Inverse Stochastic Programming with Interval Constraints

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INTRODUCTION

Inverse optimization perturb objective function to make an initial feasible solution optimal with respect to perturbed objective function while minimizing cost of perturbation. We extend inverse optimization to two state stochastic linear program since the resulting model grows with number of scenarios, we present two decomposition approaches for solving these problems.

Inverse optimization has many application areas ,and inverse problems have been studied extensively in the analysis of geophysical data. Recently inverse optimization has extended into a variety of fields of study. [Burton and Toint (1992,1994)] to predict the movements of earth quakes assuming that earth quakes move along shortest paths.

Zhang and Lin (1996) suggested a solution method for general inverse linear programs including upper and lower bound constraints based on the optimality conditions for linear programs.Their objective function was to minimize the cost of perturbation based on the L_1 norm.

Ahuja and Orlin (2001) studied inverse optimization for deterministic problems and showed that the inverse of a deterministic problems and showed that the inverse of determinisiticL.p is also an L.p.They attained the inverse feasible cost vectors using optimality condition for L.ps and minimized the cost of perturbation based on both L_1 and $L\infty$ norm.

To consider the inverse optimization problem under the weighted L_1 norm involves solving the problem according to the objective Min $\sum_{j\in J} v_j|d_j - c_j|$ where J is the variable index set ,d_j and c_j are perturbed and original objective cost coefficients , respectively, and v_j is the weights coefficients respectively. By introducing variables αj *and* βj for each variable $j \in J$,this objective is equivalent to the following problem:

Two stage stochastic linear programming (TSSLP)considers LPs in which some problem data are random In this paper extends deterministic inverse LP to TSSLP. Although many of the applications of inverse optimization are stochastic in nature, to the best of our knowledge ,deterministic version of these problems have been considered so far .With this paper ,we add this stochastic nature to inverse problems along with interval coefficients.

Min $\sum_{j\in J} v_j\ (\alpha_j + \beta_j)$

s.t.dj-cj = $\alpha_j - \beta_j$

$\alpha_j \geq 0, \beta_j \geq 0\ , j \in J$

Two stage stochastics linear programming (TSSLP), consider LP's in which some problem data are random In this case first stage decisions are made without full information on the random events while second stage decisions are taken after full information on the random variables becomes available. With this paper we add this stochast nature to inverse problems along with interval coefficients.

We consider the extensive form of two stage Stochastic linear programming (TSS LP) with a finite number of scenarios. Let J^0 denotes the index set of first stage variables. I^0 denotes the index set of first stage constrains K denotes the se of scenarios,J^k denotes the index set of second stage variable for scenario K∈K. I^kdenotes the index set of second stage constrains for scenario K∈K. The two stage Stochastic linear programming in extensive form in which the co efficient of constraints are in the interval form is defined as

$$\sum_{\substack{j\in j \\ j=1}}^{n} c_j x_j + \sum_{k=1}^{n} \sum_{\substack{j=1 \\ j\in j^k}}^{n} p^k q_j^k y_j^k - - - - - - - - -(1)$$

Subject to

$$\sum_{\substack{j\in j^0\\ j=1}}^{n} (\underline{a}_{ij}, \bar{a}_{ij}) xj \le bi, i = 1,2,----n, \quad i \in I^0 --------(2)$$

$$\sum_{\substack{j=1\\ j\in j^0}}^{n} t_{ij}^k x_j + \sum_{\substack{j=1\\ j\in J^k}}^{n} (\underline{w_{ij}^k}, \overline{w_{ij}^k})\, y_j^k \le h_i^t, \quad i \le I^k, i = 1,2,3--------(3)$$

$$x_j \ge 0 \quad y_j^k \ge 0 --------------------------(4)$$

We associate first stage constraints (2) with the dual variable ε_i and the second stage constraints (3) with the dual variable φ_i^k then the dual of extensive form with the interval coefficients is written as

$$\sum_{\substack{i=1\\ i\in I^0}}^{n} (\underline{b}_i, \overline{b_i})\varepsilon_i + \sum_{k=1}^{m} \sum_{\substack{i=1\\ i\in I^k}}^{n} (\underline{hi}^k, \bar{h}i^k)\, \varphi_i^k, i = 1,2,---- (5)$$

S.t

$$\sum_{\substack{i\in I^0\\ j=1}}^{n} (\underline{a}_{ij}, \bar{a}_{ij})\varepsilon_i + \sum_{k=1}^{m} \sum_{\substack{j=1\\ i\in I^k}}^{n} t_{ij}^k \varphi_i^k \ge Cj, j \in J^0 = 1,2,---- (6)$$

$$\sum_{j=1}^{n} (\underline{w_{ij}^k}, \overline{w_{ij}^k})\, \varphi_i^k \ge p^k q_{jk}, \quad j \in J^k, i = 1,2,3--------(7)$$

$$\varepsilon_i \ge 0 \quad \varphi_i^k \ge 0 ------------------ (8)$$

LP optimality conditions require that at optimality, a primal solution

$\{x,y^k, k \in K\}$ is feasible to the constraints 2 to 4 and a corresponding dual solution $\{\varepsilon_i, \varphi_{i\ \forall k\in K}^k\}$ is feasible to constraints (6) to (8). It is noted that the following complementary slackness conditions are satisfied.

$$\forall_i \in I^o, if \sum_{\substack{j=1\\ j\in J^o}}^{n} \left(\underline{a_{ij}}, \overline{a_{ij}}\right) xj < bithen\varepsilon_i = 0$$

$$\forall_k \in ki\epsilon I^k, if$$

$$\sum_{\substack{j=1\\ j\in J^o}}^{n} t_{ij}^k x_j + \sum_{j\epsilon J^k} (\underline{w}_{ij}^k, \overline{w}_{ij}^k)\, y_j^k < \varphi_i^k\ then \varphi_i^k = 0$$

Let B^o denotes the set of binding constraints among the first stage constraints (2) with respect to an initial primal feasible solution ($x,y,k\in K$) and B^k, $k \in k$ be the set of binding constraints among the second stage constraints(3) based on the binding constraints, the complementary slackness conditions are written as $\epsilon_i = 0\, foralli \in I^o \backslash B^o$ For any $k \in K, \varphi_i^k = 0\ fori \in I^k \backslash B^k$ TSSLP is denoted as EF(d,q)', where c_j^1 are replaced by d_j^1 and $q_j^{k'} sarereplacedbyr_j^k$.

It is assured that $x^\wedge, (y^\wedge)^k, \forall k \in K)$ is an optional solution to EF(d,q^1) iff there exists a dual solution $(\epsilon_i, \varphi_i^k \forall k \in K)$ that satisfies the constraints from 6 to 8 with dj, q_j^k replaced r_j^k and the primal dual pair satisfies the complementary slackness conditions. On combining, the dual feasibility condition with the newly established complementary slackness conditions gives the following characterization of inverse feasible cost vectors for two stage stochastic linear programming

under weighted L norm, the problem is stated as Min

$$\sum_{j\in j^o} vj^o\left|d_j - c_j\right| + \sum_{k\in k}\sum_{j\in J^k} P_k^k v_j^k \left|p_j^k - q_j^k\right| ------(9)$$

$$\sum_{i\in B^0}(\underline{a}_{ij},\bar{a}_{ij})\varepsilon_i + \sum_{k\in K}\sum_{i\in B^k} t_{ij}^k \varphi_i^k \geq dj, j \in J^0 = 1,2, ---- (10)$$

and

$$\sum_{i\in B^k}^{n}(\underline{w_{ij}^k}, \overline{w_{ij}^k})\,\varphi_i^k \geq p^k r_j^k, \quad k \in k, j \in J^k = 1,2,3 ----(11)$$

$$\varepsilon_i \geq 0 \quad i \in I^o, \quad \emptyset_i^k \geq 0 \quad k \in k, i \in I^k ----------(12)$$

The coefficients $v_j^o, j \in J^o$ and $v_j^k, j \in J^k$ denote the weight vectors associated with the first and second stage respectively. In order to linearize this non linear objective, define the following terminologies.

In the first stage,

dj-cj = $\alpha_j^0 - \beta_j^0$, -------------- (13) , where $\alpha_j^0 \geq 0$ and $\beta_j^0 \geq 0\ \forall j \in J^0$

In the second stage ,

$r_j^k - q_j^k = \alpha_j^k - \beta_j^k$ ----------------(14) , where $\alpha_j^k \geq 0$ and $\beta_j^k \geq 0\ \forall k \in K, \forall j \in J^k$

The inverse two stage stochastic linear programming under the weighted L_1 norm is to minimize the first stage weighted absolute cost of perturbation plus the expected second stage weighted absolute cost of perturbation. Now, the inverse two stage stochastic linear programming with interval coefficients in extensive form is stated as under.

Min

$$\sum_{j\in J^0} v_j^0(\alpha_j^0 + \beta_j^0) + \sum_{k\in K}\sum_{j\in J^k} v_j^k p^k(\alpha_j^0 + \beta_j^0) --------- (15)$$

Subject to

$$\sum_{i\in B^0}(\underline{a}_{ij},\bar{a}_{ij})\varepsilon_i + \sum_{k\in K}\sum_{i\in B^k} t_{ij}^k\varphi_i^k - \alpha_i^k + \beta_j^0 \geq c_j, j \in J^0 ----------(16)$$

and

$$\sum_{i\in B^k}(\underline{w}_{ij}^k,\overline{w}_{ij}^k)\varphi_i^k - p^k\alpha_i^k + p^k\beta_j^k \geq p^k q_j^k, k \in K, j \in J^k --------- (17)$$

$$\varepsilon_i \geq 0, i \in B^0, \varphi_i^k \geq 0, k \in K, i \in B^k --------------- (18)$$

$$\alpha_j^0\beta_j^0 \geq 0\ j \in J^0, \alpha_j^k, \beta_j^k \geq 0, k \in K, j \in J^k ------------- (19)$$

By defining

$$c_j^{\varepsilon_i} = c_j - \sum_{i\in B^0}(\underline{a}_{ij},\bar{a}_{ij})\varepsilon_i - \sum_{k\in K}\sum_{i\in B^k} t_{ij}^k\varphi_i^k ----------(20)$$

and

$$c_j^{\emptyset_i^k} = q_j^k - \frac{1}{p^k}\sum_{i\in B^k}(\underline{w}_{ij}^k,\overline{w}_{ij}^k)\,\varphi_i^k ------------- (21)$$

Now, the inequalities (16) and are constructed as

$-\alpha_j^0 + \beta_j^0 \geq c_j^{\varepsilon_i}, j \in J^0$ −−−−−−−−−−−−−−−(22)

$-\alpha_j^k + \beta_j^k \geq c_j^{\emptyset k}, k \in K, j \in J^k$ −−−−−−−−−−−−−−−(23)

Consider three mutually exclusive cases in two different sets.

Set. I:

case(1)

$e_j^{\varepsilon_i} > 0$

$\alpha_j^0 = 0 \quad \beta_j^0 = c_j^{\varepsilon_i} \Rightarrow d_j = c_j - c_j^{\varepsilon_i}$

Case : 2

$c_j^{\varepsilon_i} < 0$

$\alpha_j^0 = \beta_j^0 = 0 \Rightarrow d_j = c_j$

case:3

$c_j^{\varepsilon_i} = 0$

$c_j^{\varepsilon_i} = 0 \Rightarrow \alpha_j^0 = B_j^0 = 0 \Rightarrow d_j = c_j$

Set: II

Case 4

$c_j^{\emptyset_i^k} > 0$

$\alpha_j^k = 0 \quad \beta_j^k = c_j^{\phi_i^k} \Rightarrow r_j^k = q_j - c_j^{\emptyset_i^k}$

Case : 5

$c_j^{\emptyset_i^k} < 0$

$\alpha_j = \beta_j^k = 0 \Rightarrow (r_j^k) = q_j^k$

Case : 6

$c_j^{\emptyset_i^k} = 0,$

$c_j^{\emptyset_i^k} = 0 \Rightarrow \alpha_j^k = \beta_j^k = 0 \Rightarrow (r_j^k) = q_j^k$

3. Decomposition Approaches:

The inverse 2SSLP problem with interval coefficients of constraints given in the expression (15) whose constraints (16) to (19) grows with the number of scenarios \k1. This problem motivates to undergo decomposition approaches. Such as Dantzing-Wolfe (1961) decomposition relaxation [Fisher (1985)]. Or Lagrangian may be utilized. Further more $\{\alpha^k, \beta^k \forall k \in K\}$ do not appear in J^o constraints and $(\varepsilon_i, \alpha^0, \beta^0)$do not appear J^k ,$k \in k$. constraints. Therefore, the problem is relatively easy to solve when only these variables are present So$\varphi_i^k \forall\, k \in K$ are the linking variables for which Benders (1962) decomposition is appropriate.

3.1. Dantzing – Wolfe Decomposition:

Dantzing – Wolfe (1961) decomposition is an application of inverse projection to linear program with special structure. With Dantzing-Wolfe decomposition, the LP is decomposed into two set of constraints as easy and hard. Rather than solving the LP with all the variables present, the variables are added as needed. This approach uses column generation.

Observe that if one views the $(P_1,\ldots\ldots\ldots Q_k)$Variables as "first stage" variable, the resulting inverse 2SSLP may be interpreted as a TSSLP as well. So, for the inverse TSSLP, J^k, $k \in K$ are easy constraints J^0 are hard constraints, optimizing the sub problem by solving K independent LP's may be preferable to solving the entire system.

Let Let $(\phi_k, \alpha^k, \beta^k)^{\prime}$-------$(\phi_k, \alpha^k, \beta^k)^{qk}$

be the entre me points in the easy polyhedron as a combination of their extreme points and entre me rays. Substituting these into the hard constraint set and into the objective function gives the following Dantzing-Wolfe problem.

Min

$$\sum_{j\in J^0} V_j^0\left(\alpha_j^0+\beta_j^0\right)+\sum_{k\in K}\sum_{j\in J^k} v_j^k p^k\left[\sum_{s=1}^{r_k} z_s^k\left[\left(\alpha_j^k\right)^s+\left(\beta_j^k\right)^s\right]\right]------------(24)$$

s.t

$$\sum_{i\in B_0}\left(\underline{a}_{ij},\overline{a}_{ij}\right)\varepsilon_i-\sum_{k\in K}\sum_{i\in B^k} t_{ij}^k\sum_{s=1}^{r_k} z_s^k\left[\left(\varphi_i^k\right)^s-\alpha_j^0+B_j^0\geq c_j\ j\in J^0------(25)\right.$$

$$\sum_{s=1}^{q_k} z_s^k=1, k=1,2,\ldots\ldots\ldots.., k,-----------------(26)$$

$$z_s^k\geq 0, k=1,2\ldots\ldots\ldots\ldots\ldots.k.\ s=1,2\ldots\ldots\ldots\ldots\ldots\ldots.r_k\text{----------------}(27)$$

The above stated objective function given in (24) and its constraints (25) are coupling constraints while constraints (26) are convexity rows. It is noted that problem (24) has fewer constraints (25)-(27) than the original problem (15) whose constraints (16) to (18). The number of variables in the Dantzing-Wolfe problem is larger than that in the original problem, since the points in the easy polyhedral are rewritten in terms of extreme points and extreme rays.

Hence the required restricted problem can be constructed with a small sub set (^(k)) of the columns in the full problem as follows:

Min

$$\sum_{j\in J^0} V_j^0\left(\alpha_j^0+\beta_j^0\right)+\sum_{k\in K}\sum_{j\in J^k} v_j^k p^k\left[\sum_{st^\wedge(k)}^{r_k} z_s^k\left[\left(\alpha_j^k\right)^s+\left(B_j^k\right)^s\right]\right]-----(28)$$

s.t

$$\sum_{i\in B_0}\left(\underline{a}_{ij},\overline{a}_{ij}\right)\varepsilon_i-\sum_{k\in K}\sum_{i\in B^k} t_{ij}^k\sum_{st^\wedge(k)}^{r_k} z_s^k\left[\left(\varphi_i^k\right)^s-\left(\alpha_j^0+B_j^0\right)\geq c_j\ j\in J^0,(u)--(29)\right.$$

$$\sum_{s\in^\wedge(k),s\leq q_k} z_s^k=1, k=1,2\ldots\ldots\ldots\ldots\ldots.k,\left(u_0^k\right)------------(30)$$

$$z_s^k\geq 0, k=1,2\ldots\ldots\ldots\ldots\ldots.k.\ s\in{}^\wedge(k)-------(31)$$

If the reduced costs of all variables in the restricted problem are non negative, the optimal solution to the full master. Otherwise, the column with the minimum reduced cost, is added to the restricted master,in this stage in order to obtain minimum reduced cost, we have to solve the Dantzigwolfe sub problem. In this case these are K sub problem to solve instead of full problem.

consider the optimal dual multipliers is (u,u_0^k) are restricted master problem, so that the kth($k \in K$)sub problem can be written as

Min

$$\left[\sum_{j\in J^k} V_j^k p^k(\alpha_j^k + \beta_j^k) - \sum_{i\in B^k} t_{ij}^k \varphi_i^k\right] u_j - u_0^k - - - - - - - - - - - - - (32)$$

s.t

$$\textstyle\sum_{i\in B^k}(\underline{w}_{ij}^k, \overline{w}_{ij}^k)\emptyset_i^k - p^k\alpha_j^k + p^k\beta_j^k \geq p^k q_j^k \; j \in J^k - - - - - - - - -(34)$$

The Dantzig wolf algorithm terminate when a optimum solution of the sub problem is greater than or equal to zero for all k$\in K$ otherwise, the valuable with the minimum reduced cost is added to the restricted master problem.

3.2.Benders Decomposition :

Benders decomposition is a technique in mathematical programming that allows the solution of very large linear programming problems that have a special block structure. This structure often occurs in application such as stochastic programming.

Benders(1962) has proposed that decomposition variables are divided into two sets as ‘easy” and complicating (linking) variables. The problem with only easy variable is relatively easy to solve. Bender’ decomposition projects out easy variables and then solves the remaining problem with linking variables.

In this algorithm, easy variables are replaced with more constraints. The number of constraints is exponential in the number of easy variables. However constraints are added as needed basis which overcomes the problem of an exponential number of constraints.

The original problem given in (15) to (19) may be modified based on benders rule as

Min Z^0

Subject to

$$z^0 - \sum_{k\in K}\sum_{j\in J^k} V_j^k p^k(\alpha_j^k + \beta_j^k) - \sum_{j\in J^0} v_j^0(\alpha_j^0 + \beta_j^0) \geq 0 \quad - - - - - (35)$$

$$\sum_{i\in B_0}(\underline{a}_{ij}, \overline{a}_{ij})\varepsilon_i - \alpha_j^0 + \beta_j^0 \geq c_j - \sum_{k\in K}\sum_{i\in B^k} t_{ij}^k\varphi_i^k, j \in J^0 - - - - - - - (36)$$

$$-p^k\alpha_j^k + p^k\beta_j^k \geq p^k q_j^k - \sum_{i\in B^k}(\underline{w}_{ij}^k, \overline{w}_{ij}^k)\varphi_i^k, k \in K, j \in J^k - - - - - -(37)$$

By introducing optimal dual variable (u_j^0, u_j^k) in the constrains (35) to (37) the resultant constraints are given as

Min z^0

Subject to

$$z^0 \geq \sum_{j\in J^0} (u_j^{o_i})^T\,(c_j - \sum_{k\in K}\sum_{i\in B^k} t_{ij}^k \varphi_i^k + \sum_{k\in K}\sum_{j\in J^k} (u_j^{ki})^T (p^k q_j^k - \sum_{i\in B^k} (\underline{w}_{ij}^k, \overline{w}_{ij}^k)\,, i = 0,1..q \ldots (38)$$

$$0 \geq \sum_{j\in J^0} (u_j^{o_i})^T\,(c_j - \sum_{k\in K}\sum_{i\in B^k} t_{ij}^k \varphi_i^k + \sum_{k\in K}\sum_{j\in J^k} (u_j^{ki})^T (p^k q_j^k - \sum_{i\in B^k} (\underline{w}_{ij}^k, \overline{w}_{ij}^k)\,, i = q+1 \ldots r\,. (39)$$

Since BMP has a lot of constraints to optimize directly, the basic idea behind Benders decompression is to solve to relaxed master problem with only a small subset of constraints. If there is some constraints in the BMP that is violated by the solution to the relaxed master problem, the violated constraints is added to the master problem. The under stated benefits sub problem (BSP) for the inverse extensive form is solved for obtaining the violated constraints.

Max

$$\sum_{j\in J^0} (u_j^{o_i})^T\,(c_j - \sum_{k\in K}\sum_{i\in B^k} t_{ij}^k \varphi_i^k + \sum_{k\in K}\sum_{j\in J^k} (u_j^{ki})^T (p^k q_j^k - \sum_{i\in B^k} (\underline{w}_{ij}^k, \overline{w}_{ij}^k)\,, \varphi_i^k - - - - - (40)$$

st

$$\sum_{j\in J^0} (\underline{a}_{ij}, \overline{a}_{ij}) u_j^0 \leq 0 \qquad i \in B^0 - - - - - - - - - - - - - - - - (41)$$

$$u_i^0 \leq v_j^0 \qquad j \in J^0 - - - - - - - - - - - - - - - - - - (42)$$

$$u_j^k \leq p^k v_j^k, k \in K, j \in J^k - - - - - - - - - - - - - - (43)$$

$$u_j^0 \geq \ \ k \in K, j \in J^0, j \in J^k, u_j^k \geq 0 - - - - - - - - - - -(44)$$

If the solution u^i BSP is either extreme point or extreme direction then accordingly the constraint type (38) or (39) is added to the relaxed master problem. Blender decomposition algorithm iteratively generate upper and lower bounds on the optimal solution value to the original problem and is terminated when the difference between the bounds is less than or equal to a pre specified value.

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