# Projective Product of Gamma Rings with Unities and $(\sigma, \tau)$ Derivations

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***Abstract****: This paper highlights many enlightening results on $(\sigma,\tau)$-derivations and unities in the projective product of Gamma-rings. If (X, $\Gamma$ ) is the projective product of two Gamma-rings*

*( $X_1$, $\Gamma_1$) and ( $X_2$ , $\Gamma_2$), a pair of $(\sigma,\tau)$-derivations $D_1$ and $D_2$ on ( $X_1$, $\Gamma_1$) and ( $X_2$ , $\Gamma_2$) respectively can be extended to a $(\sigma,\tau)$-derivation D on (X, $\Gamma$ ). The converse problems are also studied fruitfully. The similar results can be investigated in case of the projective product of n number of Gamma- rings.*

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## 1. INTRODUCTION:

A new and prominent dimension of research in the area of Gamma-rings has been started when many prominent researchers have become able to extend many powerful technical results known for general rings to Gamma-rings. The concept of a gamma ring was first introduced by Nobusawa [6]. Barnes [10] weakened slightly the conditions in the definition of gamma ring in the sense of Nobusawa. This field , which has evolved as an extension of general ring theory does not only cover a small area with an independent life, but also serves as a unifying thread interlacing many other branches such as Banach spaces, C*-algebras, Dynamical systems, Quantum theory etc., and thus it suggests a very wide of scope of doing research. Many prominent researchers have done magnificent works on different types of derivations in the field of gamma-rings [1,2,3,4,5,7,8].

## 2. BASIC CONCEPTS:

**Definition 2.1:** A mapping $f$: (X, $\Gamma$ ) → (X, $\Gamma$ ) is called a $\Gamma$ **- automorphism** if $f$ is bijective and a $\Gamma$ - homomorphism.

**Definition 2.2:** An additive mapping $d$: (X, $\Gamma$ ) → (X, $\Gamma$ ) is called a $(\boldsymbol{\sigma},\boldsymbol{\tau})$ **derivation** if

$$d(x\alpha y) = \sigma(x)\alpha d(y) + d(x)\alpha\tau(y)\ \forall\, x, y \in X,\ \alpha \in \Gamma$$

**Definition 2.3:** Let (X, $\Gamma$ ) be a gamma ring with left and right operator rings $L$ and $R$ respectively. X is said to have a **left (or right) unity** if there exist $d_1, d_2, \dots\dots, d_n \in X$ and $\delta_1, \delta_2, \dots\dots \delta_n \in \Gamma$ such that for all $x \in$ X, $\sum_{i=1}^{n} d_i\delta_i x = x$ (or $\sum_{i=1}^{n} x\delta_i d_i = x$ ).

X is said to have a **strong left (or strong right) unity** if there exist $d \in$ X, $\delta \in \Gamma$ such that

$d\delta x = x$ or $x\delta d = x$ for all $x \in$ X.

An ideal I of X will be called **left modular (left strongly modular)** if the factor gamma ring X/I has a left unity (strong left unity). **Right modular and right strongly modular ideals** are similarly defined.

**Definition 2.4:** An ideal $A$ of X is called a **direct summand** if there exist an ideal $B$ of X such that every element of X is uniquely expressible in the form $x = a + b$ $where$ $a \in A, b \in B$, then we write X=A+B. it can be proved that if $a \in A, b \in B$ then $a\gamma b = 0$ for all $\gamma \in \Gamma$ .

**Definition 2.5:** A gamma ring (X, $\Gamma$ ) is said to be a **prime gamma ring** if $x\Gamma X\Gamma x = 0$, $with$ $x, y \in$ X implies either $x = 0$ $or$ $y = 0$.

**Definition 2.6:** Let $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ be two gamma rings. Let $X = X_1 \times X_2$ and

$\Gamma = \Gamma_1 \times \Gamma_2$. Then defining addition and multiplication on $X$ and $\Gamma$ by,

$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$ ,

$(\alpha_1, \alpha_2) + (\beta_1, \beta_2) = (\alpha_1 + \beta_1, \alpha_2 + \beta_2)$

and $(x_1, x_2)(\alpha_1, \alpha_2)(y_1, y_2) = (x_1\alpha_1 y_1, x_2\alpha_2 y_2)$

for every $(x_1, x_2), (y_1, y_2) \in X$ and $(\alpha_1, \alpha_2), (\beta_1, \beta_2) \in \Gamma$,

$(X, \Gamma)$ is a gamma ring. We call this gamma ring as the **Projective product of gamma rings**.

## 3. MAIN RESULTS:

**Theorem 3.1:** Two automorphisms on $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ give rise to an automorphim on $(X, \Gamma)$ but the converse is true for homomorphism, where $(X, \Gamma)$ is the projective product of $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$.

Proof: Let $f_1$ and $f_2$ be two automorphisms on the gamma rings $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ respectively. Then $f_1$ and $f_2$ are homomorphisms as well as bijective.

We define a mapping $f: X \to X$ by,

$f(x) = f((x_1, x_2)) = (f_1(x_1), f_2(x_2))$ for all $x = (x_1, x_2) \in X$.

Obviously $f$ is additive.

Let $x = (x_1, x_2), y = (y_1, y_2) \in X$ and $\alpha = (\alpha_1, \alpha_2) \in \Gamma$ be any elements. Then

$$\begin{aligned} f(x\alpha y) &= f((x_1, x_2)(\alpha_1, \alpha_2)(y_1, y_2)) \\ &= f((x_1\alpha_1 y_1, x_2\alpha_2 y_2)) \\ &= (f_1(x_1\alpha_1 y_1), f_2(x_2\alpha_2 y_2)) \end{aligned}$$

$= (f_1(x_1)\alpha_1 f_1(y_1), f_2(x_2)\alpha_2 f_2(y_2))$ [Since $f_1$ and $f_2$ are homomorphisms on $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ respectively]

$$\begin{aligned} &= (f_1(x_1), f_2(x_2))(\alpha_1, \alpha_2)(f_1(y_1), f_2(y_2)) \\ &= f((x_1, x_2))(\alpha_1, \alpha_2) f((y_1, y_2)) \\ &= f(x)\alpha f(y) \end{aligned}$$

So, $f(x\alpha y) = f(x)\alpha f(y)\ \forall x, y \in X \text{ and } \alpha \in \Gamma$. Thus $f$ is a homomorphism on $(X, \Gamma)$.

Again let,

$f(x) = f(y) \Rightarrow f((x_1, x_2)) = f((y_1, y_2)) \Rightarrow (f_1(x_1), f_2(x_2)) = (f_1(y_1), f_2(y_2))$

$\Rightarrow f_1(x_1) = f_1(y_1) \text{ and } f_2(x_2) = f_2(y_2)$

$\Rightarrow x_1 = y_1 \text{ and } x_2 = y_2$ [since $f_1$ and $f_2$ are one-one mappings]

$\Rightarrow (x_1, y_1) = (x_2, y_2) \Rightarrow x = y$

So $f$ is an one-one mapping.

Now to show $f$ is onto, let $y = (y_1, y_2) \in X$ be any element. Then $y_1 \in X_1 \text{ and } y_2 \in X_2$. Since $f_1$ and $f_2$ be two automorphisms on the gamma rings $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ respectively so for $y_1 \in X_1 \text{ and } y_2 \in X_2$ there exist $x_1 \in X_1 \text{ and } x_2 \in X_2$ such that $f_1(x_1) = y_1$ and $f_2(x_2) = y_2$. Thus there exist $x = (x_1, x_2)$ such that $f(x) = f((x_1, x_2)) = (f_1(x_1), f_2(x_2)) = (y_1, y_2) = y$, which shows that $f$ is onto.

Hence $f$ is an automorphism on $X$ defined by $f_1$ and $f_2$.

Converse part: Let $f$ be a $\Gamma$- automorphism on $X$. We define two mappings $f_1: X_1 \to X_1$ and $f_2: X_2 \to X_2$ by $f_1(x_1) = Ff((x_1, 0))\ \forall x_1 \in X_1$ and $f_2(x_2) = Sf((0, x_2))\ \forall x_2 \in X_2$, where $F$ and $S$ represents the first and second component of an ordered pair in $X$.

$x_1, y_1 \in X_1$ and $\alpha_1 \in \Gamma_1$ be any elements. Then we have the following

$f_1(x_1\alpha_1 y_1)$

$= Ff((x_1\alpha_1 y_1, 0))$

$= Ff((x_1\alpha_1 y_1, 0\alpha_2 0))$ , $\alpha_2 \in \Gamma_2$

$= Ff((x_1, 0)(\alpha_1, \alpha_2)(y_1, 0))$

$= Ff(x\alpha y), \text{where } x = (x_1, 0), y = (y_1, 0) \in X \text{ and } \alpha = (\alpha_1, \alpha_2) \in \Gamma$

$= F[f(x)\alpha f(y)]$ [since $f$ is an automorphism on $(X, \Gamma)$]

$= Ff(x)F\alpha Ff(y)$

$= Ff((x_1,0))F(\alpha_1,\alpha_2)Ff((y_1,0))$

$= f_1(x_1)\alpha_1 f_1(y_1)$

Thus $f_1(x_1\alpha_1 y_1) = f_1(x_1)\alpha_1 f_1(y_1)\ \forall\, x_1, y_1 \in X_1$ and $\alpha_1 \in \Gamma_1$.

Similarly we can show, $f_2(x_2\alpha_2 y_2) = f_2(x_2)\alpha_2 f_2(y_2)\ \forall\, x_2, y_2 \in X_2$ and $\alpha_2 \in \Gamma_2$.

So $f_1$ and $f_2$ are two homomorphisms on the gamma rings $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ respectively. But nothing can be said about the bijectiveness of the mappings $f_1$ and $f_2$, though $f$ is bijective. Hence the result.

**Theorem 3.2:** Two $(\sigma,\tau)$ derivation on the gamma rings $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ give rise to a $(\sigma,\tau)$ derivation on the projective product $(X, \Gamma)$ of the gamma rings $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$.

**Proof:** Let $d_1: X_1 \to X_1$ and $d_2: X_2 \to X_2$ be two $(\sigma_1,\tau_1)$ and $(\sigma_2,\tau_2)$ derivations on the gamma rings $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ respectively.

We define mappings $d: X \to X$, $\sigma: X \to X$ and $\tau: X \to X$ by

$d(x) = d((x_1,x_2)) = (\, d_1(x_1), d_2(x_2))$

$\sigma(x) = \sigma((x_1,x_2)) = (\, \sigma_1(x_1), \sigma_2(x_2))$

$\tau(x) = \tau((x_1,x_2)) = (\, \tau_1(x_1), \tau_2(x_2))$ for all $x = (x_1,x_2) \in X$

Then $d, \sigma, \tau$ are well defined as well as additive mappings.

Let $x = (x_1,x_2), y = (y_1,y_2) \in X$ and $\alpha = (\alpha_1,\alpha_2) \in \Gamma$ be any elements. Then

$d(x\alpha y)$

$= d(\,(x_1,x_2)(\alpha_1,\alpha_2)(y_1,y_2))$

$= d((x_1\alpha_1 y_1, x_2\alpha_2 y_2))$

$= (d_1(x_1\alpha_1 y_1), d_2(x_2\alpha_2 y_2))$

$= (\sigma_1(x_1)\alpha_1 d_1(y_1) + d_1(x_1)\alpha_1\tau_1(y_1), \sigma_2(x_2)\alpha_2 d_2(y_2) + d_2(x_2)\alpha_2\tau_2(y_2))$ [Since $d_1$ and $d_2$ are two $(\sigma_1,\tau_1)$ and $(\sigma_2,\tau_2)$ derivations on the gamma rings $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ respectively.

$= (\,\sigma_1(x_1)\alpha_1 d_1(y_1), \sigma_2(x_2)\alpha_2 d_2(y_2)) + (d_1(x_1)\alpha_1\tau_1(y_1), d_2(x_2)\alpha_2\tau_2(y_2))$

$= (\,\sigma_1(x_1), \sigma_2(x_2))(\alpha_1,\alpha_2)(d_1(y_1), d_2(y_2)) + (d_1(x_1), d_2(x_2))(\alpha_1,\alpha_2)(\,\tau_1(y_1), \tau_2(y_2))$

$= \sigma((x_1,x_2))(\alpha_1,\alpha_2)d((y_1,y_2)) + d((x_1,x_2))(\alpha_1,\alpha_2)\,\tau((y_1,y_2))$

$= \sigma(x)\alpha d(y) + d(x)\alpha\tau(y)$

Thus $d(x\alpha y) = \sigma(x)\alpha d(y) + d(x)\alpha\tau(y) \forall\, x, y \in X$ *and* $\alpha \in \Gamma$.

So $d$ is a $(\sigma,\tau)$ derivation on $(X, \Gamma)$ defined by $d_1$ and $d_2$.

**Conversely** if $d$ is a $(\sigma,\tau)$ derivation on $(X, \Gamma)$, the defining the mappings $d_1: X_1 \to X_1$, $\sigma_1: X_1 \to X_1$ and $\tau_1: X_1 \to X_1$ by

$d_1(x_1) = Fd((x_1,0))\ \forall\, x_1 \in X_1$

$\sigma_1(x_1) = F\sigma((x_1,0))\ \forall\, x_1 \in X_1$

$\tau_1(x_1) = F\tau((x_1,0))\ \forall\, x_1 \in X_1$, where $F$ represents the first component.

Then it can be easily shown that $d_1$ is a $(\sigma_1,\tau_1)$ derivation on $(X_1, \Gamma_1)$. Similarly we can find $d_2$ is a $(\sigma_2,\tau_2)$ derivation on $(X_2, \Gamma_2)$ and hence the result.

**Theorem 3.3:** Let $(X, \Gamma)$ be the projective product of the gamma rings $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$. Then if $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ has left unities then $(X, \Gamma)$ also has left unity and vice versa.

**Proof:** Suppose $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ has left unities.

Since $(X_1, \Gamma_1)$ has left unity so there exist $d_1, d_2, d_3, \ldots, d_n \in X_1$ and $\delta_1, \delta_2, \delta_3, \ldots, \delta_n \in \Gamma_1$ such that $\sum_{i=1}^{n} d_i\delta_i x_1 = x_1\ \ \forall\ x_1 \in X_1$.

Again since $(X_2, \Gamma_2)$ has left unity so there exist $p_1, p_2, p_3, \ldots, p_m \in X_1$ and $\rho_1, \rho_2, \rho_3, \ldots, \rho_m \in \Gamma_2$ such that $\sum_{i=1}^{m} p_i\rho_i x_2 = x_2\ \ \forall\ x_2 \in X_2$.

Without the loss of generality, let $n \geq m$ and let $x = (x_1,x_2) \in X$ be any element.

Then $x = (x_1, x_2)$

$= (\sum_{i=1}^{n} d_i\delta_i x_1, \sum_{i=1}^{m} p_i\rho_i x_2)$

$= (d_1\delta_1 x_1 + d_2\delta_2 x_1 + \ldots + d_n\delta_n x_1, p_1\rho_1 x_2 + p_2\rho_2 x_2 + \ldots + p_m\rho_m x_2)$

$$= (d_1\delta_1 x_1 + d_2\delta_2 x_1 + \cdots + d_m\delta_m x_1 + d_{m+1}\delta_{m+1} x_1 + \cdots + d_n\delta_n x_1, \; p_1\rho_1 x_2 + p_2\rho_2 x_2 + \cdots + p_m\rho_m x_2 + 00x_2 + \cdots + 00x_2)$$

$$= (d_1\delta_1 x_1, p_1\rho_1 x_2) + (d_2\delta_2 x_1, p_2\rho_2 x_2) + \cdots + (d_m\delta_m x_1, p_m\rho_m x_2) + (d_{m+1}\delta_{m+1} x_1, 00x_2) + \cdots + (d_n\delta_n x_1, 00x_2)$$

$$= (d_1, p_1)(\delta_1, \rho_1)(x_1, x_2) + (d_2, p_2)(\delta_2, \rho_2)(x_1, x_2) + \cdots + (d_m, p_m)(\delta_m, \rho_m)(x_1, x_2) + (d_{m+1}, 0)(\delta_{m+1}, 0)(x_1, x_2) + \cdots + (d_n, 0)(\delta_n, 0)(x_1, x_2)$$

$$= a_1\gamma_1 x + a_2\gamma_2 x + \cdots + a_m\gamma_m x + a_{m+1}\gamma_{m+1} x + \cdots + a_n\gamma_n x$$

$$= \sum_{i=1}^{n} a_i\gamma_i x$$

Where $a_i = \begin{cases} (d_i, p_i) & if\ i \le m \\ (d_i, 0) & if\ i > m \end{cases}$ and

$$\gamma_i = \begin{cases} (\delta_i, \rho_i) & if\ i \le m \\ (\delta_i, 0) & if\ i > m \end{cases}$$

Then each $a_i \in X$ $and$ $\gamma_i \in \Gamma$.

Thus we get elements $a_1, a_2, a_3, \ldots, a_n \in X$ and $\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_n \in \Gamma$ such that

$\sum_{i=1}^{n} a_i\gamma_i x = x$ for all $x \in X$, which implies that the projective gamma ring $(X, \Gamma)$ also has left unity if its component gamma rings have left unities.

Conversely suppose that $(X, \Gamma)$ has left unity.

Then there exist elements $u_i = (c_i, d_i) \in X$ and $\omega_i = (\alpha_i, \beta_i) \in \Gamma$, $i = 1,2, \ldots, n$ such that

$\sum_{i=1}^{n} u_i\omega_i x = x$ for all $x = (x_1, x_2) \in X$

$$=> \sum_{i=1}^{n} (c_i, d_i)(\alpha_i, \beta_i)(x_1, x_2) = (x_1, x_2)$$

$$=> \sum_{i=1}^{n} (c_i\alpha_i x_1, d_i\beta_i x_2) = (x_1, x_2)$$

$$=> (\sum_{i=1}^{n} c_i\alpha_i x_1, \sum_{i=1}^{n} d_i\beta_i x_2) = (x_1, x_2)$$

$=> \sum_{i=1}^{n} c_i\alpha_i x_1 = x_1$ $and$ $\sum_{i=1}^{n} d_i\beta_i x_2 = x_2$

Thus we get elements $c_1, c_2, c_3, \ldots, c_n \in X_1$ and $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n \in \Gamma_1$, $i = 1,2, \ldots, n$ such that

$\sum_{i=1}^{n} c_i\alpha_i x_1 = x_1$ for all $x_1 \in X_1$. So $(X_1, \Gamma_1)$ has left unity. Similarly $(X_2, \Gamma_2)$ also has left unity. That is if the projective product has left unity then the component gamma rings also has left unities and hence the result.

**Theorem 3.3 can also be proved for gamma rings having right unity as well as strong left and strong right unities.**

**Theorem 3.4:** Let $(X, \Gamma)$ be the projective product of two prime gamma rings $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ having non zero $(\sigma_1, \tau_1)$ and $(\sigma_2, \tau_2)$ derivations respectively. Then for $a \in X$, there exist a $(\sigma, \tau)$ derivation $d$ on $(X, \Gamma)$ such that if at least one of the following (i)$d(X)\Gamma\sigma(a) = 0$ or (ii)$a\Gamma d(X) = 0$ holds, then $a = 0$. Converse is also true i.e if the result holds in the projective prime gamma ring then it will hold in the component gamma rings also.

**Proof:** Let $d_1: X_1 \to X_1$ and $d_2: X_2 \to X_2$ be two non zero $(\sigma_1, \tau_1)$ and $(\sigma_2, \tau_2)$ derivations on the gamma rings $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ respectively.

We define the mappings $d: X \to X$, $\sigma: X \to X$ and $\tau: X \to X$ by

$$d(x) = d((x_1, x_2)) = (d_1(x_1), d_2(x_2))$$

$$\sigma(x) = \sigma((x_1, x_2)) = (\sigma_1(x_1), \sigma_2(x_2))$$

$\tau(x) = \tau((x_1, x_2)) = (\tau_1(x_1), \tau_2(x_2))$ for all $x = (x_1, x_2) \in X$

Then $d, \sigma, \tau$ are well defined as well as additive mappings. Since $d_1$ and $d_2$ are non zero so $d$ is also non zero. Also $d$ is a $(\sigma, \tau)$ derivation $d$ on $(X, \Gamma)$ as proved in theorem 3.2.

Let $a \in X$ be any element such that $d(X)\Gamma\sigma(a) = 0$ holds. Then we have

$d(x)\gamma\sigma(a) = 0$ for all $x = (x_1, x_2) \in X, \gamma = (\gamma_1, \gamma_2) \in \Gamma$

$=> \big(d_1(x_1), d_2(x_2)\big)(\gamma_1, \gamma_2)\big(\sigma_1(a_1), \sigma_2(a_2)\big) = 0$

$=> \big(d_1(x_1)\gamma_1\ \sigma_1(a_1), d_2(x_2)\gamma_2\ \sigma_2(a_2)\big) = 0$
$= (0,0)$

$=> d_1(x_1)\gamma_1\ \sigma_1(a_1) = 0, d_2(x_2)\gamma_2\ \sigma_2(a_2) = 0$ for all $x_1 \in X_1$, $x_2 \in X_2$ and $\gamma_1 \in \Gamma_1$, $\gamma_2 \in \Gamma_2$

$=> a_1 = 0, a_2 = 0$ [Since $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ are prime gamma rings, so the result holds]

$=> (a_1, a_2) = (0,0) = 0 => a = 0$

Similarly the other result can also be proved.

Here $(X_1, \Gamma_1)$ and $(X_2, \Gamma_2)$ are prime gamma rings, but not necessarily their projective product $(X, \Gamma)$ is a prime gamma ring. But the result holds in $(X, \Gamma)$ irrespective of whether it is a prime gamma ring or not. Thus the result holds if the component gamma rings are prime, no matter the projective product itself is a prime gamma ring or not.

Conversely if we let $(X, \Gamma)$, a prime gamma ring with a non zero $(\sigma, \tau)$ derivation $d$, the result is obvious in the component gamma rings because the components of a prime gamma ring are always prime.

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