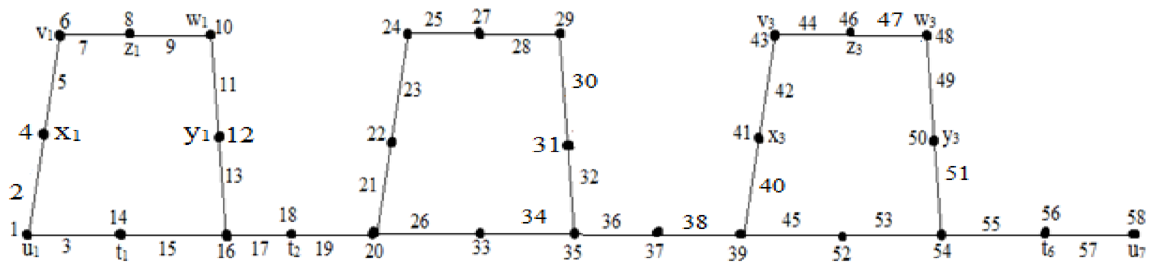


Figure: 1

From the above labeling pattern, we get $\{f(V(G))\} \cup \{f(e):e \in E(G)\} = \{1,2,\ldots,p+q\}$

$\therefore$ In this case, " f " provides a Super Geometric mean labeling of $A(Q_N)$.

Subcase (1) (b): If 'n' is even, then

Define a function $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 19i\text{-}18,\ 1 \le i \le \left(\frac{n}{2}\right)$
$f(u_{2i}) = 19i\text{-}3\ 1 \le i \le \left(\frac{n}{2}\right)$
$f(t_{2i-1}) = 19i\text{-}5,\ 1 \le i \le \left(\frac{n}{2}\right)$
$f(t_{2i}) = 19i\text{-}1,\ 1 \le i \le \left(\frac{n-2}{2}\right)$
$f(x_1) = 4$
$f(x_i) = 19i\text{-}16,\ 2 \le i \le \left(\frac{n}{2}\right)$
$f(y_i) = 19i\text{-}7,\ 1 \le i \le \left(\frac{n}{2}\right)$
$f(v_1) = 6$
$f(v_i) = 19i\text{-}14,\ 2 \le i \le \left(\frac{n}{2}\right)$
$f(z_i) = 19i\text{-}11,\ 1 \le i \le \left(\frac{n}{2}\right)$
$f(w_i) = 19i\text{-}9,\ 1 \le i \le \left(\frac{n}{2}\right)$

The labeling pattern of $S[A(Q_8)]$ is given below.

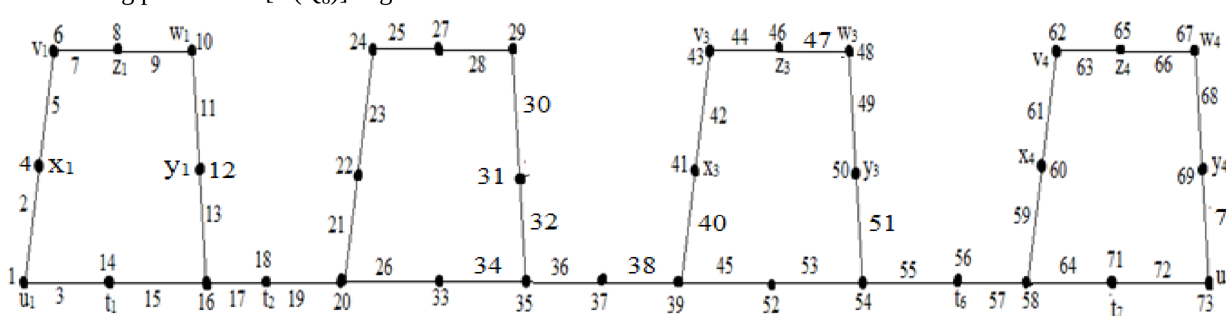


Figure: 2

From the above labeling pattern we get, $\{f(V(G))\} \cup (f(e):e \in E(G)\} = \{1,2,\ldots,p+q\}$

In this case, $A(Q_N)$ is a Super Geometric mean graph.

Case 2: If Q_n starts from u_2,

Let t_i, $1\le i\le n-1$ be the vertices which subdivide the edges u_iu_{i+1}.

Let x_i, y_i and z_i be the vertices which subdivide the edges $u_{2i}v_i$, $u_{2i+1}w_i$ and v_iw_i respectively.

Here we have to consider two subcases.

Subcase (2) (a) If 'n' is odd, then

Define a function f: V(G) $\rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 19i-18,\ 1\le i\le\left(\frac{n-1}{2}\right)+1$

$f(u_{2i}) = 19i-14,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i-1}) = 19i-16,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i}) = 19i-1,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(x_i) = 19i-12,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(y_i) = 19i-3,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(v_i) = 19i-10,\ 1\le i\le\left(\frac{n-1}{2}\right)$

` $f(z_i) = 19i-7,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(w_i) = 19i-5,\ 1\le i\le\left(\frac{n-1}{2}\right)$

The labeling pattern of $S[A(Q_7)]$ is displayed below.

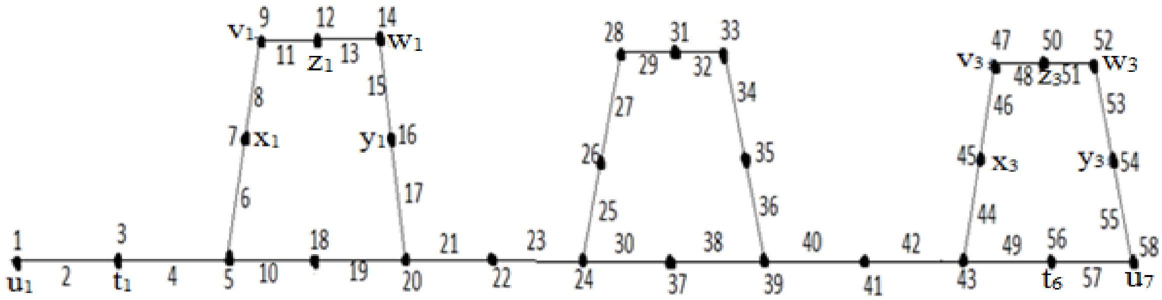


Figure: 3

From the above labeling pattern, we get $\{f(V(G))\}\cup\{f(e):e\in E(G)\}=\{1,2,\ldots,p+q\}$.

Hence $A(Q_N)$ admits a Super Geometric mean labeling.

Subcase (2) (b) If 'n' is even, then

Define a function f: V(G) $\rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 19i-18,\ 1\le i\le\left(\frac{n}{2}\right)$

$f(u_{2i}) = 19i-14,\ 1\le i\le\left(\frac{n}{2}\right)$

$f(t_{2i-1}) = 19i-16,\ 1\le i\le\left(\frac{n}{2}\right)$

$f(t_{2i}) = 19i-1,\ 1\le i\le\left(\frac{n-2}{2}\right)$

$f(x_i) = 19i-12,\ 1\le i\le\left(\frac{n-2}{2}\right)$

$f(y_i) = 19i-3,\ 1\le i\le\left(\frac{n-2}{2}\right)$

$f(v_i) = 19i-10,\ 1\le i\le\left(\frac{n-2}{2}\right)$

` $f(z_i) = 19i-7,\ 1\le i\le\left(\frac{n-2}{2}\right)$

$f(w_i) = 19i-5,\ 1\le i\le\left(\frac{n-2}{2}\right)$

The labeling pattern of $S[A(Q_8)]$ is shown below.

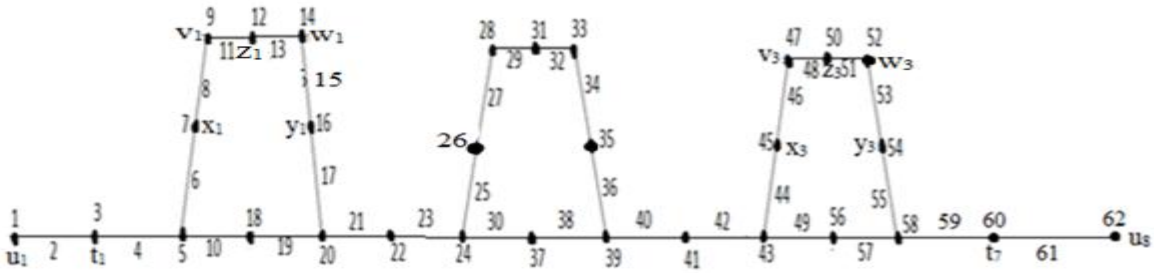


Figure: 4

From the above labeling pattern, both vertices and edges together get distinct labels from {1,2,...,p+q}.

From all the above cases, we conclude that Subdivision of Alternate Quadrilateral snake is a Super Geometric mean graph.

Theorem : 2.2

Subdivision of Double Quadrilateral snake $S[D(Q_n)]$ is a Super Geometric mean graph.

Proof:

Let $D(Q_n)$ be a Double Quadrilateral snake which is obtained from a path $P_n = u_1u_2 \dots u_n$ by joining u_i and u_{i+1} with four new vertices v_i , w_i, x_i and y_i, $1 \le i \le n-1$ by the edges u_iv_i, $u_{i+1}w_i$, v_iw_i, u_ix_i, $u_{i+1}y_i$ and x_iy_i.

Let $S[D(Q_n)] = D(Q_N) = G$ be a graph obtained by subdividing all the edges of $D(Q_n)$.

Let t_i, r_i, s_i, z_i, m_i, n_i and q_i be the vertices which subdivide the edges u_iu_{i+1}, u_iv_i, $u_{i+1}w_i$, v_iw_i, u_ix_i, $u_{i+1}y_i$ and x_iy_i respectively.

Define a function $f: V(G) \rightarrow \{1,2,\dots,p+q\}$ by,

$f(u_1) = 11$
$f(u_i) = 26i-25,\ 2 \le i \le n$
$f(t_i) = 26i-9,\ 1 \le i \le n-1$
$f(m_1) = 8$
$f(m_i) = 26i-22,\ 2 \le i \le n-1$
$f(n_1) = 9$
$f(n_i) = 26i-1,\ 2 \le i \le n-1$
$f(x_1) = 6$
$f(x_i) = 26i-18,\ 2 \le i \le n-1$
$f(y_1) = 1$
$f(y_i) = 26i-6,\ 2 \le i \le n-1$
$f(q_1) = 4$
$f(q_i) = 26i-13,\ 2 \le i \le n-1$
$f(r_1) = 14$
$f(r_i) = 26i-21,\ 2 \le i \le n-1$
$f(s_1) = 25$
$f(s_i) = 26i-4,\ 2 \le i \le n-1$
$f(v_1) = 18$
$f(v_i) = 26i-16,\ 2 \le i \le n-1$
$f(w_1) = 23$
$f(w_i) = 26i-8,\ 2 \le i \le n-1$
$f(z_1) = 20$
$f(z_i) = 26i-12,\ 2 \le i \le n-1$

From the above labeling pattern, we get $\{f(V(G))\} \cup \{f(e): e \in E(G)\} = \{1,2,\dots,p+q\}$.

Hence $D(Q_N)$ is a Super Geometric mean graph.

Example 2.3: A Super Geometric mean labeling of $S[D(Q_5)]$ is displayed below.

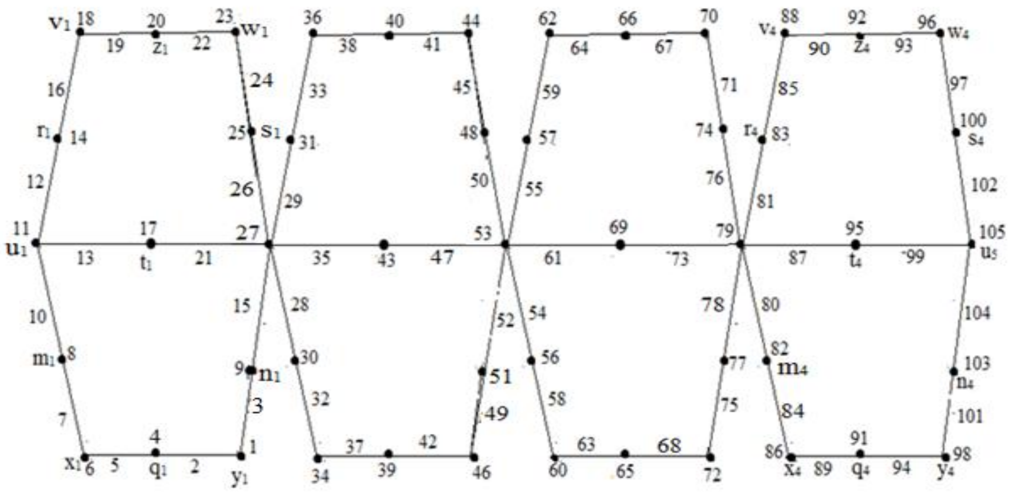


Figure: 5

Theorem : 2.4

Subdivision of Alternate Double Quadrilateral snake $S[A(D(Q_n))]$ is a Super Geometric mean graph.

Proof:

Let $A[D(Q_n)]$ be an Alternate Double Quadrilateral snake which is obtained from a path $P_n=u_1u_2\ldots u_n$ by joining u_i and u_{i+1} alternatively with four new vertices v_i, w_i, x_i and y_i.

Let $S[A(D(Q_n))] = A[D(Q_N)] = G$ be a graph obtained by subdividing all the edges of $A[D(Q_n)]$.

Here we consider two cases.

Case 1: If $D(Q_n)$ starts from u_1,

Let t_i, r_i, s_i, z_i, m_i, n_i and q_i be the vertices which subdivide the edges u_iu_{i+1}, $u_{2i-1}v_i$, $u_{2i}w_i$, v_iw_i, $u_{2i-1}x_i$, $u_{2i}y_i$ and x_iy_i respectively.

We have to consider two subcases.

Subcase (1) (a): If 'n' is odd, then

Define a function $f: V(G)\rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u_1) = 11$

$f(u_{2i-1}) = 30i-29,\ 2\le i\le\left(\frac{n-1}{2}\right)+1$

$f(u_{2i}) = 30i-3,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i-1}) = 30i-13,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i}) = 30i-1,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(m_1) = 8$

$f(m_i) = 30i-26,\ ,\ 2\le i\le\left(\frac{n-1}{2}\right)$

$f(n_1) = 9$

$f(n_i) = 30i-5,\ 2\le i\le\left(\frac{n-1}{2}\right)$

$f(x_1) = 6$

$f(x_i) = 30i-22,\ 2\le i\le\left(\frac{n-1}{2}\right)$

$f(q_1) = 4$

$f(q_i) = 30i-17,\ 2\le i\le\left(\frac{n-1}{2}\right)$

$f(y_1) = 1$

$f(y_i) = 30i-10,\ 2\le i\le\left(\frac{n-1}{2}\right)$

$f(r_1) = 14$

$f(r_i) = 30i-25,\ 2\le i\le\left(\frac{n-1}{2}\right)$

$f(s_1) = 25$

$f(s_i) = 30i-8,\ 2 \leq i \leq \left(\frac{n-1}{2}\right)$

$f(v_1) = 18$

$f(v_i) = 30i-20,\ 2 \leq i \leq \left(\frac{n-1}{2}\right)$

$f(z_1) = 20$

$f(z_i) = 30i-16,\ 2 \leq i \leq \left(\frac{n-1}{2}\right)$

$f(w_1) = 23$

$f(w_i) = 30i-12,\ 2 \leq i \leq \left(\frac{n-1}{2}\right)$

The labeling pattern of $S[A(D(Q_5))]$ is given below.

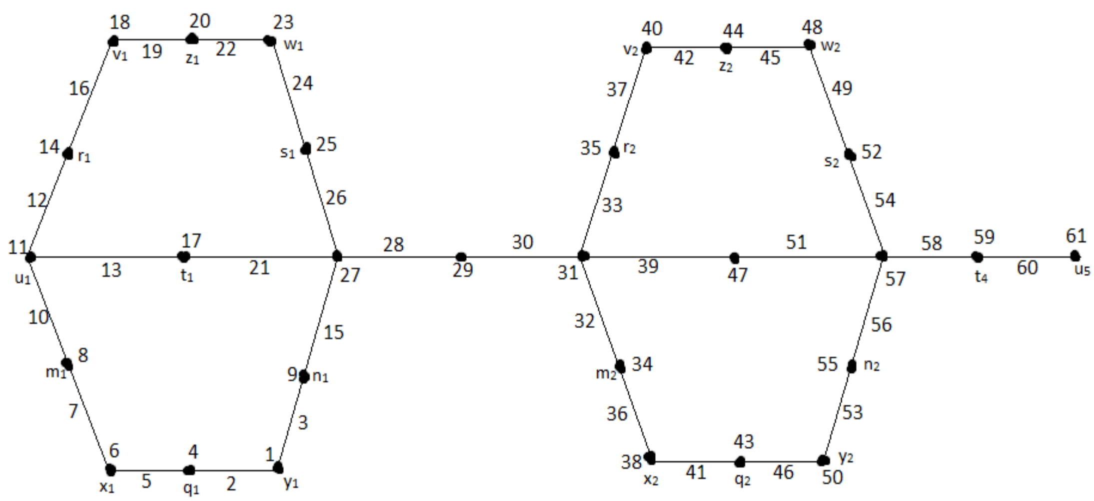


Figure: 6

From the above labeling pattern we get, $\{f(V(G))\} \cup \{f(e):e \in E(G)\} = \{1,2,\ldots,p+q\}$

In this case, " f " provides a Super Geometric mean labeling of G.

Subcase (1) (b): If 'n' is even, then

Define a function $f:V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_1) = 11$

$f(u_{2i-1}) = 30i-29,\ 2 \leq i \leq \left(\frac{n}{2}\right)$

$f(u_{2i}) = 30i-3,\ 1 \leq i \leq \left(\frac{n}{2}\right)$

$f(t_{2i-1}) = 30i-13,\ 1 \leq i \leq \left(\frac{n}{2}\right)$

$f(t_{2i}) = 30i-1,\ 1 \leq i \leq \left(\frac{n-2}{2}\right)$

$f(m_1) = 8$

$f(m_i) = 30i-26,\ 2 \leq i \leq \left(\frac{n}{2}\right)$

$f(n_1) = 9$

$f(n_i) = 30i-5,\ 2 \leq i \leq \left(\frac{n}{2}\right)$

$f(x_1) = 6$

$f(x_i) = 30i-22,\ 2 \leq i \leq \left(\frac{n}{2}\right)$

$f(q_1) = 4$

$f(q_i) = 30i-17,\ 2 \leq i \leq \left(\frac{n}{2}\right)$

$f(y_1) = 1$

$f(y_i) = 30i-10,\ 2 \leq i \leq \left(\frac{n}{2}\right)$

$f(r_1) = 14$
$f(r_i) = 30i\text{-}25,\ 2 \le i \le \left(\frac{n}{2}\right)$
$f(s_1) = 25$
$f(s_i) = 30i\text{-}8,\ 2 \le i \le \left(\frac{n}{2}\right)$
$f(v_1) = 18$
$f(v_i) = 30i\text{-}20,\ 2 \le i \le \left(\frac{n}{2}\right)$
$f(z_1) = 20$
$f(z_i) = 30i\text{-}16,\ 2 \le i \le \left(\frac{n}{2}\right)$
$f(w_1) = 23$
$f(w_i) = 30i\text{-}12,\ 2 \le i \le \left(\frac{n}{2}\right)$

The labeling pattern of $S[A(D(Q_6))]$ is shown below.

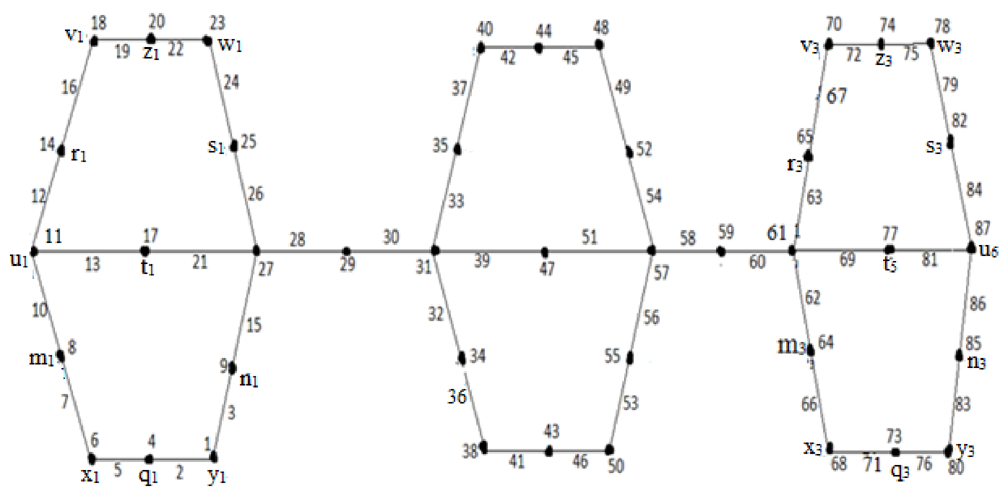


Figure: 7

From the above labeling pattern, we get $\{f(V(G))\} \cup \{f(e): e \in E(G)\} = \{1,2,\ldots,p+q\}$

Hence $A[D(Q_N)]$ admits Super Geometric mean labeling.

Case 2: If $D(Q_n)$ starts from u_2.

Let t_i, r_i, s_i, z_i, m_i, n_i and q_i be the vertices which subdivide the edges $u_i u_{i+1}$, $u_{2i}v_i$, $u_{2i+1} w_i$, v_iw_i, $u_{2i}x_i$, $u_{2i+1}y_i$ and x_iy_i respectively.

We have to consider two subcases.

Subcase (2) (a): If 'n' is odd, then

Define a function $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 30i\text{-}29,\ 1 \le i \le \left(\frac{n-1}{2}\right) + 1$
$f(u_{2i}) = 30i\text{-}25,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(t_{2i-1}) = 30i\text{-}27,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(t_{2i}) = 30i\text{-}9,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(m_i) = 30i\text{-}22,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(n_i) = 30i\text{-}1,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(x_1) = 13$
$f(x_i) = 30i\text{-}18,\ 2 \le i \le \left(\frac{n-1}{2}\right)$
$f(q_i) = 30i\text{-}13,\ 1 \le i \le \left(\frac{n-1}{2}\right)$
$f(y_i) = 30i\text{-}6,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

$f(r_i) = 30i\text{-}21,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

$f(s_i) = 30i\text{-}4,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

$f(v_i) = 30i\text{-}16,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

$f(z_i) = 30i\text{-}12,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

$f(w_i) = 30i\text{-}8,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

The labeling pattern of $S[A(D(Q_7))]$ is displayed below.

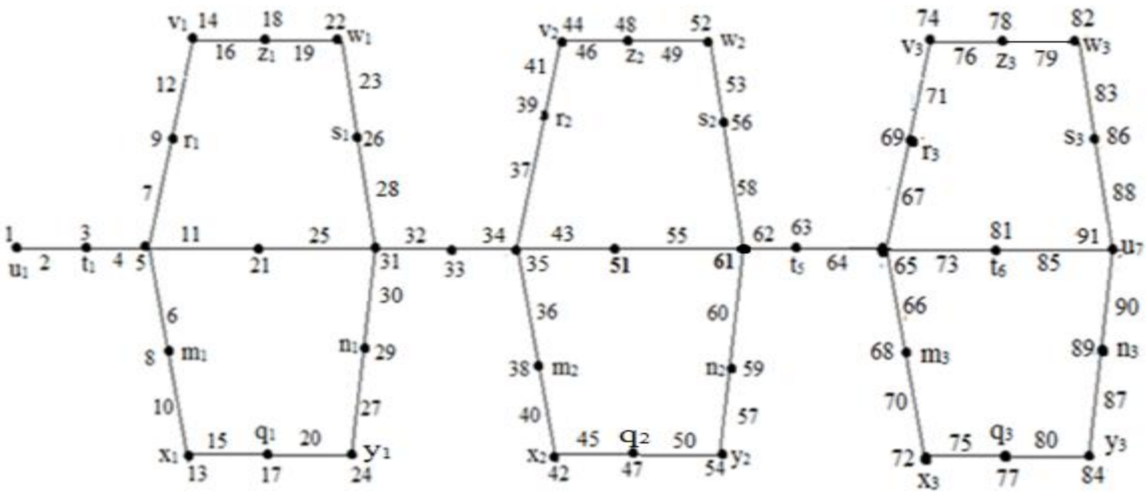


Figure: 8

From the above labeling pattern, both vertices and edges together get distinct labels from {1,2,…,p+q}

Hence $A[D(Q_N)]$ is a Super Geometric mean graph.

Subcase (2) (b): If 'n' is even, then

Define a function f: $V(G) \to \{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 30i\text{-}29,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(u_{2i}) = 30i\text{-}25,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(t_{2i-1}) = 30i\text{-}27,\ 1 \le i \le \left(\frac{n}{2}\right)$

$f(t_{2i}) = 30i\text{-}9,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(m_i) = 30i\text{-}22,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(n_i) = 30i\text{-}1,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(x_1) = 13$

$f(x_i) = 30i\text{-}18,\ 2 \le i \le \left(\frac{n-2}{2}\right)$

$f(q_i) = 30i\text{-}13,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(y_i) = 30i\text{-}6,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(r_i) = 30i\text{-}21,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(s_i) = 30i\text{-}4,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(v_i) = 30i\text{-}16,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(z_i) = 30i\text{-}12,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

$f(w_i) = 30i\text{-}8,\ 1 \le i \le \left(\frac{n-2}{2}\right)$

The labeling pattern of $S[A(D(Q_6))]$ is given below.

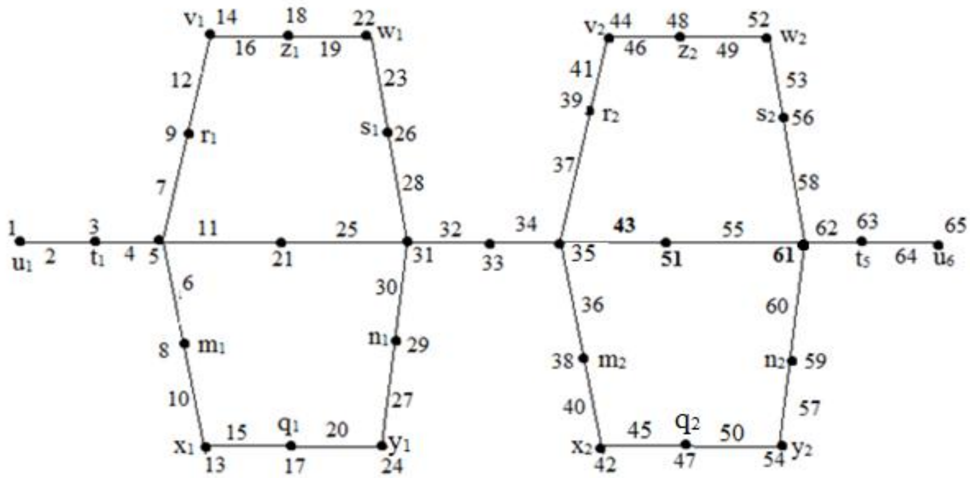


Figure: 9

From the above labeling pattern, we get $\{f(V(G))\}\cup\{f(e):e\in E(G)\}=\{1,2,\ldots,p+q\}$

This makes " f " a Super Geometric mean labeling of $A[D(Q_N)]$.

From all the above cases, we conclude that Subdivision of Alternate Double Quadrilateral snake is a Super Geometric mean graph.

Theorem : 2.5

Subdivision of Triple Quadrilateral snake $S[T(Q_n)]$ is a Super Geometric mean graph.

Proof:

Let $T(Q_n)$ be a Triple Quadrilateral snake which is obtained from a path $P_n=u_1u_2\ldots u_n$ by joining u_i and u_{i+1} with six new vertices v_i, w_i, x_i, y_i, a_i and b_i, $1\le i\le n-1$ by the edges u_iv_i, v_iw_i, w_iu_{i+1}, u_ia_i, a_ib_i, b_iu_{i+1}, u_iu_{i+1}, u_ix_i, x_iy_i and y_iu_{i+1}.

Let $S[T(Q_n)] = T(Q_N) = G$ be the graph obtained by subdividing all the edges of $T(Q_n)$.

Let t_i, m_i, n_i, q_i, r_i, s_i, z_i, l_i, g_i and k_i be the vertices which subdivide the edges u_iu_{i+1}, u_ix_i, $u_{i+1}y_i$, x_iy_i, u_iv_i, $u_{i+1}w_i$, v_iw_i, u_ia_i, u_{i+1} b_i and a_ib_i respectively.

Define a function $f: V(G) \rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u_1) = 8$

$f(u_i) = 37i-36,\ 2\le i\le n$

$f(t_i) = 37i-9,\ 1\le i\le n-1$

$f(m_i) = 37i-20,\ 1\le i\le n-1$

$f(n_i) = 37i-1,\ 1\le i\le n-1$

$f(x_1) = 20$

$f(x_i) = 37i-18,\ 2\le i\le n-1$

$f(q_1) = 26$

$f(q_i) = 37i-14,\ 2\le i\le n-1$

$f(y_1) = 31$

$f(y_i) = 37i-8,\ 2\le i\le n-1$

$f(r_1) = 12$

$f(r_i) = 37i-26,\ 2\le i\le n-1$

$f(s_i) =37i-3,\ 1\le i\le n-1$

$f(v_1) = 19$

$f(v_i)=37i-16,\ 2\le i\le n-1$

$f(z_i) = 37i-13,\ 1\le i\le n-1$

$f(w_1) =27$

$f(w_i) = 37i-7,\ 2\le i\le n-1$

$f(l_1) = 4$

$f(l_i) = 37i-34,\ 2\le i\le n-1$
$f(g_1) = 15$
$f(g_i) = 37i-21,\ 2\le i\le n-1$
$f(a_1) = 1$
$f(a_i) = 37i-31,\ 2\le i\le n-1$
$f(k_1) = 5$
$f(k_i) = 37i-28,\ 2\le i\le n-1$
$f(b_1) = 10$
$f(b_i) = 37i-25,\ 2\le i\le n-1$

From the above labeling pattern, $\{f(V(G))\}\cup\{f(e):e\in E(G)\}=\{1,2,\ldots,p+q\}$.
Hence Subdivision of Triple Quadrilateral snake is a Super Geometric mean graph.

Example 2.6: A Super Geometric mean labeling of $S[T(Q_4)]$ is shown below.

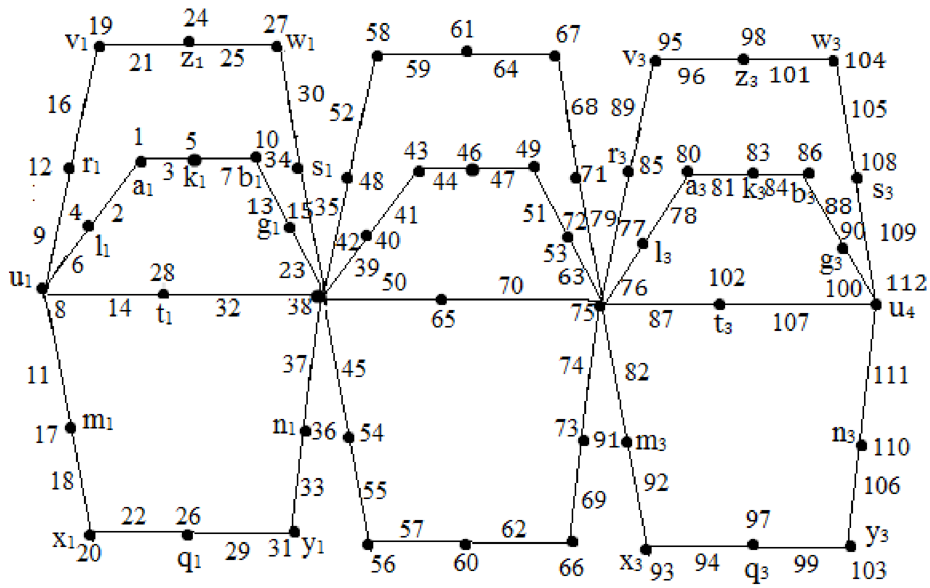


Figure: 10

Theorem : 2.7

Subdivision of Alternate Triple Quadrilateral snake $S[A(T(Q_n))]$ is a Super Geometric mean graph.

Proof:

Let $A[T(Q_n)]$ be an Alternate Triple Quadrilateral snake which is obtained from a path $P_n=u_1u_2\ldots u_n$ by joining u_i and u_{i+1} alternatively with six new vertices v_i, w_i, a_i, b_i, x_i and y_i.

Let $S[A(T(Q_n))] = A[T(Q_N)]=G$ be the graph obtained by subdividing all the edges of $A[T(Q_n)]$.

Here we consider two cases.

Case 1: If $T(Q_n)$ starts from u_1,

Let t_i, m_i, n_i, q_i, r_i, s_i, z_i, l_i. g_i and k_i be the vertices which subdivide the edges u_iu_{i+1}, $u_{2i-1}\,x_i$, $u_{2i}\,y_i$, x_iy_i, $u_{2i-1}v_i$, $u_{2i}w_i$, v_iw_i, $u_{2i-1}a_i$, $u_{2i}b_i$ and a_ib_i respectively.

We have to consider two subcases.

Subcase (1) (a): If 'n' is odd, then

Define a function $f: V(G) \rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u_1) = 8$
$f(u_{2i-1}) = 41i-40,\ 2\le i\le\left(\frac{n-1}{2}\right)+1$
$f(u_{2i}) = 41i-3,\ 1\le i\le\left(\frac{n-1}{2}\right)$
$f(t_{2i-1}) = 41i-13,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i}) = 41i-1,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

$f(m_i) = 41i-24,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

$f(n_i) = 41i-5,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

$f(x_1) = 20$

$f(x_i) = 41i-22,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

$f(q_1) = 26$

$f(q_i) = 41i-18,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

$f(y_1) = 31$

$f(y_i) = 41i-12,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

$f(r_1) = 12$

$f(r_i) = 41i-30,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

$f(s_i) = 41i-7,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

$f(v_1) = 19$

$f(v_i) = 41i-20,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

$f(z_i) = 41i-17,\ 1 \le i \le \left(\frac{n-1}{2}\right)$

$f(w_1) = 27$

$f(w_i) = 41i-11,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

$f(l_1) = 14$

$f(l_i) = 41i-38,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

$f(g_1) = 15$

$f(g_i) = 41i-25,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

$f(a_1) = 1$

$f(a_i) = 41i-35,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

$f(k_1) = 5$

$f(k_i) = 41i-32,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

$f(b_1) = 10$

$f(b_i) = 41i-29,\ 2 \le i \le \left(\frac{n-1}{2}\right)$

The labeling pattern of $S[A(T(Q_5))]$ is displayed below.

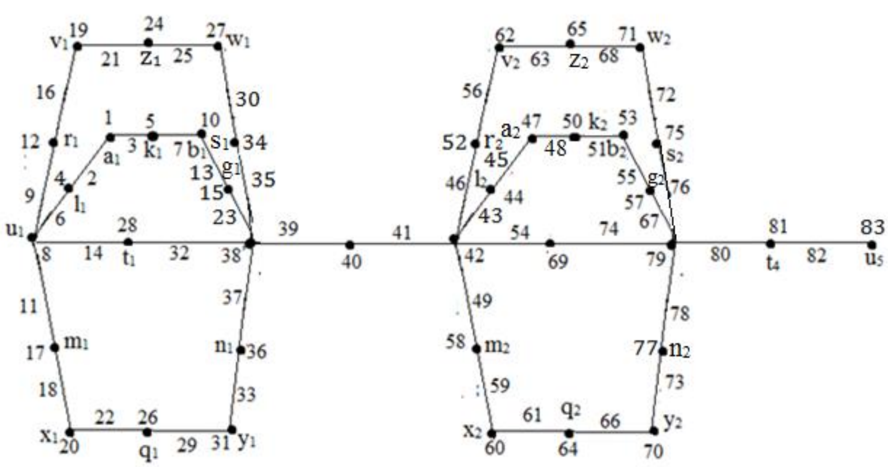


Figure: 11

From the above labeling pattern, we get $\{f(V(G))\} \cup \{f(e): e \in E(G)\} = \{1,2,\ldots,p+q\}$

Hence " f " provides a Super Geometric mean labeling of G.

Subcase (1) (b): If 'n' is even, then

Define a function f: V(G) $\rightarrow$ {1,2,…,p+q} by,

$f(u_1) = 8$

$f(u_{2i-1}) = 41i-40,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(u_{2i}) = 41i-3,\ 1\leq i\leq\left(\frac{n}{2}\right)$

$f(t_{2i-1}) = 41i-13,\ 1\leq i\leq\left(\frac{n}{2}\right)$

$f(t_{2i}) = 41i-1,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(m_i) = 41i-24,\ 1\leq i\leq\left(\frac{n}{2}\right)$

$f(n_i) = 41i-5,\ 1\leq i\leq\left(\frac{n}{2}\right)$

$f(x_1) = 20$

$f(x_i) = 41i-22,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(q_1) = 26$

$f(q_i) = 41i-18,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(y_1) = 31$

$f(y_i) = 41i-12,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(r_1) = 12$

$f(r_i) = 41i-30,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(s_i) = 41i-7,\ 1\leq i\leq\left(\frac{n}{2}\right)$

$f(v_1) = 19$

$f(v_i) = 41i-20,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(z_i) = 41i-17,\ 1\leq i\leq\left(\frac{n}{2}\right)$

$f(w_1) = 27$

$f(w_i) = 41i-11,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(l_1) = 4$

$f(l_i) = 41i-38,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(g_1) = 15$

$f(g_i) = 41i-25,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(a_1) = 1$

$f(a_i) = 41i-35,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(k_1) = 5$

$f(k_i) = 41i-32,\ 2\leq i\leq\left(\frac{n}{2}\right)$

$f(b_1) = 10$

$f(b_i) = 41i-29,\ 2\leq i\leq\left(\frac{n}{2}\right)$

The labeling pattern of $S[A(T(Q_6))]$ is given below.

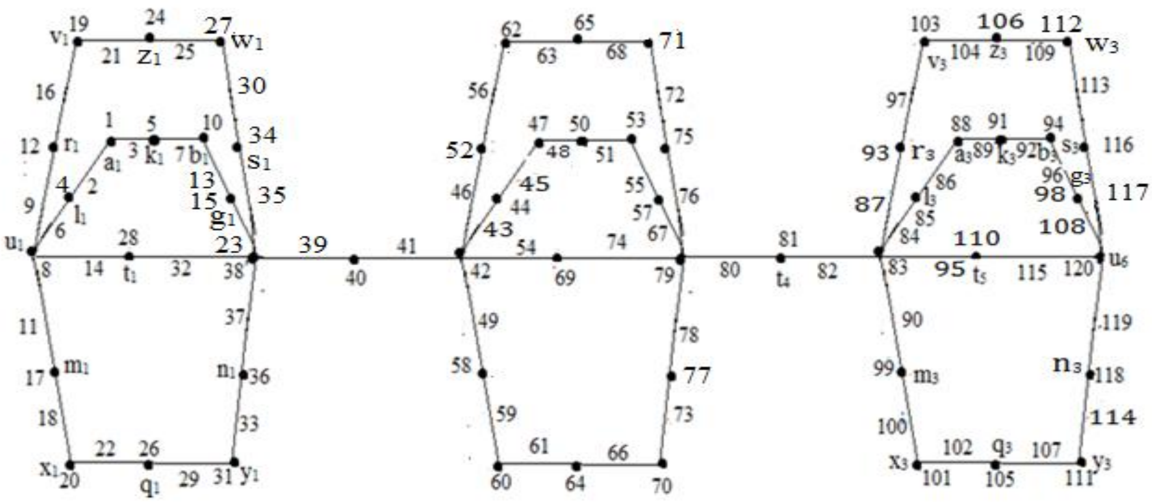


Figure: 12

From the above labeling pattern, we get $\{f(V(G))\}\cup\{f(e):e\in E(G)] = \{1,2,\ldots,p+q\}$

Hence G admits Super Geometric mean labeling.

Case 2: If $T(Q_n)$ starts from u_2,

Let t_i, m_i, n_i, q_i, r_i, s_i, z_i, l_i, g_i and k_i be the vertices which subdivide the edges u_iu_{i+1}, $u_{2i}x_i$, $u_{2i+1}\,y_i$, x_iy_i, $u_{2i}v_i$, $u_{2i+1}w_i$, v_iw_i, $u_{2i}a_i$, $u_{2i+1}\,b_i$ and a_ib_i respectively.

We have to consider two subcases.

Subcase (2) (a): If 'n' is odd, then

Define a function $f: V(G)\rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 41i-40,\ 1\le i\le\left(\frac{n-1}{2}\right)+1$

$f(u_{2i}) = 41i-36,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i-1}) = 41i-38,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(t_{2i}) = 41i-9,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(m_i) = 41i-20,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(n_i) = 41i-1,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(x_i) = 41i-18,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(q_i) = 41i-14,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(y_i) = 41i-8,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(r_i) = 41i-26,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(s_i) = 41i-3,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(v_i) = 41i-16,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(z_i) = 41i-13,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(w_i) = 41i-7,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(l_i) = 41i-34,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(g_i) = 41i-21,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(a_i) = 41i-31,\ 1\le i\le\left(\frac{n-1}{2}\right)$

$f(k_1) = 14$

$f(k_i) = 41i\text{-}28,\ 2\leq i\leq\left(\frac{n-1}{2}\right)$

$f(b_1) = 17$

$f(b_i) = 41i\text{-}25,\ 2\leq i\leq\left(\frac{n-1}{2}\right)$

The labeling pattern of $S[A(T(Q_5))$ is shown below.

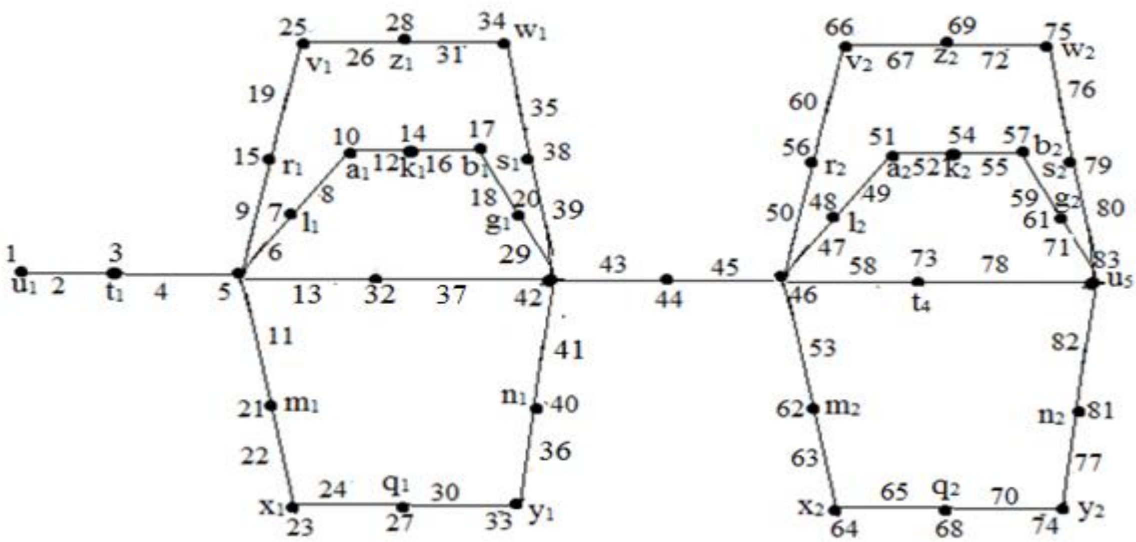


Figure : 13

From the above labeling pattern, both vertices and edges together get distinct labels from {1,2,…,p+q}. This makes “ f ” a Super Geometric mean labeling of G.

Subcase (2) (a): If ‘n’ is even, then

Define a function $f: V(G)\rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 41i\text{-}40,\ 1\leq i\leq\left(\frac{n}{2}\right)$

$f(u_{2i}) = 41i\text{-}36,\ 1\leq i\leq\left(\frac{n}{2}\right)$

$f(t_{2i-1}) = 41i\text{-}38,\ 1\leq i\leq\left(\frac{n}{2}\right)$

$f(t_{2i}) = 41i\text{-}9,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(m_i) = 41i\text{-}20,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(n_i) = 41i\text{-}1,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(x_i) = 41i\text{-}18,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(q_i) = 41i\text{-}14,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(y_i) = 41i\text{-}8,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(r_i) = 41i\text{-}26,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(s_i) = 41i\text{-}3,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(v_i) = 41i\text{-}16,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(z_i) = 41i\text{-}13,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(w_i) = 41i\text{-}7,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(l_i) = 41i\text{-}34,\ 1\leq i\leq\left(\frac{n-2}{2}\right)$

$f(g_i) = 41i\text{-}21,\ 1\le i\le\left(\frac{n-2}{2}\right)$
$f(a_i) = 41i\text{-}31,\ 1\le i\le\left(\frac{n-2}{2}\right)$
$f(k_1) = 14$
$f(k_i) = 41i\text{-}28,\ 2\le i\le\left(\frac{n-2}{2}\right)$
$f(b_1) = 17$
$f(b_i) = 41i\text{-}25,\ 2\le i\le\left(\frac{n-2}{2}\right)$
The labeling pattern of $S[A(T(Q_8)]$ is shown below.

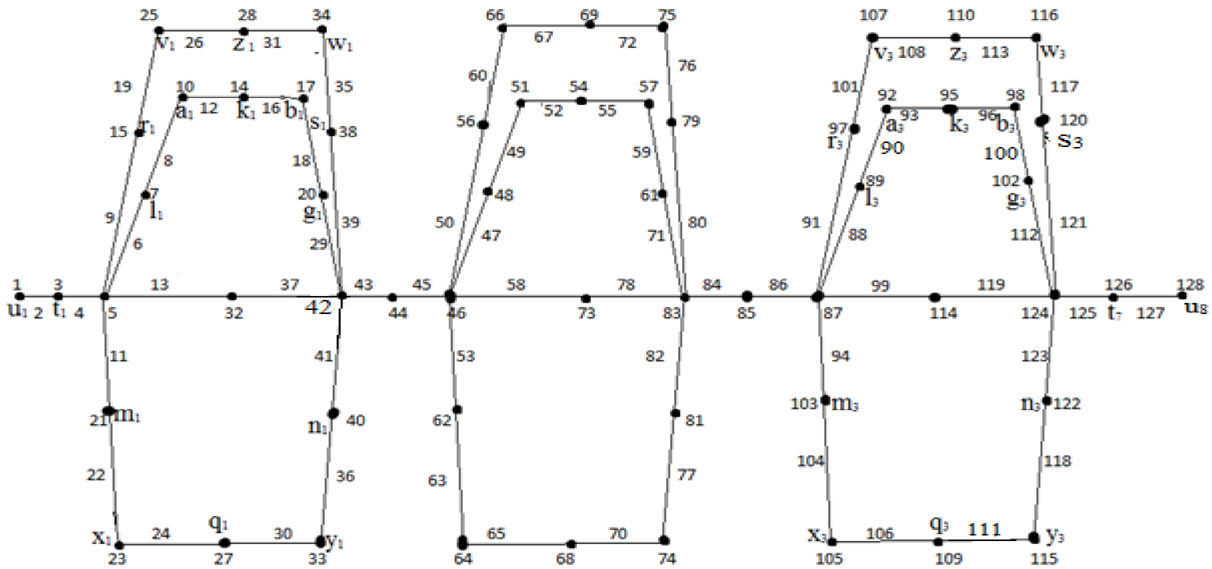


Figure: 14

From the above labeling pattern we get, $\{f(V(G))\}\cup\{f(e):e\in E(G)\} = \{1,2,\ldots,p+q\}$
Hence G admits a Super Geometric mean labeling.
From all the above cases, we conclude that Subdivision of Alternate Triple Quadrilateral snake is a Super Geometric mean graph.

References:-

[1] Gallian.J.A. **"A Dynamic survey of graph labeling".** The electronic Journal of Combinatorics 2011, 18 #DS6.

[2] Harary.F., 1988, **"Graph theory"** Narosa publishing House, New Delhi.

[3] C. Jayasekaran, S.S.Sandhya and C. David Raj, "**Some Results on Super Harmonic mean graphs",** International Journal of Mathematics Trends and Technology, Vol.6(3)(2014), 215-224.

[4] Somasundaram.S and Ponraj.R, 2003 **"Mean labeling of graphs"** National Academy of Science letters Vol.26, p.210-213.

[5] Somasundaram.S, Ponraj.R and S.S.Sandhya "**Harmonic mean labeling of graphs",** Communicated to Journal of Combinatorial Mathematics and Combinatorial Computing.

[6] S.Somasundaram, R. Ponraj and P.Vidhyarani "**Geometric mean labeling of graphs"** Bulletin of Pure and Applied Sciences, 30E (2),(2011), p.153-160

[7] S.S.Sandhya, E.Ebin Raja Merly and B.Shiny **"Super Geometric mean labeling on Double Quadrilateral snake graphs".** Asia Pacific Journal of Research, Vol:1, Issue XXI, Jan 2015, ISSN: 2320 – 5504, E-ISSN-2347-4793