Analysis of the Nonlinear Singular Systems using Adomian Decomposition Method

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Abstract — *The Adomian Decomposition Method (ADM) is used to analysis the nonlinear singular systems of time-invariant and time-varying cases [14]. The obtained discrete solutions using ADM and Single-term Haar wavelet series (STHW) are compared with the exact solutions of the nonlinear singular systems of time-invariant and time-varying cases. It is found that the solution obtained using ADM is closer to the exact solutions of the nonlinear singular systems of time-invariant and time-varying cases. Error graphs for discrete and exact solutions are presented in a graphical form to highlight the efficiency of this method. This ADM can be easily implemented in a digital computer and the solution can be obtained for any length of time.*

Keywords — *Nonlinear singular systems of time-invariant and time-varying cases, Singular systems, Adomian Decomposition Method, Single-term Haar wavelet series.*

I. Introduction

There is an ever widening use of simulation, design and analysis packages. Frequently many mathematical models appearing as appearing as mixed system of nonlinear algebraic and differential equations or what is called the nonlinear singular systems. This has been one of the driving forces behind the work on singular systems over the last decade.

The development of singular nonlinear systems has been studied by some researchers [2, 5-6]. But no were closed-form solution available in that paper. In some analysis of neural networks, both singular systems [7] and nonlinear systems [19] have been used. The multipliers and algebraic interconnections between singular systems and nonlinear systems are allowed in dynamical systems. For singular bilinear systems, Lewis *et al*. [9] have been discussed extensively in the literature. However, the solution given by Lewis *et al.* [10] only applies for the time-invariant case.

In this paper we developed numerical methods for addressing nonlinear singular systems of time-invariant and time-varying cases by an application of the Adomian Decomposition Method which was studied by Sekar and team of his researchers [12-13, 15-17]. Recently, Sekar *et al*. [14] discussed the nonlinear singular systems of time-invariant and time-varying cases using STHW. In this paper, the same nonlinear singular systems of time-invariant and time-varying cases problem was considered (discussed by Sekar *et al.* [14]) but present a different approach using the Adomian Decomposition Method with more accuracy for nonlinear singular systems of time-invariant and time-varying cases.

II. Adomian Decomposition Method

Suppose k is a positive integer and $f_1, f_2, \ldots, f_k$ are k real continuous functions defined on some domain G. To obtain k differentiable functions $y_1, y_2, \ldots, y_k$ defined on the interval I such that $(t, y_1(t), y_2(t), \ldots, y_k(t)) \in G$ for $t \in I$.

Let us consider the problems in the following system of ordinary differential equations:

$$\frac{dy_i(t)}{dt} = f_i(t, y_1(t), y_2(t), \ldots, y_k(t)) \;,$$

$$y_i(t)\,|_{t=0} = \beta_i \qquad (1)$$

where β_i is a specified constant vector, $y_i(t)$ is the solution vector for $i = 1, 2, \ldots, k$. In the decomposition method, (1) is approximated by the operators in the form: $Ly_i(t) = f_i(t, y_1(t), y_2(t), \ldots, y_k(t))$ where L is the first order operator defined by $L = d/dt$ and $i = 1, 2, \ldots, k$.

Assuming the inverse operator of L is L^{-1} which is invertible and denoted by $L^{-1}(.) = \int_{t_0}^{t} (.)\, dt$, then applying L^{-1} to $L\, y_i(t)$ yields

$$L^{-1}Ly_i(t) = L^{-1} f_i(t, y_1(t), y_2(t), \ldots, y_k(t))$$

where $i = 1, 2, \ldots, k$. Thus

$$y_i(t) = y_i(t_0) + L^{-1} f_i(t, y_1(t), y_2(t), \ldots, y_k(t)) .$$

Hence the decomposition method consists of representing $y_i(t)$ in the decomposition series form given by

$$y_i(t) = \sum_{n=0}^{\infty} f_{i,n}(t, y_1(t), y_2(t), \ldots, y_k(t))$$

where the components $y_{i,n}$, $n \geq 1$ and $i = 1, 2, \ldots, k$ can be computed readily in a recursive manner. Then the series solution is obtained as

$$y_i(t) = y_{i,0}(t) + \sum_{n=1}^{\infty} \{L^{-1} f_{i,n}(t, y_1(t), y_2(t), \ldots, y_k(t))\}$$

For a detailed explanation of decomposition method and a general formula of Adomian polynomials, we refer reader to [Adomian 1].

III. Nonlinear Singular Systems

The time-invariant non-linear singular systems of the form is considered

$$K\dot{x}(t) = Ax(t) + f(x(t)) \tag{5.1}$$

with $x(0) = x_0$, where K is an $n \times n$ singular matrix, A is an $n \times n$ matrix, $x(t)$ is an n - state vector and f is an "n" vector function. In order to make the above system (5.1) as time-varying case some of the components (not necessarily all the elements) in the system (5.1) are converted, as time-varying and then the system will be of the following form

$$K(t)\dot{x}(t) = A(t)x(t) + f(x(t)) \tag{5.2}$$

with $x(0) = x_0$, where this $K(t)$ is an $n \times n$ singular matrix, $A(t)$ is an $n \times n$ matrix, $x(t)$ is an n - state vector and *f* is an "n" vector function.

IV. Numerical Examples for Time-Invariant

Considering the following time-invariant nonlinear singular system (Campbell [3,4], Lin and Yang [11])

$$K = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \text{ and } f(x(t)) = \begin{bmatrix} 0 \\ -x^2 \end{bmatrix}$$

with initial condition $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

The exact solutions are $x_1(t) = -t$, $x_2(t) = \dfrac{t^2}{2}$

Time t	Approximate solution x_1-values				
	Exact Solutions	STHW Solutions	STHW Error	ADM Solutions	ADM Error
0	0	0	0	0	0
0.25	-0.25	-0.25002	0.00002	-0.2499998	2E-07
0.5	-0.50	-0.50007	0.00007	-0.4999993	7E-07
0.75	-0.75	-0.75009	0.00009	-0.7499991	9E-07
1	-1.00	-1.00014	0.00014	-0.999986	0.000014
1.25	-1.25	-1.25017	0.00017	-1.249983	0.000017
1.5	-1.50	-1.50019	0.00019	-1.499981	0.000019
1.75	-1.75	-1.75022	0.00022	-1.749978	0.000022
2	-2.00	-2.00026	0.00026	-1.999974	0.000026

Table 1 Solutions for time-invariant system for various values of "x_1".

The results (approximate solutions) obtained using STHW and ADM (with step size time $t = 0.25$) along with the exact solutions and its absolute errors between them are calculated and are presented in tables 1-2. To highlight the efficiency of the ADM and to distinguish the effect of the errors in accordance with the exact solutions, a graphical representation is presented in Fig. 1-2 for selected values of "x_1" and "x_2", using three-dimensional effect. In the Fig. 1-2 Series-1 mention the STHW Error and Series-2 mention ADM Error.

Time t	Approximate solution x_2-values				
	Exact Solutions	STHW Solutions	STHW Error	ADM Solutions	ADM Error
0	0	0	0	0	0
0.25	0.031	0.031002	0.000002	0.031	2E-08
0.5	0.125	0.125007	0.000007	0.1250001	7E-08
0.75	0.281	0.281009	0.000009	0.281009	0.000009
1	0.500	0.500014	0.000014	0.5000014	0.0000014
1.25	0.781	0.781017	0.000017	0.7810017	0.0000017
1.5	1.125	1.125019	0.000019	1.1250019	0.0000019
1.75	1.531	1.531022	0.000022	1.5310022	0.0000022
2	2.000	2.000026	0.000026	2.0000026	0.0000026

Table 2 Solutions for time-invariant system for various values of "x_2".

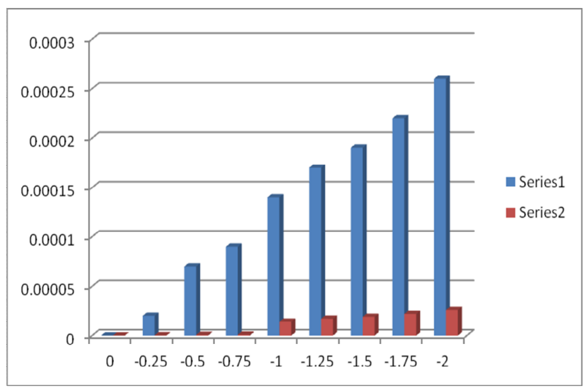

Fig.1 Error graph for "x_1" at various time intervals

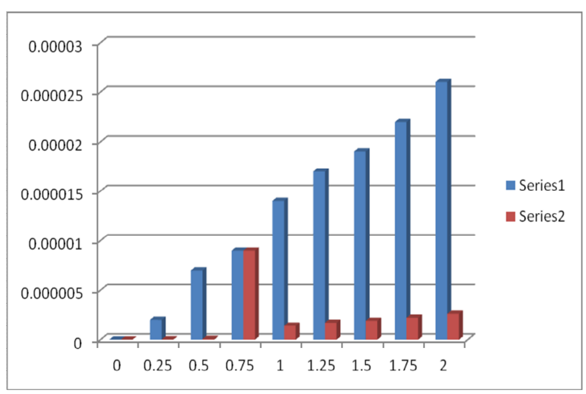

Fig.2 Error graph for "x_2" at various time intervals

V. NUMERICAL EXAMPLES FOR TIME-VARYING

Considering the time-varying nonlinear singular system of the following form (Hsiao and Wang [8] and Sepehrian and Razzaghi [18])

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & t^2 \\ 0 & 0 & 0 \end{bmatrix} \dot{x}(t) = \begin{bmatrix} tx_1(t)+x_2(t) \\ \exp(t)x_1(t)x_2(t) \\ x_2(t)(x_1(t)+x_3(t)) \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2t^2\exp(-t) \\ 0 \end{bmatrix},$$

with initial condition

$$x(0) = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}.$$

The exact solutions are

$$x(t) = \begin{bmatrix} 2\exp(-t)(1-t) \\ t^2\exp(-t) \\ -2\exp(-t)(1-t) \end{bmatrix}$$

Time t	Approximate solution x_1-values				
	Exact Solutions	STHW Solutions	STHW Error	ADM Solutions	ADM Error
0	2	2	0	2	0
0.25	0.778801	0.778805	4E-06	0.778801	4.00E-08
0.5	0	0.000006	6E-06	6E-08	6.00E-08
0.75	-0.47236	-0.47239	3E-05	-0.4723597	3.00E-07
1	-0.73575	-0.73579	4E-05	-0.7357496	4.00E-07
1.25	-0.85951	-0.85956	5E-05	-0.8595095	5.00E-07
1.5	-0.89252	-0.89258	6E-05	-0.8925194	6.00E-07
1.75	-0.86886	-0.86893	7E-05	-0.8688593	7.00E-07
2	-0.81201	-0.81209	8E-05	-0.8120092	8.00E-07

Table 3 Solutions for time-varying system at various values of "x_1".

Time t	Approximate solution x_2-values				
	Exact Solutions	STHW Solutions	STHW Error	ADM Solutions	ADM Error
0	0	0	0	0	0
0.25	0.048675	0.048676	1E-06	0.048675	1.00E-08
0.5	0.151632	0.151634	2E-06	0.151632	2.00E-08
0.75	0.265706	0.265709	3E-06	0.265706	3.00E-08
1	0.367879	0.367883	4E-06	0.367879	4.00E-08
1.25	0.447663	0.447667	4E-06	0.447663	4.00E-08
1.5	0.502042	0.502048	6E-06	0.5020421	6.00E-08
1.75	0.532182	0.532189	7E-06	0.5321821	7.00E-08
2	0.541341	0.541349	8E-06	0.5413411	8.00E-08

Table 4 Solutions for time-varying system at various values of "x_2".

Time t	Approximate solution x_3-values				
	Exact Solutions	STHW Solutions	STHW Error	ADM Solutions	ADM Error
0	-2	-2	0	-2	0
0.25	-0.77880	-0.77881	1E-05	-0.7787999	1.00E-07
0.5	0	0	0	0	0
0.75	0.472366	0.472367	1E-06	0.472366	1.00E-08
1	0.735758	0.735760	2E-06	0.735758	2.00E-08
1.25	0.859514	0.859517	3E-06	0.859514	3.00E-08
1.5	0.892521	0.892525	4E-06	0.892521	4.00E-08
1.75	0.868869	0.868876	7E-06	0.8688691	7.00E-08
2	0.812011	0.812019	8E-06	0.8120111	8.00E-08

Table 5 Solutions for time-varying system at various values of "x_3".

The results (approximate solutions) obtained using STHW and ADM (with step size time $t = 0.25$) along with the exact solutions and the absolute errors between them are calculated and presented in tables 3-5. To highlight the efficiency of the ADM and to distinguish the effect of the errors in accordance with the exact solutions, a graphical representation is presented in Fig. 3-5 for selected values of "x_1, x_2" and "x_3", using three-dimensional effect. In the Fig. 3-5 Series-1 mention the STHW Error and Series-2 mention ADM Error.

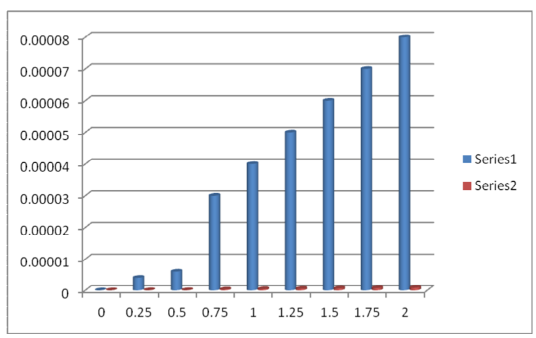

Fig.3 Error graph for "x_1" at various time intervals for time varying case

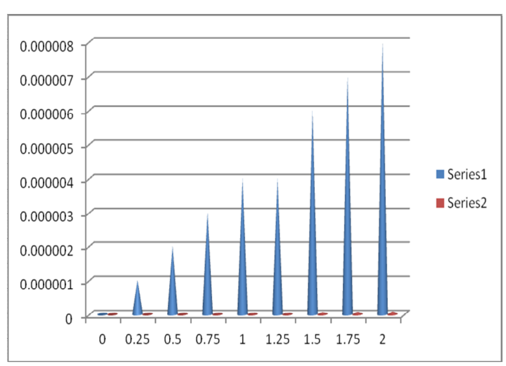

Fig.4 Error graph for "x_2" at various time intervals for time varying case

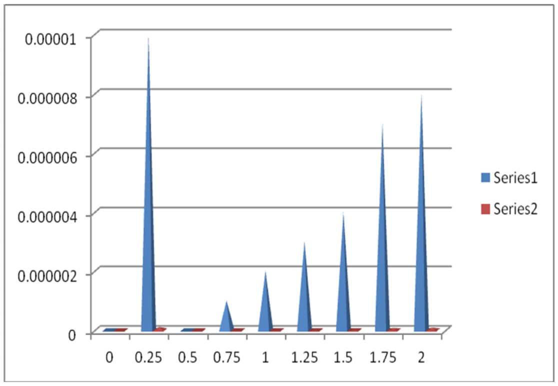

Fig.5 Error graph for " x_3 " at various time intervals for time varying case

VI. Conclusions

The obtained results (discrete solutions) of the nonlinear singular systems for time-invariant and time-varying cases show that the ADM works well for finding the state vector. From tables 1–5, we can observe that for most of the time intervals, the absolute error is less (almost no error) in the ADM when compared to the STHW, which yields a small error, along with the exact solutions. From Fig. 1–5, one can predict that the error is much less in the ADM when compared to the STHW methods discussed by Sekar *et al*. [14].

Hence the ADM is more suitable for studying the time-invariant and time-varying nonlinear singular systems.

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