The Deficient Discret Quartic Spline Interpolation over Uniform Mesh

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Abstract

In this paper we have studied existence, uniqueness and convergence properties of deficient discrete quartic spline interpolation over uniform which agree with function and two interior points boundary points and first difference at interior points.

Key Words – Deficient, Discrete, Quartic Spline, Interpolation

1. INTRODUCTION.

Discrete splines have been introduced by Mangasarian and Schumaker [7] in connection with certain studies of minimization problems involving differences They have a closed connection with best summation formula in [8] which is especial case of abstract theory of best of approximation of linear functions. Malcolm [6] has used discrete splines to compute nonlinear splines iteratively discrete Cubic splines which interpolates given functional of a uniform mesh have been studied by Lyche [11]. These resuls were generalized by Dikshit and Rana [2] for nonuniform mesh. It has been observed that deficient spline mor useful than usual spline observed they require less continuity requirement at the mesh points. In the direction of some constrictive aspect of discrete splines we refer to Astor and Duris [1],Jia [9] and Schumaker [10], Rana and Dubey [5,] .Dubey and Paroha [4] and Dubey and Nigam[3]

EXISTENCE AND UNIQUENESS

Let a mesh on [01] be defined by

$$P : 0 = x_0 < x_1 < \ldots\ldots\ldots\ldots x_n = 1$$

Such that

$$x_i - x_{i-1} = P \qquad \text{for} \qquad i = 0\ 1, 2, \ldots\ldots\ldots n-1$$

Through h will represent given position real number. For a given function f, we introduced the following interpolatory conditions

$$s(\alpha_i) = f(\alpha_i) \quad \alpha_i = x_i + \frac{1}{3}\mathrm{P} \tag{2.2}$$

$$s(\beta_i) = f(\beta_i) \quad \beta_i = x_i + \frac{1}{2}P \tag{2.3}$$

$$D_h^{\{1\}} s\{\gamma_i\} = D_h^{\{1\}} s\{\gamma_i\} \quad \gamma_i = x_i + \theta P \quad 0 < \theta < 1 \quad (2.4)$$

and boundary conditions

$$s(x_0, h) = f(x_0, h) \qquad (2.5)$$
$$s(x_n, h) = f(x_n, h)$$

The class $D(m, r, p, h)$ of deficient discrete spline of degree-m with deficiency r is the set of all continuous function $s(x, h)$ such that for $i = 0, 1, \ldots\ldots\ldots, n,-1$ the restriction s_i of $s(x, h)$ on $[x_i, x_{i+1}]$is a polynomial of degree m or less and

$$D_h^{\{i\}} s_i (x_i, h) = D_h^{\{i\}} s_{i+1} (x_i, h), \; j = 0, 1, \ldots\ldots\ldots m\text{-}r\text{-}1 \qquad (2.6)$$

Where the difference operator

$D_h^{(i)}$ for a function is defined by

$$D_h^{\{0\}} f(x) = f(x)$$

$$D_h^{\{1\}} f(x) = \frac{f(x+h) - f(x-h)}{2h}$$

We pose the following

Problem A : Given h > 0 for what restriction in θ there exist an unique $s(x, h) \;\epsilon\, S^*(4, 1, P, h)$ which satisfies the condition.(2.2) - (2.5)

Let P(t) be the quartic polynomial on [0, 1] then we can show that

$$P(t) = P\left(\frac{1}{3}\right) Q_1(t) + P\left(\frac{1}{2}\right) Q_2(t) + P'(\theta) Q_3(t) + P(0) Q_4(t) + P(1) Q_5(t) \qquad (2.7)$$

Where

$$Q_1(t) = \left[\left\{36\,\theta^3 - \frac{81}{2}\theta^2 + 9\theta + \left(36\theta - \frac{27}{2}\right)h^2\right\}t + \left\{-108\theta^3 - \frac{189\theta^2}{2} - \frac{9}{2} + \left(-108\theta + \frac{63}{2}\right)h^2\right\}t^2 + \left\{72\theta^3 - 63\theta + \frac{27}{2} + 72\theta h^2\right\}t^3\right] + \{54\theta - 9 - 54\theta^2 - 1\infty h^2\}z^4/A$$

$$Q_2(t) = \left[\left\{\frac{-64\theta^3}{3} + \frac{64\theta^2}{3} - \frac{32\theta}{9} + \left(-\frac{64\theta}{3} + \frac{64}{9}\right)h^2\right\}t + t\left\{\frac{256}{3}\theta^3 + \frac{48}{27} + \left(\frac{256\theta}{3} - \frac{208}{9}\right)h^2 - \frac{208}{3}\theta^2\right\} + t^3\left\{-64\theta^3 + \frac{416\theta}{9} - \frac{64}{9} - \frac{64}{1}\theta h^2\right\} + t^4\left\{48\theta^2 + 16h^2 - \frac{128\theta}{3} + \frac{16}{3}\right\}\right]/A$$

$$Q_3(t) = \left\{\frac{-t}{9} + \frac{2t^2}{3} - \frac{11t^3}{9} + \frac{2t^4}{3}\right\}/A$$

$$Q_4(t) = \left[1 + \left\{-16\theta^3 + 20\theta^2 - \frac{50\theta}{9} + \left(-16\theta + \frac{20}{3}\right)h^2\right\}t + \left\{\left(\frac{88\theta}{3} - \frac{85}{9}\right)h^2 + \left(\frac{88}{3}\theta^3 - \frac{85}{3}\theta^2\right) + \frac{25}{9}\right\}t^2 + \left\{-16\theta^3 + \frac{170}{9}\theta - \frac{20}{3} - 16\theta.h^2\right\}t^3 + \left\{4 - \frac{44}{3}\theta + 12\theta^2 + 4h^2\right\}t^4\right]/A$$

$$Q_5(z) = \left[\left\{\frac{4}{3}\theta^3 - \frac{5}{6}\theta^2 + \frac{\theta}{9} + \left(\frac{4}{3}\theta - \frac{5}{18}\right)h^2\right\}t + \left\{\frac{-20}{3}\theta^3 + \frac{19}{6}\theta^2 - \frac{1}{18} - \left(\frac{20}{3}\theta - \frac{19}{18}\right)h^2\right\}t^2 + \left\{8\theta^3 + \frac{5}{18} - \frac{38\theta}{18} + 8\theta h^2\right\}t^3 + \left\{\frac{10}{3}\theta - 6\theta^2 - 2h^2 - \frac{1}{3}\right\}t^4\right]/A$$

Where

$$A = \left[8\theta^3 - \frac{4}{3}\theta^2\frac{4}{3}\theta - \frac{1}{9} + \left(\frac{8}{3}\theta - \frac{11}{9}\right)h^2\right]$$

Now we are set to answer the problem "A" in the following

Theorem 2.1 : for any h>0 there exist an unique deficient discrete quartic splines *s(x, h)*ϵ *D(4, 1, P, h)* which satisfies the condition (2.2) - (2.4)

Prof :- Denoting $(x-x_i)/P_i$ by t, $0 \le t \le 1$ we can write (2.5) in the form of the restriction $s_i\ (x, h)$ as he deficient descrete quartic spline $s\ (x, h)$ on $[x_i,x_{i+1}]$ as follows

$$s_i(x,h) = f(\alpha_i)Q_1(t) + f(\beta_i)Q_2(t) + P\quad f(\gamma_i)Q_3(t) + s_i(x_i)Q_4(t) + s_i(x_{i+1})Q_5(t) \tag{2.6}$$

From equation (2.6) we can easity verified that $s_i(x, h)$ is quartic on $[x_i, x_{i+1}]$ for i = 0, 1, …………n-1 satisfying (2.2) - (2.4) we apply the continuity of first difference of $s_i(x, h)$ at x_i in (2.1) to see that

$$\left[\left\{\frac{16}{3}\theta^3 - \frac{34}{3}\theta^2 + \frac{68}{9}\theta - \frac{14}{9} + h^2\left(\frac{16}{3}\theta - \frac{34}{9}\right)\right\}P^2 + h^2\left\{16\theta^3 - 48\theta^2 + \frac{358\theta}{9} + (16\theta - 16)h^2 + \frac{28}{3}\right\}\right]s_{i-1} + \left\{-12\theta^3 + \frac{37\theta^2}{2} + \frac{64\theta}{9} + \frac{11}{18} + h^2\left(\frac{27}{6} - 12\theta\right)\right\}P^2 + \left\{t - 8\theta^2 + \frac{14}{18} - \frac{101\theta}{9} + 24\theta^2 - (8\theta - 8)h^2\right\}h^2 + \left[\left\{-16\theta^3 + 20\theta^2 - \frac{50\theta}{9} + \left(-16\theta + \frac{20}{3}\right)h^2\right\}P_i^2 + \left(-16\theta^3 + \frac{170}{9}\theta - \frac{20}{3} - 16\theta.h^2\right)h^2\right]s_i + s_{i+1}\left[\left\{\frac{4}{3}\theta^3 - \frac{5}{6}\theta^2 + \frac{\theta}{9} + \left(\frac{4\theta}{3} - \frac{5}{18}\right)h^2\right\}P_i^2 + \left\{8\theta^3 + \frac{5}{18} - \frac{38\theta}{18} + 8\theta h^2\right\}h^2 = F_i\right] \quad i = 1, 2, \ldots n-1 \tag{2.7}$$

Where $A_1(\theta,h) =$

$$\left[\left\{\frac{16}{3}\theta^3-\frac{34}{3}\theta^2+\frac{68}{9}\theta-\frac{14}{9}+h^2\left(\frac{16}{3}\theta-\frac{34}{9}\right)\right\}P^2+h^2\left\{16\theta^3-48\theta^2+\frac{358\theta}{9}+(16\theta-16)h^2+\frac{28}{3}\right\}\right]$$

$A_2(\theta,h)$ = [P^2{$-28\theta^3$ +$\frac{77}{2}\theta^2$ - $\frac{114}{9}$ θ + $\frac{11}{8}$ +(-28θ +$\frac{77}{6}$) h^2 +{$-24\theta^3$ +$\frac{79}{9}\theta$ $-\frac{101}{9}$ + $24\theta^2$ - (24θ - 8)h^2}h^2]

$A_3(\theta,h)$ = [P^2{$\frac{4}{3}\theta^2$ -$\frac{5}{6}\theta^2$ +$\frac{\theta}{9}$ +h^2($\frac{4}{3}\theta$ -$\frac{5}{18}$)} +{$8\theta^3$+$\frac{5}{18}$ -Type equation here.$\frac{38}{18}\theta$ +$8\theta h^2$}h^2 and

$F_i = \left\{36\theta^3+36\theta-\frac{9}{2}-\frac{135}{1}\theta^2+\left(36\theta+\frac{45}{2}\right)h^2\right\}P^2+\left\{72\theta^3-153\theta-\frac{45}{2}-216\theta^2+72(\theta-1)h^2\right\}f(\alpha_{i-1})-$ $\left\{36\theta^3-\frac{81\theta^2}{2}+9\theta+\left(36\theta-\frac{27}{2}\right)h^2\right\}P^2+\left\{72\theta^3-63\theta+\frac{27}{2}+72\theta.h^2\right\}h^2\, f(\alpha_i)+\left[\left\{\left(\frac{-128}{30}\theta^3-\frac{224}{3}\theta^2-\frac{320\theta}{9}+\right.\right.\right.$ $\left.\frac{32}{9}\right)+\left(\frac{-128\theta}{3}+\frac{224}{9}\right)h^2\Big\}P^2+\left\{-64\theta^3-\frac{1120\theta}{9}+192\theta^2+\frac{128}{9}+64(1-\theta)h^2\right\}h^2\Big]\, f(\beta_{i-1})-f(\beta_i)\left[\left\{\frac{-16\theta^3}{3}+\right.\right.$ $\frac{64\theta^2}{2}-\frac{32\theta}{9}+\left(\frac{-64\theta}{3}+\frac{64}{9}\right)h^2\Big\}P^2+\left(-64\theta^3+\frac{416}{9}\theta-\frac{64}{9}-64\theta h^2\right)h^2\Big]+f_{i-1}\left[\frac{2}{9}P^2+\frac{13}{9}h^2\right]\, D_h^{\{1\}}f(\gamma_i)+$ $D_h^{\{1\}}f(\gamma_i)\,P\ \ \left[\frac{1}{9}P\text{^}2+\frac{11}{9}h^2\right]$

We have to consider particular value of θ, $0<\theta<1$ *those satisfied* diagonally dominant properties i.e absolute value of coefficient dominant over the sum of the absolute values of coefficient matrix of system is diagonally of (2.7) in the form AM =F where M and F are the trancepose of the single column vector [m_1, m_2,…,m_n] and [F_1,F_2….F_n] respectively and the square matrix of system of equation (2.7) is diagonally dominant and hence invertible.

Now in this section we shall obtain the precise estimate of error bounds
for discrete quartic spline interpolant for s of theorem 2.1 i.e .e = f-s over the discrete interval [0,1].

ERROR BOUNDS

It may be observed that system of equation (2.7) may be written as

$$A(h).\ M(h) = F \qquad (3.1)$$

Where A(h) is coefficient matric and $M(h) = s_i(x, h)$ However as already shown in proof of theorem 2.1 A(h) is invertible. Denoting the inverse of A(h) by $A^{-1}(h)$ we note that row max norm || $A^{-1}(h)$ || satisfies the following inequality

Where,

$$\| A^{-1}(h) \| \le y(h) \qquad (3.2)$$

Where,

$$Y(h) = \max \{C_i(h)\}^{-1}$$

For convenience we assume in this section that 1=Mh, where M is positive integer. It is also assume that he mesh point $\{x_i\}$ are such that $x_i \epsilon [0, 1]$ for i=1, 2, ……..h where discrete interval [0, 1]h is the set of points {0, h, 2h,………Mh} for a function F and two district points x_1, x_2 in it's domain the divided difference is defined by

$$f[x_1, x_2] = \frac{f(x_1) - f(x_2)}{(x_1 - x_2)}.$$

For convineance we write $f^{\{1\}}$ for $D_h^{\{1\}}$ f and w(f, p) is the modules of continouty of f. The discrete norm of a function f over interval [0, 1] is defined by

$$||f|| = \frac{max}{x\epsilon[0,1]} |f(x)| \tag{3.3}$$

Without assuming any smoothness condition on data f, we shall obtain in the following bounds of error function

$E(x) = s(x, h) – f(x)$ over the discrete interval $[0, 1]h$

Theorem : Suppose s(x, h) is the deficient discrete quartic spline interpolation of theorem 2.1 then

$|| e(x) || \leq y(h)\, k (P, h)\, w (f^{\{1\}}, P)$ _(3. 4)

$|| e(x) || \leq k^* (P, h)\, w(f^{\{1\}}, P)$ ___________(3.5) and

$|| e^1(x) || \leq k^{**} (P, h)\, w(f^{\{1\}}, P)$ _(3.6)

Where $k(P, h)$ $k^* (P, h)$ and $k^{**} (P, h)$ are some positive function of p and h

Proof : writing $f_i \{x\} = f_i$ we notice that the equation (3.1) may be written as

$A(h).\ e(x) = f_i (h) – A(h) f_i = bi$ say ____________ (3.7)

Put $e(x) = s(x, h) - f_i$

We need the following result due to lyche [11] to estimate R.H.S. of (3.7)

Lemma 3.1- Let $\{a_i\}_{i=1}^{m}$ and $\{b_j\}_{j=1}^{n}$ be given of non negative real numbers such that

$$\Sigma a_i = \Sigma b_j$$

Then for any real valued function f defined on discrete interval [0, 1]h we have

$$\sum_{i=1}^{m} a_i [x_{i,0}, x_{i,1}, \dots \dots . x_{ik}]\ f - \sum_{j=1}^{n} bj\ [y_{j0},\ y_{j1}, \dots \dots \dots . y_{jk}]\ f| < h\left[f^{(k)}, 1 - kh|\Sigma a_i|k\right| \tag{3.8}$$

Where x_{ik} , y_{jk} $\epsilon [0, 1]$ for internal values of i, j and k.

Replacing s(x.h) by $e(x_i,h)$ in equality (2.6) to get

$e(x) = e(x_i)\ Q_4(t) + e(x_{i+1})\ Q(x_{i+1}) + M_i (t)$ (3.13)

where $M_i(t) = f(\alpha_i)\ Q_{1(t)} + f(\beta_i)\ Q_2(t)\ P\ f^{\{1\}}(\gamma_i)\ Q_3(t) + f(x_{i-1})Q_4(t) + f(x_i)\ Q_5(t) – f(x)$

A little computation shows that $M_i(f)$ in (3.13) may be rewritten in the form of divided difference as follows

$$M_i(t) = \left|\sum_{i=1}^{3} a_i\,[x_{i0}, x_{i1}]f - \sum_{j=1}^{3} bj\big[y_{j0}, y_{j1}\big]f\right| \qquad (3.14)$$

It may be observed that R.H.S. of (3.7) is written as

$$(L_i) = \left|\sum_{i=1}^{5} a_i\,[x_{i0}, x_{i1}]f - \sum_{j=1}^{3} bj\big[y_{j0}, y_{j1}\big]f\right|$$

Where,

$$a_1 = P\ \left\{\frac{20}{3}\theta^3 - \frac{20}{9} + \frac{17}{36} - \frac{43}{12}\theta^2 - \left(\frac{10}{3}\theta + \frac{43}{36}\right)h^2\right\}P^2 - h^2\left\{4\theta^3 + \frac{257\theta}{18} - \frac{149}{36} - 12\theta^2 + 4(\theta-1)h^2\right\}$$

$$a_2 = P\ \left\{\frac{16}{3}\theta^3 - \frac{34}{3}\theta^2 + \frac{68\theta}{9} - \frac{14}{9} + h^2\left(\frac{16}{3}\theta - \frac{34}{9}\right)\right\}P^2 + h^2\left\{16\theta^3 - 48\theta^2 + \frac{20}{3} + \frac{358\theta}{9} + 16(\theta-1)h^2\right\}$$

$$a_3 = P_{i-1}\left\{\frac{2}{9}P_{i-1}^2 + \frac{19}{9}h^2\right\}$$

$$a_4 = P_i\left[\left\{\frac{64}{18}\theta^3 - \frac{64}{18}\theta^2 + \frac{32\theta}{54} + \left(\frac{64}{18}\theta - \frac{64}{54}\right)h^2\right\}P^2 + \left\{\frac{64}{18}\theta^3 - \frac{416}{54}\theta + \frac{64}{54} + \frac{64}{6}\theta h^2\right\}h^2\right]$$

$$a_5 = P\left[\left\{\frac{-8}{9}\theta^3 - \frac{10}{18}\theta^2 - \frac{2\theta}{27} - \left(\frac{8\theta}{9} - \frac{10}{54}\right)h^2\right\}P^2 + \left\{-\frac{16}{3}\theta^3 - \frac{10}{54} + \frac{76}{18}\theta - \frac{16}{3}\theta h^2\right\}h^2\right]$$

$$b_1 = P\ \left[\left\{6\theta + 6\theta - \frac{3}{4} - \frac{135}{12}\theta^2 + \left(6\theta - \frac{45}{12}\right)h^2\right\}P^2 + h^2\left\{12\theta^3 + \frac{153\theta}{6} - \frac{45}{12} - 36\theta^2 + (12\theta - 12)h^2\right\}\right]$$

$$b_2 = P^3\left[\left\{\frac{8}{3}\theta^3 - \frac{11}{3}\theta^2 + \frac{4}{3}\theta + \left(\frac{8\theta}{3} - \frac{11}{3}\right)h^2\right\}\right]$$

$$b_3 = P\ \left[\left\{\frac{8}{3}\theta^3 - 3\theta^2 + \frac{14}{27}\theta + \left(\frac{8\theta}{3} - 3\right)h^2\right\}P^2 + \left\{\frac{16}{3}\theta^3 - \frac{170\theta}{27} + \frac{20}{9} + \left(\frac{16}{3}\theta h^2\right)\right\}\right]$$

$$b_4 = P^3\left[-\frac{1}{9}\right]$$

$$b_5 = P^3\left(\frac{11}{9}\right)h^2$$

and

$x_{10} = P_{i-1}$, $\quad x_{11} = x_i = x_{20} = x_{21}$

$x_{20} = x_{i-1}$, $\quad x_{30} = \gamma_{i-1} - h$, $\quad x_{31} = \gamma_{i-1} + h$

$x_{40} = \alpha_i$, $\quad x_{41} = \beta_i$ $\quad x_{50} = \alpha_i$ $\quad x_{51} = \alpha_{i-1}$

$y_{10} = \alpha_{i-1}$ $\quad y_{11} = \beta_{i-1}$ $\quad y_{20} = x_i$ $\quad y_{21} = \alpha_i$

$y_{30} = x_i$ $\quad y_{31} = \alpha_i$ $\quad y_{40} = y_i - h$ $\quad y_{41} = y_i + h$

$y_{50} = x_i - h$ $\quad y_{51} = y_i + h$

$$\sum_{i=1}^{5} a_n = \sum_{j=1}^{5} b_j = \Big[\Big\{\frac{26}{3}\theta^3 - \frac{-189}{12}\theta^2 + \frac{22}{3}\theta - \frac{1}{9} + \Big(\frac{10\theta}{3} - \frac{1}{12}\Big)h^2\Big\}P^2$$
$$+ h^2\Big\{12\theta^3 + \frac{153\theta}{6} - \frac{45}{12} - \frac{36}{1}\theta^2 + 12(\theta-1)h^2\Big\} + P_i\Big\{\frac{8}{3}\theta^3 - 3\theta^2 + \frac{14}{27}\theta + \Big(\frac{8\theta}{3} - 3\Big)h^2\Big\}P^2$$
$$+ \Big\{\frac{16}{3}\theta^3 - \frac{170\theta}{27} + 1 + \frac{16}{3}\theta h^2\Big\}h^2\Big]$$

$= k\,(P,h)$ *(Say)* _(3.10)

Thus apply Lemna 3.1 suitable in (3.10) for m=n=4 and k=1 to see that

$$|\,L_i\,| \le k\,(P, h)\, w\,(f^{\{1\}}|1-P\,|\,) \qquad (3.11)$$

Now using the equation (3.2) and (3.11) in (3.7) we get

$$||\,e(x_i)\,|| < y(h)\, k(P, h)\, w\,(f\{1\}, |\,1-P\,|) \qquad \_(3.12)$$

Thus in equality (3.4) of theorem (3.1) to obtain the bound of *e(x)* we replace $s_i(x, h)$ by $e(x_i)$ in equality (2.6) to get

$$e(x) = e(x_i)\, Q_4(t) + e(x_{i+1})\, Q_5(t) + M_i(f) \qquad (3.13)$$

Where,

$$M_i(f) = f(\alpha_i)\, Q_1(t) + f(\beta_i)\, Q_2(t) + P\, f^{\{1\}}(\gamma_i)\, Q_3(t) + f(x_{i-1})\, Q_4(t) + f(x_i)\, Q_5(t) - f(x)$$

A Little conversation show that Mi(f) in (3.13) may be written in the form of divided difference as follows

$$|M_i(f)| = \textstyle\sum_{i=1}^{3} a_i[x_{i0}, x_{i1}]f - \sum_{j=1}^{3} b_j[y_{i0}, y_{i1}]f \qquad (3.14)$$

Where,

$$a_1 = \Big[t\Big(-6\theta^3 + \frac{81}{12}\theta^2 - \frac{3}{2}\theta - \Big(6\theta - \frac{27}{12}\Big)h^2 + t^2\Big\{\frac{108}{6}\theta^3 - \frac{189}{12}\theta^2 + \frac{3}{4} - h^2\Big(-18\theta + \frac{21}{4}\Big)\Big\}$$
$$+ t^3\Big\{-12\theta^3 - \frac{21\theta}{2}\theta^2 - \frac{9}{4} - 12\theta h^2\Big\} + t^4\Big\{-9\theta + \frac{3}{2} + 9\theta^2 + 3h^2\Big\}\Big]$$

$$a_2 = P\;\Big[\Big\{t(8\theta^3 - 10\theta^2 + \frac{25}{9}\theta + \Big(8\theta - \frac{10}{3}\Big)h^2\Big\} + \Big\{\frac{-44}{3}\theta^3 + \frac{85}{6}\theta^2 + \frac{25}{18} - \Big(\frac{44}{3}\theta - \frac{85}{18}\Big)h^2\Big\}t^2$$
$$+ \Big\{8\theta^3 - \frac{170}{18}\theta + \frac{10}{3} + 8\theta h^2\Big\}t^3 + \Big\{-2 + \frac{29}{3}\theta - 6\theta^2 - 2h^2\Big\} + 4\Big]$$

$$a_3 = P\;\Big\{\frac{1}{9}t + \frac{2}{3}t^2 - \frac{11}{9}t^3 + \frac{2}{3}t^4\Big\}$$

$$b_1 = P \left[t\left\{\frac{-2}{3}\theta^3 + \frac{15}{12}\theta^2 - \frac{\theta}{18} + \left(\frac{2}{3}\theta + \frac{5}{36}\right)h^2\right\} + \left\{\frac{10}{3}\theta^3 - \frac{19}{12}\theta^2 + \frac{1}{36} + \left(\frac{10}{3}\theta + \frac{19}{36}\right)h^2\right\}t^2 - \left\{4\theta^3 + \frac{5}{36} - \frac{19\theta}{18} + 4\theta h^2\right\}t^3 - \left(\frac{5}{3}\theta - 3\theta^2 - h^2 - \frac{1}{6}\right)t^4\right]$$

$$b_2 = P \quad t\left\{\frac{8}{3}\theta^3 - \frac{11}{3}\theta^2 + \frac{4\theta}{3} - \frac{1}{9} + \left(\frac{8}{3}\theta - \frac{11}{9}\right)h^2\right\}$$

And

$x_{10} = \alpha_i$ $\quad x_{11} = \beta_i = x_{21}$

$x_{20} = x_{i-1}$ $\quad x_{30} = \gamma_1 - h$ $\quad x_{31} = \text{Type equation here.}\ \gamma_{i+1}$

$y_{10} = \beta_i$ $\quad y_{11} = x_i$ $\quad y_{20} = x_{i-1}$ $\quad y_{21} = x$

Clearly

$$\sum_{i=1}^{3} a_i = \sum_{j=1}^{2} b_j$$

$$= \left[\left\{2\theta^3 - \frac{39}{12}\theta^2 + \frac{29}{18}\theta - \frac{1}{9} + \left(2\theta - \frac{13}{12}\right)h^2\right\}t + t^2\left\{\frac{20}{6}\theta^3 - \frac{19}{12}\theta^2 + \frac{1}{36} + \left(\frac{10}{3}\theta - \frac{19}{36}\right)h^2\right\} + \left(-4\theta^3 + \frac{19}{18}\theta - \frac{5}{36} - 4\theta h^2\right)t^3 + \left(\frac{-5}{6} + \frac{2}{3}\theta + 3\theta^2 + h^2\right)t^4\right]$$

$= K^*(P, h)$

Therefore applying lemma (3.1) for m=3, n=2 and k=1 we get

$$| M_i(f) | < K^*(P, h)\, w(f^{\{1\}}, P) \qquad (3.15)$$

Finally applying bounds of (3.12) and (3.15) in (3.13) we get inequality (3.5) w how (proceed to obtain an upper bound for $e^{\{1\}}(x)$ for this we use first difference operator in (2.6) and get

$P\, D_h^{\{1\}} s_i(x, h) = f(\alpha_i)\, Q_1^{\{1\}}(t) + f(\beta_i)\, Q_2^{\{1\}}(t) + Pf^{\{1\}}(\gamma_i)\, Q_3^{\{1\}}(t) + s_{i-1}(x)\, Q_4^{\{1\}}(t) + s_i(x)\, Q_5^{\{1\}}(t)$

Replace $s_i(x)$ by $e(x_i)$ we get

$= P\, e^{\{1\}}(x) = e_{i-1}\, Q_4^{\{1\}}(t) + e_i\, Q_5^{\{1\}}(t) + U_i(f)$

Where,

$U_i(f) = f(\alpha_i)\, Q_1^{\{1\}}(t) + f(\beta_i)\, Q_2^{\{1\}}(t) + \beta_i f^{\{1\}}(\gamma_i)\, Q_3^{\{1\}}(t) + f_{i-1}\, Q_4^{\{1\}}(t) + f_i\, Q_5^{\{1\}}(t) - Pf^{\{1\}}(x)$

Now rewriting $U_i(f)$ in terms of Divided difference we have

$$|U_i(f)| = \left|\sum_{i=1}^{3} a_i[x_{i0}, x_{i1}]_f - \sum_{j=1}^{2} b_j[y_{j0}, y_{j1}]_f\right|$$

Where,

$$a_1 = P \ \left\{8\theta^3 - 10\theta^2 + \frac{25}{9}\theta - h^2\left(-8\theta + \frac{10}{3}\right)\right\} + t\left\{\frac{-88}{3}\theta^3 + \frac{85}{3}\theta^2 - \frac{25}{9} - \left(\frac{88}{3}\theta - \frac{85}{9}\right)h^2\right\}$$
$$+ (3t^2 + h^2)\left\{8\theta^3 - \frac{85\theta}{3} + \frac{10}{3} + 8\theta h^2\right\} + \left\{-2 + \frac{22}{3}\theta - 6\theta^2 - 2h^2\right\}(4t)(t^2 + h^2)$$

$$a_2 = P \ \left[\frac{2}{3}\theta^3 - \frac{5}{12}\theta^2 + \frac{\theta}{18} + h^2\left(\frac{2}{3}\theta - \frac{5}{36}\right) + \left\{\frac{-20}{3}\theta^3 + \frac{19}{6}\theta^2 - \frac{1}{18} - \left(\frac{20}{3}\theta - \frac{19}{18}\right)h^2\right\}2t\right.$$
$$\left. + \left\{4\theta^3 + \frac{5}{36} - \frac{19}{18}\theta + 4\theta h^2\right\}(t^2 + h^2) \ + \left\{\frac{5}{3}\theta - 3\theta^2 - h^2 - \frac{1}{6}\right\}4t(t^2 + h^2)\right]$$

$$a_3 = P \ \left\{\frac{-1}{9} + \frac{4}{3}t - \frac{11}{9}(3t^2 + h^2) + \frac{8}{3}t(t^2 + h^2)\right\}$$

$$b_1 = P \ \left[6\theta^3 - \frac{81}{12}\theta^2 + \frac{3}{2}\theta + \left(6\theta - \frac{9}{4}\right)h^2 + t\left\{-36\theta^3 + \frac{63}{2}\theta^2 - \frac{3}{2} + \left(-36\theta + \frac{3}{2}\right)h^2\right\}\right.$$
$$\left. + \left\{12\theta^3 - \frac{21}{2}\theta + \frac{9}{4} + 12\theta h^2\right\} (3t^2 + h^2) + \left(9\theta - \frac{3}{2} - 9\theta^2 - 3h^2\right) 4t\,(t^2 + h^2)\right]$$

$$b_2 = P \ \left[\frac{8}{3}\theta^3 - \frac{11}{3}\theta^2 + \frac{4}{3}\theta - \frac{1}{9} + \left(\frac{8}{3}\theta - \frac{11}{9}\right)h^2\right]$$

It can e easily seen that

$$\sum_{i=1}^{3} a_i = \sum_{j=1}^{2} b_j$$

$$= P \ \left[\frac{26}{3}\theta^3 - \frac{125}{2}\theta^2 + \frac{17}{6}\theta + \left(\frac{26}{3}\theta - \frac{125}{36}\right)h^2 + t\left\{-36\theta^3 + \frac{63}{2}\theta^5 - \frac{3}{2} + \left(-36\theta + \frac{21}{2}\right)h^2\right\}\right.$$
$$\left. + \left\{12\theta^3 - \frac{21}{2}\theta + \frac{9}{4} + 12\theta h^2\right\} + \left\{9\theta - \frac{3}{2} - 9\theta^2 - 3h^2\right\} 4t(t^2 + h^2)\right]$$

and

$y_{10} = \alpha_i$ $\quad$ $y_{11} = \beta_i$

$y_{20} = x+h$ $\quad$ $y_2 = x-h$ $x_{10} = \beta_i$ $x_{11} = x_i$

$x_{30} = \gamma_i-h$ $\quad$ $x_{20} = \beta_i$ $x_{21} = x_{i+1}$ $\quad$ $x_{30} = \gamma_i - h$, $x_{21 = x_{i+1}}$, $x_{31 = \gamma_i} + h$

Using (3.15) and (3.17) in (3.16) we get inequality (3.6)

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