Optimal EPQ Model with Weibully Distributed Deterioration Rate and Time Varying IHC

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Abstract - *Many real life situations, the stock level of the inventoried items is continuously depleting due to the combined effects of its demand and deterioration. This paper develops an optimal lot-sizing EPQ model for Weibull deteriorated items with constant rate of demand and time-varying holding cost over a finite planning horizon. Specifically a 3-parameter Weibull distribution is used to represent the deterioration rate. Shortages are permitted and are completely backlogged. Numerical example along with sensitivity analysis is given to support the model.*

Keywords *- Inventory, Time-varying holding cost, 3-parameter Weibull deterioration, Fully backlogged shortages.*

I. INTRODUCTION

One of the assumptions in the traditional inventory model is that the items preserved their physical characteristics while they are kept or stored in the inventory. This assumption is evidently true for most of the items, but not for all. In any company, the major problem a supplier confronts is the control and maintenance of inventories of deteriorating items. From the past one and a half decade, inventory problems for deteriorating items have been widely studied, as most physical goods deteriorate with time. Deterioration is defined as the decay, damage, vaporization, obsolescence, pilferage, loss of utility or loss of marginal value of the products that results in decrease in their original properties of the goods to satisfy the demand. Deterioration rate is almost nil in some of the items like hardware, glassware, toys, steel etc. but for the items like food grains, vegetables, fish, medicines, gasoline, alcohol, radioactive chemicals, fashion goods, electronic substances etc. have finite shelf life and deteriorate rapidly over time. Owing to this fact, controlling and maintaining the inventory of deteriorating items become a challenging problem for decision makers.

Whitin [20] first initiated the research in this direction by considering fashion goods deteriorating the end of a prescribed storage period. Ghare and Schrader [7] developed an inventory model with a constant rate of deterioration. Shah and Jaiswal [17] developed an order level inventory model for items deteriorating at a constant rate. Aggarwal [1] by rectifying the work of Shah and Jaiswal [17] calculated the average inventory holding cost. Then Dave and Patel [6] developed an inventory model for replenishment. Further Dave[5] discussed an inventory model with variable instantaneous demand, discrete opportunities for replenishment and shortages. An heuristic model with time-proportional demand was discussed by Bahari-Kashani [2].

Goswami and Chaudhuri [8] developed an EOQ model for deteriorating items with shortages and linear trend in demand.

After this researchers started developing inventory models with time-dependent deterioration rate. By using a two-parameter Weibull distribution Cover and Philip [4] represented the distribution of time to deterioration. Then Philip [13] extended this model by taking three-parameter Weibull distribution of time to deterioration. Wee [19] considered deteriorating inventory model for quantity discount, pricing and partial backordering. Goyal and Giri [11] gave recent trend of modelling in deteriorating inventory.They classified inventory models on the basis of demand variations and various other conditions. Sarkar and Chakrabarty [16] developed EPQ model using Weibull distribution for electronic items in which holding cost is time varying and same work continued with exponential demand for permissible delay in payment. Sugapriya and Jeyaraman [18] considered the EPQ model for non-instantaneous deteriorating item with variable holding cost.

Recently, Gothi and Chatterji [9] have developed an inventory model for imperfect items under constant demand rate with time varying IHC. Parmar and Gothi [15] developed an EPQ model for deteriorating items using three parameter weibull distribution with constant production rate and time varying holding cost. Parmar and Gothi [14] developed an order level inventory model for deteriorating itmes under quadratic demand with time dependent IHC. Gothi and Parmar [10] had also developed an order level inventory model for deteriorating items under quadratic demand with time dependent IHC and partial backlogging.

Kawale and Bansode [12] have developed an inventory model for time varying holding cost and weibull distribution for deterioration with fully backlogged shortages. In this paper, it is observed that the p.d.f. $f(t) = \alpha\beta(\mu-\gamma)^{\beta-1}e^{-\alpha(\mu-\gamma)^{\beta}}$ is taken which is constant and so deterioration rate is also constant which seems to be an old work.

In this paper, an effort has been made to redevelop Sunil Kanwale and Pravin Bansode's [11] EPQ model of deteriorating items with three parameter Weibull deterioration rate $\theta = \alpha\beta(t-\mu)^{\beta-1}$ and with constant demand and production rates; time varying holding cost and fully backlogged shortages.

Mathematical model and solution procedures are derived followed by numerical examples, graphical and sensitivity analysis to demonstrate the effects of changing parameter values on the optimal solution of the system. The paper ends with concluding remarks.

II. NOTATIONS

The following notations are used for developing the model:

1. Q(t) : Inventory level of the product at time t (≥ 0).
2. d : Demand rate.
3. p : Rate of production per unit time (>d).
4. k : Production cost per unit.
5. A : Operating cost.
6. C_h : Inventory holding cost per unit per unit time.
7. C_d : Deterioration cost per unit per unit time.
8. C_s : Shortage cost per unit per unit time.
9. T : Duration of a cycle.
10. TC : Total cost per unit time.

III. ASSUMPTIONS

The model is developed under the following assumptions:

1. Demand rate is known and finite.
2. Production rate is known and finite which is always greater than the demand rate.
3. Shortages occur and they are completely backlogged.
4. An infinite planning horizon is assumed.
5. Once a unit of the product is produced, it is available to meet the demand.
6. As soon as the production stops, products start deterioration.
7. Time to deteriorate follows three parameter Weibull distribution.
8. No replacement or repairs for the deteriorated items will be done.

IV. MATHEMTICAL MODEL AND ANALYSIS

The distribution of time to deteriorate is a random variable which follows a three parameter Weibull distribution. The probability density function for three parameter Weibull distribution is given by

$$f(t) = \alpha\beta(t-\mu)^{\beta-1}e^{-\alpha(t-\mu)^{\beta}}$$

where $t \geq \mu,\ 0 < \alpha < 1,\ \beta, \mu > 0$.

The instantaneous rate of deterioration $\theta(t)$ of the non-deteriorated inventory at time t can be obtained from $\theta(t) = \dfrac{f(t)}{1-F(t)}$, where $F(t) = 1 - e^{-\alpha(t-\mu)^{\beta}}$ is the cumulative distribution function for the three parameter Weibull distribution. Thus, the instantaneous rate of deterioration of the on-hand inventory is $\theta(t) = \alpha\beta\,(t-\mu)^{\beta-1}$. The probability density function represents the distribution of the time to deteriorate which may be decreasing, constant or increasing rate of deterioration. The three parameter Weibull distribution is suitable for items with any initial value of the rate of deterioration (Begum at el. [3]).

This paper assumes that the production starts at time $t = 0$ and stops at time $t = \mu$. During $[0, \mu]$ the inventory is built up at the rate of $p - d$, there is no deterioration at this interval and stock level attains a level Q_1. During $[\mu, t_1]$, the inventory level gradually decreases because of deterioration and supply is also there with discounted rate and the stock falls to zero level till time $t = t_1$. Then shortages occur and get accumulated to the level S in the interval $[t_1, t_2]$. The production starts again at time $t = t_2$ to fulfill the backlog by the time $t = T$ to attain the stock level upto zero. The cycle then repeats itself after time $t = T$.

The pictorial presentation is shown in the Figure – 1.

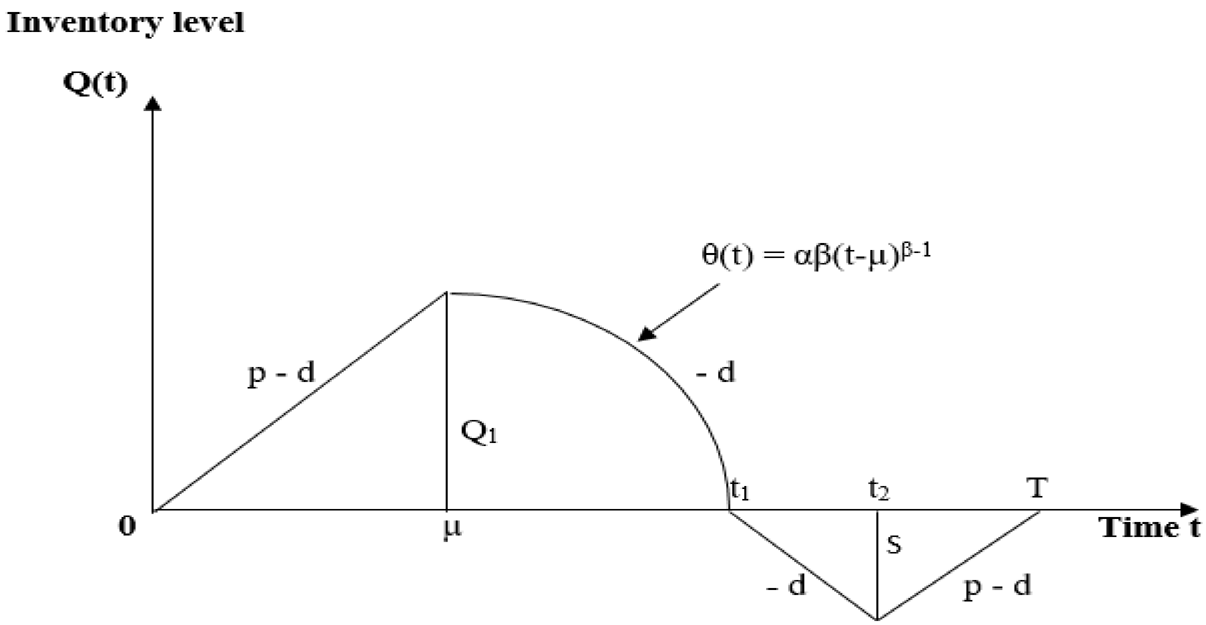

Figure 1: Graphical presentation of the inventory system

The differential equations which describe the instantaneous state of Q(t) over the period (0, T) are given by

$$\frac{dQ(t)}{dt} = p - d \qquad (0 \le t \le \mu) \qquad .....(1)$$

$$\frac{dQ(t)}{dt} + \alpha\beta(t-\mu)^{\beta-1}Q(t) = -d \qquad (\mu \le t \le t_1) \qquad .....(2)$$

$$\frac{dQ(t)}{dt} = -d \qquad (t_1 \le t \le t_2) \qquad .....(3)$$

$$\frac{dQ(t)}{dt} = p - d \qquad (t_2 \le t \le T) \qquad .....(4)$$

Under the boundary conditions $Q(0) = 0$, $Q(\mu) = Q_1$, $Q(t_1) = 0$, $Q(t_2) = -S$ and $Q(T) = 0$ solutions of equations (1) to (4) are given by

$$Q(t) = (p-d)t \qquad (0 \le t \le \mu) \qquad .....(5)$$

$$Q\;\;t\;\; = d\left[(t_1-\mu) + \frac{\alpha}{\beta+1}(t_1-\mu)^{\beta+1}\right] \qquad (\mu \le t \le t_1) \qquad .....(6)$$

$$Q(t) = -d(t-t_1) \qquad (t_1 \le t \le t_2) \qquad .....(7)$$

$$Q(t) = -S + (p-d)(t-t_2) \qquad (t_2 \le t \le T) \qquad \text{.....(8)}$$

Putting $Q(t_2)$ = – S in equation (7), we get

$$\Rightarrow S = d(t_2 - t_1)$$

Putting Q(T) = 0 in equation (8), we get

$$\Rightarrow S = (p-d)(T-t_2)$$

Equation (7) and (8) coincide at t = t_2 hence

$$\Rightarrow T = \frac{pt_2 - dt_1}{p-d}$$

The total cost per unit time comprises of the following costs

1) Production Cost

$$PC = pk(\mu + T - t_2) \qquad \text{.....(9)}$$

2) Setup Cost

$$OC = A \qquad \text{.....(10)}$$

3) Holding Cost

$$IHC = \int_0^{\mu}(h+rt)Q(t)dt + \int_{\mu}^{t_1}(h+rt)Q(t)dt$$

$$= \left\{ \begin{array}{l} \int_0^{\mu} (h+rt)(p-d)t \ dt \\ + d\int_{\mu}^{t_1} (h+r\mu)+r(t-\mu) \left[\begin{array}{l} (t_1-\mu)-(t-\mu) \\ +\dfrac{\alpha}{\beta+1} \ (t_1-\mu)^{\beta+1} - (t-\mu)^{\beta+1} \\ -\alpha \ (t_1-\mu)-(t-\mu) \ (t-\mu)^{\beta} \end{array} \right] dt \end{array} \right\}$$

$$\therefore IHC = \left\{ \begin{array}{l} (p-d)\left[\frac{1}{2}h\mu^2 + \frac{1}{3}r\mu^3\right] \\ +d\left[\begin{array}{l} (h+r\mu)\left\{ \frac{(t_1-\mu)^2}{2} + \frac{\alpha}{\beta+1}\frac{(t_1-\mu)^{\beta+2}}{2} - \frac{\alpha}{(\beta+1)(\beta+2)}(t_1-\mu)^{\beta+2} \right\} \\ +r\left\{ \frac{(t_1-\mu)^3}{2} + \frac{\alpha}{\beta+1}(t_1-\mu)^{\beta+2} - \frac{\alpha}{(\beta+2)(\beta+3)}(t_1-\mu)^{\beta+3} \right\} \end{array} \right] \end{array} \right\}$$

.....(11)

4) Deterioration Cost

$$DC = C_d \int_{\mu}^{t_1} \alpha\beta(t-\mu)^{\beta-1}Q(t)dt$$

$$\Rightarrow DC = C_d d\frac{\alpha}{\beta+1}\; t_1-\mu \;^{\beta+1}$$ .....(12)

5) Shortage Cost

$$SC = -C_s \int_{t_1}^{t_2} Q(t)dt$$

$$\Rightarrow SC = C_s d\frac{(t_2-t_1)^2}{2}$$ .....(13)

Hence, the average total cost for the time period [0,T] is given by

$$TC\;\; T \;\; = \frac{1}{T}\;\; PC + OC + IHC + DC + SC$$

$$TC\;\;T\;=\frac{1}{(pt_2-dt_1)^2}\left[\begin{array}{l} pk\left(\mu+\dfrac{pt_2-dt_1}{p-d}-t_2\right)+A+(p-d)\left\{\dfrac{1}{2}h\mu^2+\dfrac{1}{3}r\mu^3\right\} \\ +d\left[\begin{array}{l}(h+r\mu)\left\{\dfrac{(t_1-\mu)^2}{2}+\dfrac{\alpha}{\beta+1}\dfrac{(t_1-\mu)^{\beta+2}}{2}-\dfrac{\alpha}{(\beta+1)(\beta+2)}(t_1-\mu)^{\beta+2}\right\} \\ +r\left\{\dfrac{(t_1-\mu)^3}{2}+\dfrac{\alpha}{\beta+1}(t_1-\mu)^{\beta+2}-\dfrac{\alpha}{(\beta+2)(\beta+3)}(t_1-\mu)^{\beta+3}\right\}\end{array}\right] \\ +C_d d\dfrac{\alpha}{\beta+1}\;t_1-\mu\;^{\beta+1}+\dfrac{1}{2}C_s d(t_2-t_1)^2 \end{array}\right]$$

…..(14)

μ*, t_1* and t_2* are the optimum values of μ, t_1 and t_2 respectively, which minimize the cost function TC and they are the solutions of the equations $\frac{\partial TC}{\partial \mu}=0,\;\; \frac{\partial TC}{\partial t_1}=0,\;\; \frac{\partial TC}{\partial t_2}=0,$ satisfying the sufficient conditionH > 0, at μ*, t_1* and t_2* where

$$H=\begin{vmatrix} \dfrac{\partial^2 TC}{\partial \mu^2} & \dfrac{\partial^2 TC}{\partial \mu \partial t_1} & \dfrac{\partial^2 TC}{\partial \mu \partial t_2} \\ & \dfrac{\partial^2 TC}{\partial t_1^2} & \dfrac{\partial^2 TC}{\partial t_1 \partial t_2} \\ & & \dfrac{\partial^2 TC}{\partial t_2^2} \end{vmatrix}$$ is Hessian determinant. …..(15)

V. NUMERICAL EXAMPLE

Let us consider the following example to illustrate the above developed model, taking A = 500, p = 50, d = 25, k = 1, α = 0.02, β = 2, h = 2, r = 1, C_d = 1, C_s = 2 (with appropriate units).

The optimal values of μ, t_1 and t_2 are μ* = 0.6983193493, t_1* = 2.094466538, t_2* = 5.959701114 units and the optimal total cost per unit time TC = 121.6308644 units.

VI. SENSITIVITY ANALYSIS

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. Here, we study the sensitivity for the cycle length T and total cost per time unit TC with respect to the changes in the values of the parameters A, p, d, k,α, β, h, r, $C_{d,}$ and C_s.

The sensitivity analysis is performed by considering variation in each one of the above parameters keeping all other remaining parameters as fixed. The last column of the **Table – 1** gives the percentage changes in TC as compared to the original solution for the relevant costs.

Table – 1: Partial Sensitivity Analysis

Para-meters	Change	μ	t_1	t_2	TC	% TC
A	450	0.6497004	2.01285888	5.66922575	116.409172	- 4.29
	475	0.6745138	2.05445726	5.81661924	119.054050	- 2.12
	525	0.7212076	2.13302376	6.09881332	124.144739	2.07
	550	0.7432568	2.17024813	6.23425535	126.600181	4.09
p	46	0.8026992	2.09819018	6.01214554	114.340286	- 5.99
	48	0.7485104	2.09634486	5.98490287	118.163369	- 2.85
	53	0.6295683	2.09162145	5.92519389	126.264178	3.81
	56	0.5675723	2.08877247	5.89404522	130.324514	7.15
d	17	0.4413641	2.39997355	6.92683390	118.582746	- 2.51
	20	0.5429811	2.26597779	6.49117252	121.404674	- 0.19
	27	0.7567441	2.03842823	5.79364252	120.279523	- 1.11
	30	0.8411634	1.96400501	5.58020213	116.788731	- 3.98
k	0.8	0.7876776	2.11193926	5.98969045	116.943780	- 3.85
	0.9	0.7440563	2.10382004	5.97572890	119.297721	- 1.92
	1.1	0.6501843	2.08374113	5.94143095	123.942246	1.90
	1.2	0.5992976	2.07147184	5.92070170	126.230747	3.78
α	0.010	0.6917269	2.10412869	5.96682419	121.567387	- 0.05
	0.015	0.6950776	2.09923587	5.96321342	121.599439	- 0.03
	0.030	0.7045005	2.08527448	5.95295204	121.691939	0.05
	0.040	0.7103145	2.07650899	5.94654086	121.750797	0.10
β	1.8	0.6972112	2.09615466	5.96126058	121.627648	- 0.0026
	1.9	0.6977599	2.09531706	5.96048644	121.629234	- 0.0013
	2.1	0.6988897	2.09360246	5.95890399	121.632539	0.0014
	2.2	0.6994711	2.09272429	5.95809456	121.634257	0.0028
h	1.8	0.7073381	2.16226550	6.00167622	120.985268	- 0.53
	1.9	0.7031763	2.12808808	5.98063739	121.313733	- 0.26
	2.1	0.6929036	2.06142780	5.93891591	121.937203	0.25
	2.2	0.6870389	2.02899036	5.91832029	122.233248	0.50
r	0.8	0.7536853	2.22767348	6.06482753	120.928851	- 0.58
	0.9	0.7244714	2.15754208	6.00934557	121.295087	- 0.28
	1.1	0.6747085	2.03728020	5.91490226	121.940552	0.25
	1.2	0.6532366	1.98507637	5.87418228	122.227648	0.49
C_d	0.8	0.6973604	2.09554677	5.96041123	121.621611	- 0.0076
	0.9	0.6978408	2.09500604	5.96005575	121.626243	- 0.0038
	1.1	0.6987960	2.09392827	5.95934730	121.635476	0.0038
	1.2	0.6992709	2.09339122	5.95899432	121.640077	0.0076
C_s	1.8	0.6615765	2.03275491	6.15141379	117.669825	- 3.26
	1.9	0.6804340	2.06439815	6.05139935	119.691279	- 1.59
	2.1	0.7153208	2.12309879	5.87528079	123.494778	1.53
	2.2	0.7315152	2.15041584	5.87528079	125.305930	3.02

VII. GRAPHICAL PRESENTATION

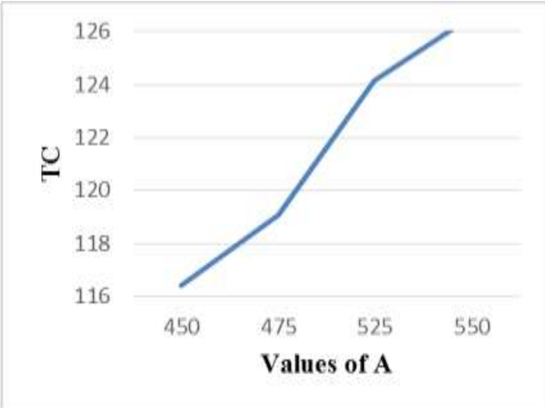

Figure – 2

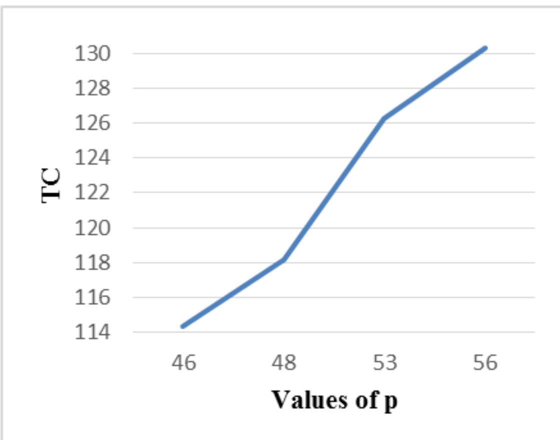

Figure – 3

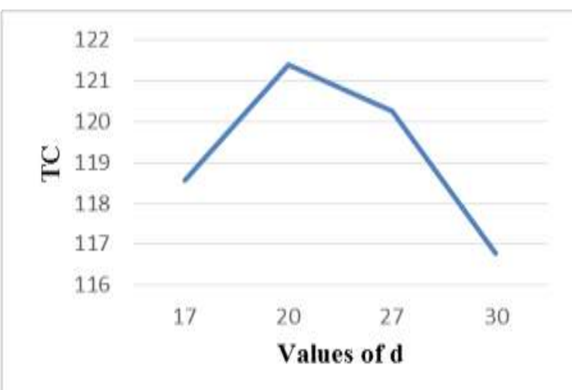

Figure – 4

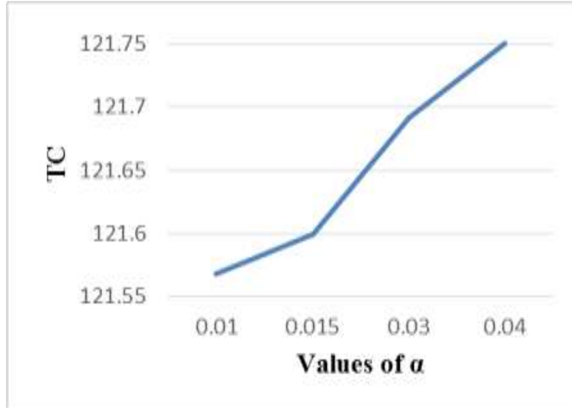

Figure – 5

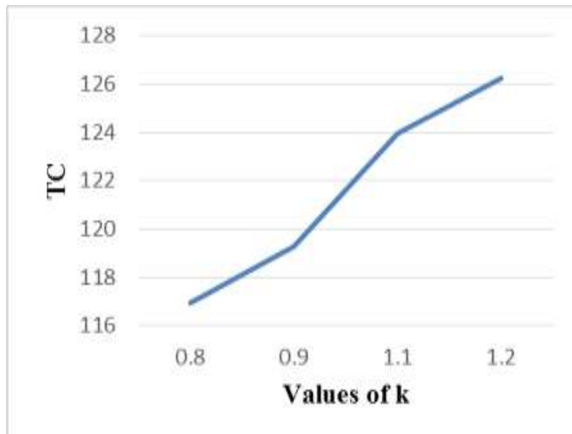

Figure – 6

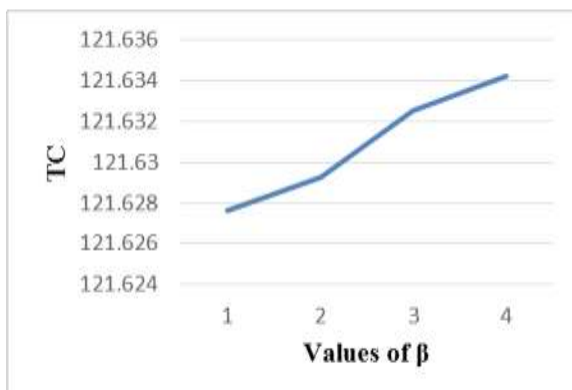

Figure – 7

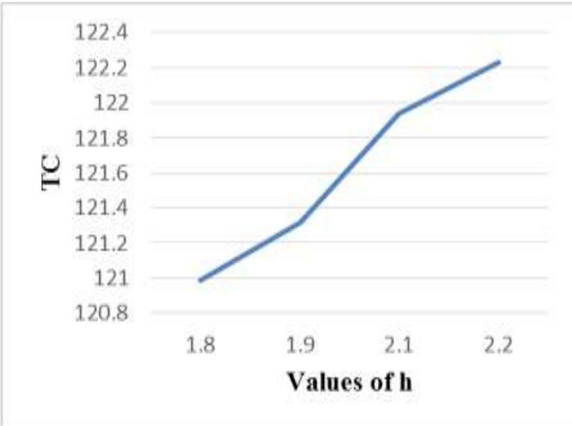

Figure – 8

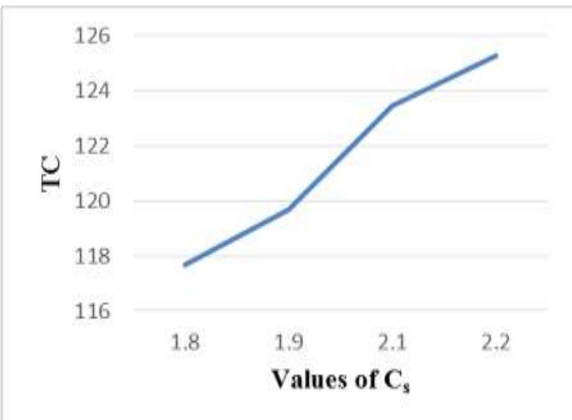

Figure – 9

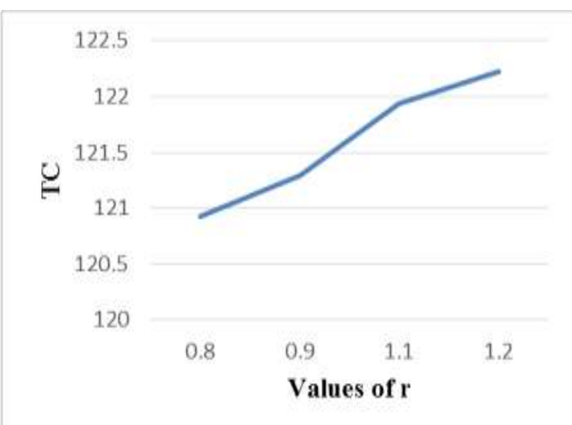

Figure – 10

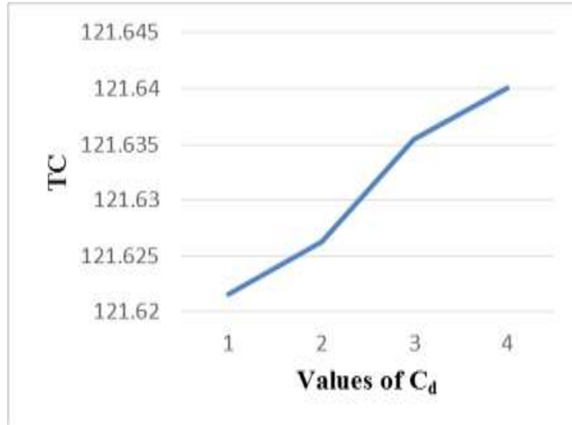

Figure – 11

VIII. CONCLUSION

- From the Partial Sensitivity Analysis table we can conclude that as the Operating Cost and Rate of Production increases, Total Cost also increases.
- From figure 4 it is analysed that as demand rate increases Total Cost of Production increases upto a certain level and then it decreases and then as the production stops and deterioration starts Total Cost decreases continuously.
- From figure 5 and figure 7 it is observed that as the scale parameter and shape parameter increases, the Total Cost again increases.
- From figure 8 and figure 10 it is observed that as h and r of Inventory Holding Cost increase, Total Cost increases.
- From figure 9 and figure 11 it is observed that as the Shortage cost and Deterioration cost increase, Total Cost increases.
- From figure 2, 3 and 6 it is observed that as operating cost, production rate and production cost per unit time incresease, the Total Cost increases.

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