An Inventory Model for Optimum Ordering Interval for Constant Deterioration with Discrete in Time with Shortages

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ABSTRACT:

In this paper an inventory model for constant deterioration with discrete in time with shortage had been proposed, demand rate increase by a constant percentage during each time interval the proposed model will be constrained bionomial geometric programming model with constant Deterioration and shortages. This implies that for any exponentially growing quantity, the larger the increase in quantity, the faster it grows. A numerical analysis of the proposed model has been presented. Production rate is considered as finite and approximation procedure is used to solve the model

KEYWORDS:

Deterioration, shortage, inventory control, Demand Rate, Periodic, Exponential Growth,

INTRODUCTION:

In our country the harvest of food grain like paddy, wheat etc is periodic. There are large number of landless people are there in rural areas, so the demand will be constant throughout the years. During the lasts few year inventory problems involving time variables demand pattern received, attention of several researchers. [6]Discuss a finite horizon inventory problems with variable demand [4] First solved the classical no shortage inventory problems analytically with linear trend in demand over a finite time horizon.

ASSUMPTION & NOTATIONS

(1) The demand rate is assumed as price dependent d (p) and λ is a constant fraction.

(2) The replenishment rate is infinite.

(3) The lead time is zero and shortages are allowed and back logged.

(4) I (t) is the inventory at time 't'

(5) I_w (t) is the inventory without decay at time.

(6) Q_T is the order quantity and Q_1 is the inventory which is increased.

(7) C is the unit purchasing cost, h is the holding cost, C_1 is ordering cost per order and C_2 is shortage cost.

THE MODEL:

The difference enquation, which describes inventory leavel at time t of the system, is

$$\Delta I(t) = -\lambda I(t) - d(p) \quad \text{.............(1)}$$

Or

$\Delta I(t) + \lambda I(t) = -d(p)$

Or $I(t+I) - I(t) + \lambda I(t) = -d(p)$

$I(t+I) - (I-\lambda)I(t) = -d(p)$

Multiplying by $(1-\lambda)^{-(t+1)}$ both sides

$(1-\lambda)^{-(t+1)}I(t+1) - (1-\lambda)^{-t}I(t) = -d(p)(1-\lambda)^{-(t+1)}$

Or

$\Delta\{1-\lambda)^{-1}I(t)\} = -d(p)(1-\lambda)^{-(t+1)}$

Or $(1-\lambda)^{-1}$ I(t) = $\Delta^{-1}(p)(1-\lambda)^{-(t+1)} \Rightarrow \Delta^{-1}d(p)(1-\lambda)^{-(t+1)}$

Or $(1-\lambda)^{-1}$ I(t) = -d(p)[$(1-\lambda)^{-(t+1)} + (1-\lambda)^{-t} + (1-\lambda)^{-(t-1)} + \ldots..] + C$

All = 0, I(t) = I(0)

$(1-\lambda)^{-1}I(t) = \frac{-d(p)(I-\lambda)^2}{\lambda} + C$

$\therefore I(t)(1-\lambda)^{-1} = I(0) + \frac{d(p)}{\lambda} - \frac{d(p)}{\lambda}(1-\lambda)^{-t}$ …………..(2)

(2) t = 0,1,2,…………T

In case of no decay, the difference equation

$\Delta I_w(t) = -d(p)$ …………..(3)

On using initial condition, Iw(t) = I(0)at t = 0

$I_w(t) = -d(p)t + I(0)$ ……………..(4)

The stock loss due to decay is

$Z(t) = I_w(t) - I(t)$ ……………..(5)

Or Z(t) = I(0)-d(p).tI(t) ……………..(6)

Substituting the value of I(0),from (2)

Z(t) = $\{(1-\lambda)^{-1}I(t) - \frac{d(p)}{\lambda} + \frac{d(p)}{\lambda}(1-\lambda)^{-1}\} - d(p)t - I(t)$

Z(t) = $\{1(t) + -\frac{d(p)}{\lambda}\}\{(1-\lambda)^{1} - 1\} - d(p)t$

From (2)

$$Z(t) = \frac{d(p)}{\lambda}\{1-\lambda)^{-T}-1\} - d(p).T \quad \ldots\ldots\ldots\ldots(7)$$

For t = T

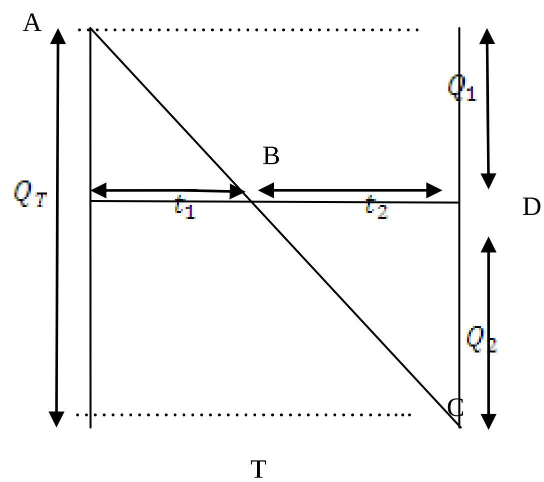
I(T) being zero in case of instantaneous replenishment

Shortage quantity:-

$$\because Q_T = Q_1 + Q_2 \quad \ldots\ldots\ldots\ldots\ldots(8)$$

By similar triangles

$$\frac{t_1}{T} = \frac{Q_1}{Q_1} \; and \; \frac{t_2}{T} = \frac{Q_2}{Q_1} \quad \ldots\ldots\ldots\ldots\ldots(9)$$



By ΔBCD shortage quantity

$$S(Q) = {}^1/_2\, Q_2.t_2$$

Form (9) $t_1 = \frac{T}{Q_T}Q_1, \quad t_2 = \frac{T}{Q_T}Q_2$

Shortage quantity S(Q) $= \frac{1}{2}(Q_T - Q_1)\frac{T}{Q_T}Q_2$

$$S(Q) = \frac{1}{2}\frac{T}{Q_T}(Q_T - Q_1)^2$$

Or $S(Q) = \frac{1}{2}T(Q_T - 2Q_1)$ (10) (Neglecting higher powers)

Order quantity :-

$$Q_T = Z(T) + d(b).T + S(Q)$$

$$Q_T = \frac{d(p)}{\lambda}[(1-\lambda)^{-T}-1] - d(p).T + d(p)T + \frac{1}{2}\frac{T}{Q_T}(Q_T - Q_1)^2$$

Or $\quad Q_T = \frac{d(p)}{\lambda}[(1-\lambda)^{-T}-1] + \frac{1}{2}\frac{T}{Q_T}(Q_T - Q_1)^2$

Or $\quad Q_1 - \frac{1}{2}\frac{T}{Q_T}(Q_T - Q_1)^2 = \frac{d(p)}{\lambda}[(1-\lambda)^{-T}-1]$

$$Q_T = \frac{2}{2-T}[\frac{d(p)}{\lambda}\{(1-\lambda)^{-T}-1\} - TQ_1]$$

In cash of instantaneous I(0) = Q_T

From (2)

$$(1-\lambda)^{-t}I(t) - \frac{d(p)}{\lambda} + \frac{d(p)}{\lambda}(1-\lambda)^{-t} = \frac{2}{2-T}\left[\frac{d(p)}{\lambda}\{(1-\lambda)^{-T}-1\} - TQ_1\right]$$

Or, $I(t) = \frac{d(p)}{\lambda}\left[-(1-\lambda)^{t}\frac{T}{2-T} + \frac{2}{2-T}(1-\lambda)^{t-T}1 - \frac{2T}{2-T}(1-\lambda)^{t}Q_1 - 1\right]$ –(12)

The average number of units in inventory per unit time during a cycle is

$$I_{\wedge}I(T) = \frac{d(p)}{\lambda(T+1)}\sum_{t=0}^{T} I(t)$$

$$\therefore I_{\wedge}I(T) = \frac{d(p)}{\lambda(T+1)}\left[\frac{T}{T-2}(1-\lambda)^{T} - \frac{2}{2-T}(1-\lambda)^{-T} + \frac{2T}{2-T} - \frac{2T}{2-T}Q_1[(1-\lambda)^{T}-1]\right] \text{--(13)}$$

Total cost C(T,P) = $\frac{C_1}{T} + \frac{C}{T}Q_T + h.I_{\wedge}(T) + C_2 S(Q)$

C(T,P) =

$$\frac{C_1}{T} + \frac{C}{T}\left[\frac{2}{2-T}\frac{d(p)}{\lambda}\{(1-\lambda)^{-T}-1\} - \frac{2T}{2-T}Q_1\right]$$

$$+h\left[\frac{d(p)}{\lambda(T+1)}\left\{\frac{T}{T-2}(1-\lambda)^{-T} - \frac{2}{2-T}(1-\lambda)^{-T} + \frac{2T}{2-T}\right\} - \frac{2T}{2-T}Q_1\{(1-\lambda)^{T}-1\}\right] +$$

$$\frac{C_2T}{2}\left[\frac{1}{2-T}\frac{d(p)}{\lambda}\{(1-\lambda)^{T}-1\} - \frac{2}{2-T}Q_1\right] \qquad \text{---------(14)}$$

Since T must be a non negative integer, the condition for C(T,P) to have absolute minimum at $T = T^*$ is

$$\Delta c(T^* - 1, P) \le 0 \le \Delta C(T^*, P) \qquad ----(15)$$

Where $\Delta C(T,P) = C(T+1,P) - C(T,P)$

And $\Delta C(T^*P) = (T^*1,P) - C\,(T,P) \ge 0$

Or $\phi(T^*) \le \frac{C_1\lambda}{d(p)T^*(T^*+1)} + \frac{2C\lambda Q_1}{d(p)(2-T^*)} + \frac{2h\lambda Q_1}{d(p)}\left[\frac{T^*+1}{1-T^*}\{(1-\lambda)^{T^*+1}-1\} - \frac{T^*}{2-T^*}\{(1-\lambda)^{T^*}-1\}\right]$

$+\frac{4C_2\lambda Q_1}{d(p)(1-T^*)(2-T^*)} \le \phi(T^*+1)$

8.5 NUMERICAL EXAMPLE

C1= Rs. 300 per order

H= 0.50 Per unit per week

d(p) = 50 - .5p.

let C_2 =0.5, Q_1= 3, units, then

P	$\lambda = 0.02$	C	C(T,P) without shortage	With shortage
40	$T^* = 5$	10	443	405.52
60	$T^* = 6$	10	275	170

putting these value in equation (17) , we get T^* = 5 and 6

CONCLUSION

In this study we present an inventory model with price dependent demand and constant rate of deterioration with shortages. The model developed in this paper can be extended incorporating more realistic assumption such as stochastic nature of demand. Variable rate of deterioration and demand as a function of time and price both.

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