1 -Near Mean Cordial Labeling of $D_2(P_n)$, $P_n(+)N_m$ (when n is even) , Jelly Fish $J(m,n)$ graphs

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Abstract— *Let G=(V,E) be a simple graph. A surjective function $f:V\to\{0,1,2\}$ is said to be a 1-Near Mean Cordial labeling if for each edge uv, the induced map*

$$f(uv)=0 \text{ if } \frac{f(u)+f(v)}{2} \text{ is an integer}, \quad =1 \text{ Otherwise}$$

*Satisfies the condition $|e_f(0)-e_f(1)|\le 1$ where $e_f(0)$ is the number of edges with 0 label and $e_f(1)$ is the number of edges with 1 label. G is said to be 1-Near Mean Cordial Graph if it has a 1-Near Mean Cordial Labeling. In this paper we proved that $D_2(P_n)$, $P_n(+)N_m$ (When n is even)**,** Jelly Fish $J(m,n)$ graphs are 1-Near Mean Cordial Graphs.*

Keywords— 1-Near Mean Cordial Labeling, 1-Near Mean Cordial Graph
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1. Introduction

Let us consider the graphs to be finite, undirected and simple. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively. The cardinality of V(G) and E(G) are respectively called order and size of G. Labeling of graphs has enormous application in many practical problems involved in circuit designing, communication network, astronomy etc. [1]. The concept of mean cordial labeling was introduced by Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram in the year 2012 in [5]. Some results on Mean Cordial Labeling was discussed in [3,4,5,6]. Let f be a function V (G) to $\{0,1,2\}$. For each edge uv of G assign the label $\frac{f(u)+f(v)}{2}$. f is called a mean cordial labeling of G if $|v_f(i)-v_f(j)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$, $i,j\in\{0,1,2\}$ where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x $(x=0,1,2)$ respectively. A graph with a mean cordial labeling is called mean cordial graph. K.Palani, J.Rejila Jeya Surya [2] introduced a new concept called 1-Near Mean Cordial labeling and investigated the 1-Near Mean Cordial Labeling behavior of Paths, Combs, Fans and Crowns. Terms defined here are used as in F. Harary [7].

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2. PRELIMINARIES

K.Palani,J.Rejila Jeya Surya [2] define the concept of 1-Near Mean Cordial labeling as follows

Let $G=(V,E)$ be a simple graph. A surjective function $f:V\to\{0,1,2\}$ is said to be 1-Near Mean Cordial Labeling if for each edge uv, the induced map

$$f^*(uv)=0 \text{ if } \frac{f(u)+f(v)}{2} \text{ is an integer}, \quad = 1 \text{ otherwise}$$

Satisfies the condition $|e_f(0)-e_f(1)|\le 1$ where $e_f(0)$ is the number of edges with zero label and $e_f(1)$ is the number of edges with one label. G is said to be a 1-Near Mean Cordial Graph if it has a 1-Near Mean cordial labeling. In this paper we proved that $D_2(P_n)$**,** $P_n(+)N_m$ (When n is even)**,** Jelly Fish $J(m,n)$ graphs are 1-near mean cordial graphs

Definition :2.1. : $D_2(P_n)$ is the graph of two copies of path graph P_n consisting of vertices u_i and v_i for $1\le i\le n$ .Hence $D_2(P_n)$ consists of $V(G)=2n$ and $E(G)=4(n-1)$.

Definition :2.2.: $P_n(+)N_m$ is the graph with order of vertices m+n and size of edges 2m+n-1.

Definition :2.3: Jelly Fish $J(m,n)$ is a graph with order of vertices m+n+4 and sizes of edges is m+n+5.

3. Main Results

Theorem.3.1: Graph $D_2(P_n)$ is 1-Near Mean Cordial graph

Proof: Let $G=(V,E)$ be a simple graph and let G be $D_2(P_n)$

Let $V(G)=\{u_i, v_i : 1 \le i \le n\}$,

$E(G)=\{(u_iu_{i+1}):1\le i\le n-1\}\cup\{(v_iv_{i+1}):1\le i\le n-1\}\cup\{(v_iu_{i+1}):1\le i\le n-1\}\cup\{(u_iv_{i+1}):1\le i\le n-1\}$

Define $f: V \to \{0,1,2\}$ by

$f(u_i)=0$ if $i \equiv 1 \bmod 2$

$=2$ if $i \equiv 0 \bmod 2$

$f(v_i)=1$ if $i \equiv 1 \bmod 2$

$=2$ if $i \equiv 0 \bmod 2$

The induced edge labeling are

$f^*(u_i, v_{i+1})=0$ for i is odd, $=1$ for i is even, $1 \le i \le n-1$

$f^*(v_i, u_{i+1})=1$ for i is odd, $=0$ for is even, $1 \le i \le n-1$

$f^*(u_i, u_{i+1})=0$ for $1 \le i \le n-1$

$f^*(v_i, v_{i+1})=1$ for $1 \le i \le n-1$

Hence, the graph satisfies the condition $|e_f(0)-e_f(1)| \le 1$

Therefore, the graph $D_2(P_n)$ is a 1-Near Mean Cordial Graph.

Illustration :

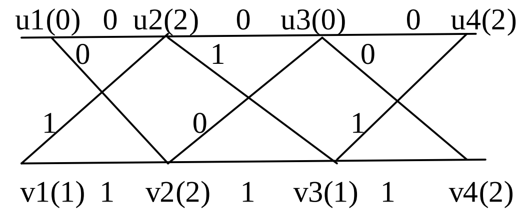

Fig.3.1.1: $D_2(P_4)$

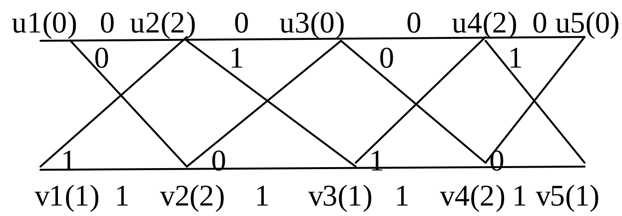

Fig.3.1.1: $D_2(P_5)$

Theorem.3.2: The Graph $P_n(+)N_m$ for all $n,m \ge 1$ is a 1-Near Mean Cordial Graph for n is even

Proof : Let $G=(V,E)$ be a simple graph and let G be $P_n(+)N_m$.

Let $V(G)=\{\{u_i:1\le i\le n\},\{v_i:1\le i\le m\}\}$

and let

$E(G)=\{(u_1v_i):1\le i\le m\}\cup\{(u_nv_i):1\le i\le m\}\cup\{(u_iu_{i+1}):1\le i\le n-1\}$

Define $f: V \to \{0,1,2\}$ by

$f(u_i)=2$ for i is even

$f(u_i)=$ 1 or 0 for i is odd

We assign alternatively for odd vertices with 1 and 0; for $1 \le i \le n$

$f(v_i)=1$ when $i \equiv 1 \bmod 2$

$=2$ when $i \equiv 0 \bmod 2$

Hence the induced edge labeling are

$f^*(v_iu_1)=0$ when i is odd

$=1$ when i is even

$f^*(v_i u_n) = 1$ when i is odd

$= 0$ when i is even

For $f^*(u_i, u_{i+1})$ is a sequence of 0's and 1's

For $P_4(+)N_m$,; $e_f(0)$ is one more than $e_f(1)$

Other wise for all n even $P_n(+)N_m$ either $e_f(1)$ is one more than $e_f(0)$ or $e_f(0)$ is one more than $e_f(1)$.

Hence it satisfies the condition $|e_f(0) - e_f(1)| \le 1$

Therefore the graph $P_n(+)N_m$ is a 1-Near Mean Cordial graph when n is even and for all $n, m \ge 1$

Illustration :

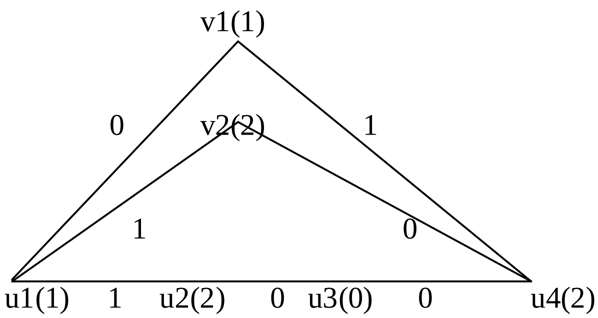

Fig.3.2.1: $P_4(+)N_2$

Illustration :

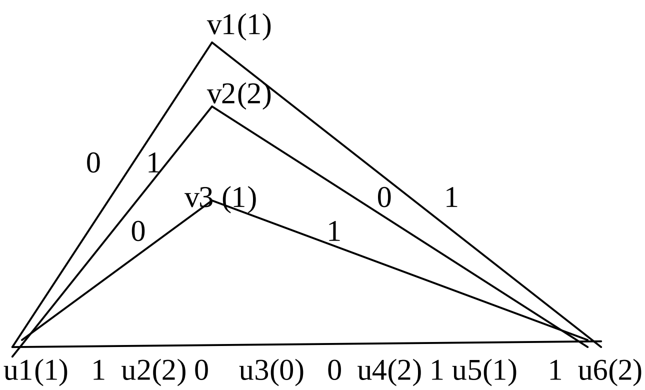

Fig.3.2.2: $P_6(+)N_3$

Theorem .3.3: Jelly Fish $J(m,n)$ graph is 1-Near Mean Cordial Graph

Proof: Let $G = (V, E)$ is a simple graph and let $G = J(m,n)$ be a Jelly Fish graph. Let the vertices of G be defined as $V(G) = V_1 \cup V_2$ where $V_1 = \{x, u, y, v\}$ and $V_2 = \{u_i, v_j; 1 \le i \le m, 1 \le j \le n\}$ and the edges $E = E_1 \cup E_2$ where $E_1 = \{xu, uy, yv, xy, vx\}$

$E_2 = \{uu_i, vv_j; 1 \le i \le m, 1 \le j \le n\}$

For labeling the vertices let us assign

$f(x) = 0$; $f(u) = 1$; $f(y) = 1$; $f(v) = 1$

$f(u_i) = 1$ if $i \equiv 1 \bmod 2$
$= 2$ if $i \equiv 0 \bmod 2$ } for $1 \le i \le m$

$f(v_j) = 1$ if $j \equiv 1 \bmod 2$
$= 2$ if $j \equiv 0 \bmod 2$ } for $1 \le j \le n$

Then the induced labeling for edges

$f^*(uu_i) = 0$ if $i \equiv 1 \bmod 2$
$= 1$ if $i \equiv 0 \bmod 2$ } for $1 \le i \le m$

$f^*(vv_j) = 0$ if $j \equiv 1 \bmod 2$
$= 1$ if $j \equiv 0 \bmod 2$ } for $1 \le j \le n$

Also we have $f(xu) = 1$; $f(uy) = 0$;
$f(yv) = 0$; $f(xv) = 1$; $f(xy) = 1$

We can have the following cases

Case.1 When both m,n are equal and odd

We find that $e_f(0)$ is one more than $e_f(1)$

Case.2 When both m,n are odd , $m \succ n$

We find that $e_f(0)$ is one more than $e_f(1)$

Case.3 When both m.n are odd, $m \prec n$

We find that $e_f(0)$ is one more than $e_f(1)$

Case.4 When both m,n are equal and even

We find that $e_f(1)$ is one more than $e_f(0)$

Case.5 When both m.n are even , $m \succ n$

We find that $e_f(1)$ is one more than $e_f(0)$

Case.6 When both m.n are even, $m \prec n$

We find that $e_f(1)$ is one more than $e_f(0)$

Case.7 When m is even , n is odd

We find that $e_f(0) = e_f(1)$

Case.8 When m is odd, n is even

We find that $e_f(0) = e_f(1)$

Clearly in all the cases mentioned above it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1.

Hence the Jelly Fish $J(m,n)$ graph is 1- Near Mean Cordial graph.

Illustration :

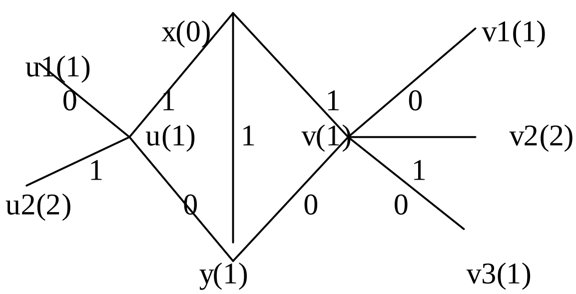

Fig.3.3.1 : $J(2,3)$-Jelly Fish

Illustration :

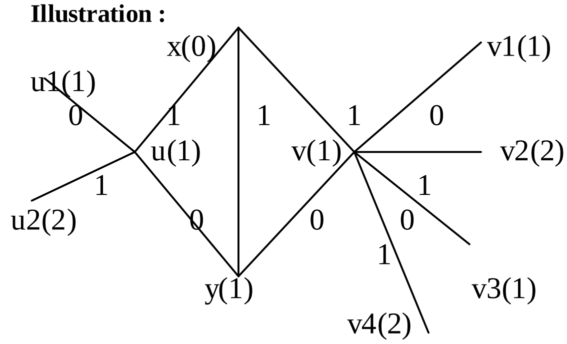

Fig:3.3.1 $J(2,4)$-Jelly Fish

REFERENCES

[1]. Gallian J.A, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics 6(2001)#DS6

[2]. K.Palani,J.Rejila Jeya Surya, 1-Near Mean Cordial Labeling of Graphs,IJMA-6(7),July 2015 PP15-20

[3]. K.Manimekalai, K.Thirusangu, Pair Sum Labeling of Some Special Graphs ,International Journal of Computing Algorithm, Vol.02, October 2013, Page:251-256

[4]. A.Nellai Murugan and G.Esther, Some Results on Mean Cordial Graphs, International Journal of Mathematics Trends and Technology, ISSN: 2231-5373, Volume 11, No.2, July 2014

[5]. Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram, Mean Cordial Labeling of Graphs, Open Journal of Discrete Mathematics 2012,2,145-148.

[6]. Albert William I, Indra Rajasingh and S.Roy, Mean Cordial Labeling of Certain Graphs, Journal of Computer and Mathematical Sciences, Vol. 4, Issue 4,31 August, 2013 Pages(202-321)

[7]. F.Harary, Graph Theory,Addison-Wesley Publishing Company Inc, USA,1969