Some results on confirmation measures

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Abstract: *our expectations for the future are influenced by our past experiences. A person always wants to confirm his proposition on the basis of some given facts. The process for making a proposition confirmed can be dealt with the confirmation measures. Because of the random elements in the data, confirmation measure depends mainly on the logic of probability. This paper is an attempt to study confirmation measures and probability measures and also to establish their relationship. Where it was observed that degree of confirmation can be expressed in terms of probability distribution functions.*

Key words: *Confirmation measure, probability measure, degree of confirmation*

1. **Introduction:** A confirmation measure is defined by the function:

   Cf: P(X)→ [0,1]

   Which assigns to each subset of X a number in the unit interval [0,1]. P(X) is called family of subsets of X or power set. When this number is assigned to a subset A ϵ P(X) then cf(A) represents the degree of confirmation that a given element of X belongs to the subset A. The subset to which we assign the highest value represents our best guess regarding the particular element in question.

2. **Axioms of confirmation measures:**

   In order to qualify as confirmation measure, the function cf must satisfy certain properties similar to the usual axioms of probability. The following are axioms of confirmation measures.

   Axioms:

   i. Cf (ϕ) = 0 (axiom of lower boun) When φ is the empty set.

   ii. Cf (X) = 1 (axiom of upper bound)

   iii. For every sets A, B ϵ P(X), if A ⊆ B then cf (A) ≤ cf (B)

   iv. We introduce an additional axiom for confirmation as

   $Cf\ (A_1 \cup A_2 \cup --- \cup A_n) \geq \Sigma cf\ (A_i) - \Sigma\ cf\ (A_i \cap A_j) + --- (-1)^{n+1}\ cf\ (A_1 \cap A_2\ ---- \cap A_n)$ ---(1)

   Where n =1,2,3---------

   For n= 2, this axiom reduces to

   $Cf(A_1 \cup A_2) \geq cf\ (A_1) + cf\ (A_2) - cf\ (A_1 \cap A_2)$ --------------------(2)

Cf (A) represents the degree of confirmation, based on available evidence that a given element of X belongs to the set A. When the set A_1, A_2---------An in axiom (IV) are pair wise disjoint $(A_i \cap A_j) = ϕ$ for all i, j ϵ N such that i ≠ j. The axiom requires that the degree of confirmation associated with the union of sets is not smaller than the sum of degree of confirmation corresponding to individual sets. The basic axiom of confirmation measure is thus a weaker version of additive axiom of probability theory. This implies that probability measures are special cases of confirmation measures for which the inequality (1) is required.

Now, let $A_1 = A$ and $A_2 = \bar{A}$, putting in equation (2) we get the following fundamental property of confirmation measures as,

$Cf(A) + cf(\bar{A}) \leq 1$

We can also show that axiom (IV) of confirmation measures satisfies axiom (III).

Let $A \subseteq B$ and let $C = B - A$. Then $A \cup C = B$ and $A \cap C = \phi$. Now applying A and C to axiom (IV) for n=2 in equation (2), we have,

$cf(A \cup C) = cf(B) \geq cf(A) + cf(C) - cf(A \cap C)$

Since, $A \cap C = \phi$ and $cf(\phi) = 0$ by axiom (1)

We get, $cf(B) \geq cf(A) + cf(C)$

And hence, $cf(B) \geq cf(A)$

3. Basic Assignments:

Every confirmation measure can be expressed in terms of a function $m : P(X) \rightarrow [0,1]$

Such that $m(\phi) = 0$ and $\sum_{A \epsilon P(X)} m(A) = 1$-------------------(3)

Where m(A) is either the degree of evidence supporting the claim that a specific element of X belongs to the set A or the degree to which we are confirm that such claim is warranted. Since equation (3) is similar to equation for probability distribution, the function m is usually called a basic probability assignment. Given a basic assignment m, a confirmation measure can be expressed as,

$cf(A) = \sum_{B \subseteq A} m(B)$; for all $A \epsilon P(X)$ -----------------(4)

whereas m(A) represents the degree of confirmation that the element in question belongs to the set A alone, cf(A) represents the total evidence or confirmation that the element belongs to A as well as various special subsets of A.

4. Probability Measures:

We can show that when axiom (IV) of confirmation measure is replaced with a stronger axiom:

$cf(A \cup B) = cf(A) + cf(B)$, for $A \cap B = \phi$------------------(5)

We obtain a special type of confirmation measures usually known as probability measures.

The following theorem presents the fundamental property of probability measures as special types of confirmation measures.

Theorem: A confirmation measure cf on a finite power set P(x) is a probability measure if and only if its basic assignment m is given by $m(\{x\}) = cf(\{x\})$ and $m(A) = 0$ for all subsets of x that are not singletons.

Proof: Let cf is a probability measure. For empty set ϕ, m(ϕ) = 0 by definition.

Let A ≠ ϕ and let A = $\{x_1, x_2\text{--------------------}x_n\}$

Then by repeated application of equation (5), we get

cf(A) = cf($\{x_1\}$) + cf ($\{x_2, x_3\text{-------------}x_n\}$)

= cf($\{x_1\}$) + cf ($\{x_2\}$) + cf ($\{x_3, x_4\text{-------------}x_n\}$)

= cf ($\{x_1\}$) + cf ($\{x_2\}$) +--------------------cf($\{x_n\}$)

Since, cf({x}) = m({x}) for any x ϵ X

Therefore, by equation (4) we have, cf(A) = $\sum_{i=1} m(\{xi\})$

Hence, cf is defined in terms of basic assignment that focuses only on singletons.

Assume that a basic assignment m is given such that $\sum_{x\epsilon X} m(\{x\}) = 1$

Then for any sets A, B ϵ P(X) such that A∩B=ϕ

We have cf(A) + cf (B) = $\sum_{x\epsilon A} m(\{x\})$+ $\sum_{x\epsilon B} m(\{x\})$

=$\sum_{x\epsilon AUB} m(\{x\})$ = cf(AUB)

Therefore, cf is a probability measure, which proves our theorem.

Probability measures on finite sets are thus fully represented by a function

$$p:x \rightarrow [0,1]$$

such that p(x) = m({x}). This function is usually called probability distribution function. For all AϵP(X) this can be written as cf(A)= $\sum_{x\epsilon A} p(x)$.

5. Conclusion:

We have tried to explain the concept of confirmation measure through axiom (1) to (IV) and have found relationship between degree of confirmation and probability measure. We have obtained the result that degree of confirmation can be expressed in terms of probability distribution function.

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