

# Inverse Complementary Domination Graph

S.Pethanachi Selvam<sup>#1</sup>, S.Padmashini<sup>\*2</sup>

<sup>#1</sup> Associate Professor, Department of Mathematics, The Standard Fireworks Rajaratnam College for Women, Sivakasi-626123, TamilNadu, India.

<sup>\*2</sup> II M.Sc, Department of Mathematics, The Standard Fireworks Rajaratnam College for Women, Sivakasi-626123, TamilNadu, India.

**Abstract** — In this paper, we introduce Complementary Dominating Set and Inverse Complementary Dominating Set. We prove some properties connecting complementary domination number and Inverse complementary domination number. We also prove the bounds of the inverse complementary domination number and the relation between complementary domination number and inverse domination number.

**Keywords** — Complementary dominating set, inverse complementary dominating set.

## I. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Let  $G=(V,E)$  be a graph where the symbols  $V$  &  $E$  denote the vertex set and edge set. For all other terminology and notations, we follow Harary [1] and the definitions related to dominations are referred from T.W.Haynes [2].

V.R. Kulli [3,4] introduced the co-total dominating number of a graph and M.H. Muddebihal[5] introduced Inverse Line Domination graph.

By the motivation of the papers, we introduce the complementary domination number, total complementary domination number, co-total complementary domination number, connected complementary domination number and inverse complementary domination number.

In this paper, we prove the bounds of inverse complementary domination number of  $G$  and the relation between complementary domination number and inverse domination number of  $G$ .

## II. DEFINITIONS

### Definition 2.1

A set  $K \subset V(\overline{G})$  is said to be a Complementary Dominating Set (shortly written by CDS), if every vertex in  $V-K$  is adjacent to some vertex in  $K$ . The minimum cardinality of vertices in such a set is called Complementary Domination number of  $\overline{G}$  and is denoted by  $\gamma(\overline{G})$ . A graph having CDS is called Complementary domination graph.

### Definition 2.2

Let  $G$  and  $\overline{G}$  be connected graphs. A complementary dominating set  $K$  is called total complementary

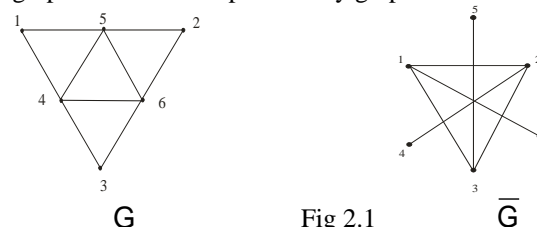
dominating set, if for every vertex  $v \in V(\overline{G})$ , there exist a vertex  $u \in K$ , where  $u \neq v$  such that  $u$  is adjacent to  $v$ . The minimum cardinality of total CDS is the total complementary domination number and is denoted by  $\gamma_t(\overline{G})$ .

### Definition 2.3

A complementary dominating set  $K \subset V(\overline{G})$  is a connected complementary dominating set, if the induced subgraph  $\langle K \rangle$  has no isolated vertices. The connected complementary domination number is the minimum cardinality of connected complementary dominating set and is denoted by  $\gamma_c(\overline{G})$ .

### Note:

Every connected complementary dominating set is a complementary dominating set. But the converse need not be true. For example, consider the graph  $G$  and its complementary graph  $\overline{G}$ .



Here  $\{1,2,3\}$  is the connected CDS which is also a CDS. Also  $\{4,5,6\}$  is the CDS but not the connected CDS.

### Definition 2.4

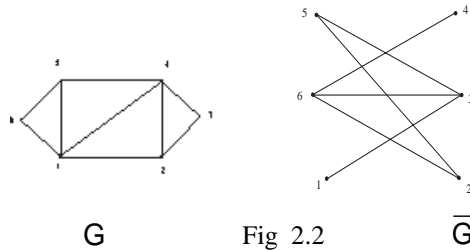
A complementary dominating set  $K \subset V(\overline{G})$  is a co-total complementary dominating set, if the induced subgraph  $\langle V-K \rangle$  has no isolated vertices.

The co-total complementary domination number is the minimum cardinality of a co-total complementary dominating set of  $\overline{G}$  and is denoted by  $\gamma_{ct}(\overline{G})$ .

### Note:

Every co-total CDS is also a CDS. But converse need not be true.

For example, consider a graph  $G$  and its complementary graph  $\overline{G}$ .



Here  $\{4,5,1\}$  is the cototal CDS which is also a CDS. Also  $\{3,6\}$  is a CDS, but not a cototal CDS.

### Definition 2.5

Let  $K$  be a CDS of  $\bar{G}$ . If  $V(\bar{G}) - K$  contains another CDS namely  $K^{-1}$  then  $K^{-1}$  is called the inverse complementary dominating set w.r.t  $K$  (shortly written as inverse CDS)

The minimum cardinality of inverse CDS is called inverse complementary domination number and is denoted by  $\gamma^{-1}(\bar{G})$ .

**Note:**

(i) Every inverse complementary dominating set is also a complementary dominating set.

## III.RESULTS

We can easily observe the following results.

### Proposition 3.1

For any graph  $G$ ,

- (i)  $\gamma(G) \leq \gamma(\bar{G})$  if  $\Delta(G) > \Delta(\bar{G})$
- (ii)  $\gamma(G) \geq \gamma(\bar{G})$  if  $\Delta(G) < \Delta(\bar{G})$

### Proposition 3.2

For any connected graph  $\bar{G}$ ,

- (i)  $\gamma(\bar{G}) \leq \gamma^{-1}(\bar{G}) \leq p$ .
- (ii)  $\gamma(\bar{G}) + \gamma^{-1}(\bar{G}) \leq p$ .
- (iii) If  $\gamma^{-1}(\bar{G}) = 1$  then  $\gamma(\bar{G}) = 1$ . But the converse need not be true.

### Proposition 3.3

If any vertex of a graph  $G$  is of degree  $p-1$ , then the inverse complementary dominating set does not exist.

## IV. MAIN RESULTS

### Theorem 4.1

- (i) For any cycle  $C_n$  with  $n > 3$  vertices,  
 $\gamma(\bar{C}_n) = 2 = \gamma^{-1}(\bar{C}_n)$ .
- (ii) For any wheel graph  $W_p$  ( $p > 3$ ),  
 $\gamma(\bar{W}_p) = 3$ .
- (iii) For any  $n$ -pan graph  $\mathcal{P}_n$  with  $n \geq 3$   
 $\gamma(\bar{\mathcal{P}}_n) = 2$  and  $\gamma^{-1}(\bar{\mathcal{P}}_n) \geq 2$

- (iv) For any complete bipartite graph  $K_{n,n}$ , where  $n > 1$ ,

$$\gamma(\bar{K}_{n,n}) = 2 \text{ and } \gamma^{-1}(\bar{K}_{n,n}) = 2.$$

- (v) For any  $n$  – barbell graph  $B_n$  ( $n \geq 3$ ),

$$\gamma(\bar{B}_n) = 2 = \gamma^{-1}(\bar{B}_n)$$

- (vi) For any  $n$  – sun graph  $\mathcal{S}_n$  ( $n \geq 3$ ),

$$\gamma(\bar{\mathcal{S}}_n) = 2 = \gamma^{-1}(\bar{\mathcal{S}}_n)$$

**Proof:**

Proof is obvious.

### Theorem 4.2

For any connected graph  $\bar{G}$ ,  $\gamma^{-1}(\bar{G}) = 1$  iff  $\bar{G}$  has at least two vertices of degree  $|V(\bar{G})| - 1$ .

**Proof:**

Let  $\bar{G}$  be a connected graph with at least two vertices of degree  $|V(\bar{G})| - 1$ . Then  $\gamma(\bar{G})$  exist. (ie)  $\bar{G}$  has complementary dominating set such that  $\gamma(\bar{G}) = 1$ . Also  $\bar{G}$  has another complementary dominating set which is the inverse complementary dominating set such that  $\gamma^{-1}(\bar{G}) = 1$ , by our hypothesis.

Conversely, if  $\gamma(\bar{G}) = 1$ , then the graph  $\bar{G}$  has at least one vertex of degree  $|V(\bar{G})| - 1$ . As  $\gamma^{-1}(\bar{G}) = 1$ , then  $\bar{G}$  has at least two vertices of degree  $|V(\bar{G})| - 1$ .

### Theorem 4.3

Let  $G$  and  $\bar{G}$  be connected graphs. If  $\gamma^{-1}(\bar{G})$  exists,

$$\text{then } \left\lfloor \frac{p}{\Delta(\bar{G}) + 1} \right\rfloor \leq \gamma^{-1}(\bar{G}) \leq p - \delta(\bar{G}).$$

**Proof:**

Let  $G$  and  $\bar{G}$  be connected graphs.

If  $\gamma^{-1}(\bar{G})$  exist, then

$$\delta(\bar{G}) + \gamma^{-1}(\bar{G}) \leq \gamma(\bar{G}) + \gamma^{-1}(\bar{G}), \quad \text{as}$$

$$\delta(\bar{G}) \leq \gamma(\bar{G}) \leq p, \text{ by proposition 3.2}$$

Thus, we've  $\gamma^{-1}(\bar{G}) \leq p - \delta(\bar{G})$  ----- (a)

To prove another relation, let us consider the following two cases.

**Case(i)** If  $p = \Delta(\bar{G}) + 1$  and  $\gamma^{-1}(\bar{G}) \geq 1$ ,

$$\text{then } \left\lfloor \frac{p}{\Delta(\bar{G})+1} \right\rfloor = 1 \leq \gamma^{-1}(\bar{G})$$

**Case(ii)** If  $p > \Delta(\bar{G}) + 1$  and  $\gamma^{-1}(\bar{G}) \geq 1$ ,

$$\text{then } \left\lfloor \frac{p}{\Delta(\bar{G})+1} \right\rfloor \leq \gamma^{-1}(\bar{G})$$

From the above two cases, we have

$$\left\lfloor \frac{p}{\Delta(\bar{G})+1} \right\rfloor \leq \gamma^{-1}(\bar{G}) \text{----- (b)}$$

$$\text{Hence } \left\lfloor \frac{p}{\Delta(\bar{G})+1} \right\rfloor \leq \gamma^{-1}(\bar{G}) \leq p - \delta(\bar{G})$$

**Theorem 4.4**

Let  $G$  be a graph with end vertices. Then its CDS is one of the end vertices and its adjacent vertex in  $G$ .

**Proof:**

Let  $G$  be a graph with end vertices. Without loss of generality, let us assume that the end vertex be  $i=1$ . Then in  $\bar{G}$ , the end vertex is of degree  $p-2$  and it is adjacent to every vertex of  $\bar{G}$  except the adjacent vertex in  $G$ . Thus the complementary dominating set is the end vertex and its adjacent vertex. Therefore, the result is true for  $i=1$ . Similarly the result is true for  $i>1$ .

**Theorem 4.5**

For any cycle  $C_n$ ,

$$\gamma(\bar{G}) = \gamma^{-1}(\bar{G}) \leq \gamma(G) \leq \gamma^{-1}(G)$$

**Proof:**

Proof is obvious.

**Theorem 4.6**

Let  $G$  be a graph with at least one vertex of maximum degree  $p-2$ . Then

$$\gamma(\bar{G}) + \gamma^{-1}(G) \leq p$$

**Proof:**

Let us prove the result by the following cases.

**Case(i)** Let  $G$  be a graph with at least one vertex of degree  $p-2$  having no end vertex. Then

$$\gamma^{-1}(G) \leq \gamma^{-1}(\bar{G}).$$

$$\gamma^{-1}(G) + \gamma(\bar{G}) \leq \gamma^{-1}(\bar{G}) + \gamma(\bar{G}) \leq p.$$

$$\gamma^{-1}(G) + \gamma(\bar{G}) \leq p.$$

**Case(ii)** Let  $G$  be a graph with at least one vertex of degree  $p-2$  having end vertices. Then  $\gamma(\bar{G}) = 2$  and  $\gamma(G) = 2$ .

As  $\gamma(G) + \gamma^{-1}(G) \leq p$ ,  $\gamma^{-1}(G) \leq p - 2$ .

$$\text{Therefore } \gamma^{-1}(G) + \gamma(\bar{G}) \leq p - 2 + \gamma(\bar{G})$$

$$\text{Thus } \gamma(\bar{G}) + \gamma^{-1}(G) \leq p.$$

**REFERENCES**

- [1] Harary.F., Graph theory, Adison Wesley, Reading mass (1972).
- [2] Haynes T.W., Hedetniemi S.T and Slater P.J., Fundamentals of domination in graphs, New York, Marcel Dekker, Inc.,(1998).
- [3] Kulli V.R., Janakiram. B and Iyer R.R., The Cototal domination number of a graph, J.Disc.Math.Sci and Cry.,2(1992),179-184.
- [4] Kulli V.R, Text Book of "Theory of domination in graphs", Vishwa International Publications (2010).
- [5] Muddebihal M.H. Panfarosh U.A and Anim R.Sedamkar. Inverse Line domination graph, IJMA (5) 2014, 23-28.R.