# Introducing a New Standard Formula for Finding Prime Numbers

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Abstract – This article introduces a new standard formula for finding prime numbers and shows the various methods of determining its solution set. The formula is standard in three ways. Firstly, it reveals the natural location of prime numbers on the sequence of natural numbers. Secondly, there is no prime other than 2 and 3 on the endless sequence of natural numbers that it can skip or fail to locate. Thirdly, it provides a basis upon which other formulas for locating primes can be discovered.

The formula is  $P = 3n_{so} \pm 2$ , where  $n_{so}$  is any special odd number equal to or greater than I, which numbers belong to appropriate solution sets for the formula. The plus and minus operations have each a unique solution set of endless elements.

The variable  $n_{so}$  represents specific odd numbers that satisfies the formula. If appropriate solution sets are identified and their elements used to replace the variable, each and every value to be obtained will be a prime. If elements of these solution sets are systematically substituted for the variable, one after another in their endless chain of succession, the formula will yield each and every succeeding prime beginning with prime 5 and going on without end.

In order to be used effectively, the formula is split it into two complementary ones. These separate but complementary formulas are as follows;

(1)  $P_1 = 3n_{so} + 2$  where  $n_{so}$  is any specific odd number equal to or greater than 1, which

numbers belong to an endless appropriate set of solutions for this particular formula,

(2)  $P_2 = 3n_{so} - 2$  where  $n_{so}$  is any specific odd number equal to or greater than 3, which numbers belong to an appropriate endless set of solutions for this particular formula,

The two formulas complement each other, or take

turns in locating each and every prime on the sequences of natural numbers. Each formulas finds its own unique set of primes, and thereby revealing an unknown fact that there are two different sets of primes. The first set is the set of First Half Pair Primes (FHPPs) which the first formula finds. Such primes extend from 5 and continue endlessly in a hidden perfect regularity. This set of primes is as follows;  $SFHPP = \{5, 11, 17, 23, 29, 41,$ 47, 53, 59, 71, 83, 89, 101,... }.The second set is the set of Second Half Pair Primes (SHPPs) which the second formula finds. Such primes begin from prime 7 and continue endlessly in a hidden perfect regularity. This set of primes is as follows; SSHPP = {7, 13, 19, 31, 37, 43, 61, 67, 73, 79, 97, 103, 109,...}. Each formula has its own unique solution set of endless elements on the sequence of odd numbers. In either case, solution set elements are those odd numbers on the sequence that are not 'related' to any prime on the sequence of natural numbers. If any odd number other than such numbers is substituted for the variable in the new primes' formula, the result will be a composite odd number whose initial divisor is a prime or primes to which that odd number 'relates'.

For example, with regard to the first half pair primes formula, if any odd number whose last digit is 1 other than 1 itself, is substituted for the variable, the value of the expression will be a composite odd number whose initial divisor is prime 5. If any odd number of the form (14N + 11) - 14 where N is any natural number, is substituted for the variable, the value of the expression will be a composite odd number divisible by prime 7. If any odd number of the form (22N + 25) - 22, where N is any natural number, is substituted for the variable in the formula, the value of the expression will be a composite odd

number divisible by prime 11. If any number of the form (26N + 21) - 26, where N is any natural number is used, the value of the expression will be a composite odd number divisible by prime 13.

With regard to the second half pair primes formula, if any odd number whose last digit is 9 including 9 itself is substituted for the variable, the value of the expression will be a composite odd number whose initial divisor is prime 5. If any number of the form (14N + 17) - 14 where N is any natural number, replaces the variable, the value of the expression will be a composite odd number divisible by prime 7. If any number of the form (22N + 41) - 22 where N is any natural number, replaces the variable, the value of the expression will be a composite odd number divisible by prime 11. If any number of the form (26N + 57) - 26 where N is any natural number is used, the value of the expression will be a composite odd number divisible by prime 13.

On the other hand, if appropriate odd numbers not 'related' to any prime are used for either case the values of the expressions will be definite primes.

The means of isolating elements of the solution sets from none elements on the sequence of odd numbers include the use of both formulas and tables of systematic structures. The article identified two types of such tables. These are those that show the distribution of none substitute elements on the sequence of odd numbers and those that indicate such numbers' numerical positions on it.

Summary – This article introduces a new standard formula for finding prime numbers and provides the various methods of determining its solution set. The formula is standard in three ways. Firstly, it reveals the natural location of prime numbers on the sequence of natural numbers. Secondly, there is no prime other than 2 and 3 on the endless sequence of natural numbers that it can skip or fail to locate. Thirdly, it provides a basis upon which other formulas for locating primes can be discovered.

The formula is  $P = 3n_{so} \pm 2$ , where  $n_{so}$  is any special odd number equal to or greater than 1, which numbers belong to appropriate solution sets for the formula.

The variable n<sub>so</sub> represent specific odd numbers that satisfies the formula. The depressed 'so' at the baseline of the variable emphasizes the fact that it is not any natural number that can be used as a substitute for the variable, but only specific odd numbers that are elements of appropriate solution sets for the formula. If appropriate solution sets are identified and their elements used to replace the variable, each and every value to be obtained will be a prime. If elements of these solution sets are systematically substituted for the variable, one after another in their endless chain of succession, the formula will yield each and every succeeding prime beginning with prime 5 and going on without end.

The formula is effectively used by splitting it into two complementary ones. These separate but complementary formulas are as follows;

 $P_1 = 3n_{so} + 2$  where  $n_{so}$  is any specific odd number equal to or greater than 1, which numbers belong to an endless appropriate set of solutions for this particular formula,

 $P_2 = 3n_{so} - 2$  where  $n_{so}$  is any specific odd number equal to or greater than 3, which numbers belong to an appropriate endless set of solutions for this particular formula,

The two are complementary in the sense that they complement each other, or take turns in locating each and every prime on the sequences of natural numbers. In actual fact, each of the two formulas finds its own unique set of primes, and thereby revealing an unknown fact that there are two different sets of primes. These are; the set of first half pair primes (SFHPP) and the set second half pair primes

(SSHPP).

The first formula is for finding First Half Pair Primes (FHPPs). Such primes extend from 5 and continue endlessly in a hidden perfect regularity. This set of primes is as follows; SFHPP = {5, 11, 17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101,...}. The second formula is for finding Second Half Pair Primes (SHPPs). Such primes begin from prime 7 and continue endlessly in a hidden perfect regularity. This set of primes is as follows; SSHPP = {7, 13, 19, 31, 37, 43, 61, 67, 73, 79, 97, 103, 109,...}.

Each of the two complementary formulas has its own unique solution set of endless elements. The elements of the solution sets for the two formulas can be identified by using any appropriate method. In this text, the method used is that of identifying and eliminating non-substitute elements from the sequence of odd numbers to leave only elements of the solution sets up to any given extent/ any selected section of the sequence. In this regard the text displays various tables that show the distribution of non-substitute elements on the sequence, and formulas that may be used to locate such elements on any section of the sequence. Two types of tables have been identified, these are, those that reveal the distribution of actual non-substitutes on the sequence and those that do so indirectly by revealing only their numerical positions on it.

With regards to the FHPPs formula, the table that shows the distribution of its variable's non-substitutes is table 4 below (It is table 4 because it is the forth table in the main text of this article)

Table 4: Table of Natural distribution of odd numbers that must not be substituted for  $n_{so}$  in the formula for finding first half pair primes ( $P_1 = 3n_{so} + 2$ ).

G.12	G.10	G.22	G.34	G.46	G.58	G.70	G.82	G.94	G.106	G.118	G.130	
G.14	11	25	39	53	67	81	95	109	123	137	151	•••
G.26	21	47	73	99	125	151	177	203	229	255	281	•••
G.38	31	69	107	145	183	221	259	297	335	373	411	•••
G.50	41	91	141	191	241	291	341	391	441	491	541	•••
G.62	51	113	175	237	299	361	423	485	547	609	671	•••
G.74	61	135	209	283	357	431	505	579	653	727	801	•••
G.86	71	157	243	329	415	501	587	673	759	845	931	•••
G.98	81	179	277	375	473	571	669	767	865	963	1061	•••
G.110	91	201	311	421	531	641	751	861	971	1081	1191	•••
G.122	101	223	345	467	589	711	833	955	1077	1199	1321	•••
G.134	111	245	379	513	647	781	915	1049	1183	1317	1451	•••
G.146	121	267	413	559	705	851	997	1143	1289	1435	1581	•••
•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	

The table above is a display of an easy to appreciate pattern of endless rows and columns of odd numbers, inclusive of primes and composites, none of which must be substituted for variable  $n_{\rm so}$  in the formula for finding FHPPs. It continues endlessly in an ascending order of perfect regularity, as shown by headers in the first row and column.

We are able to use table 4 above to determine elements of the solution set for the FHPPs formula because it indicates for us which numbers on the sequence of odd numbers are not elements of the solution set. In other words, the solution set for the first complementary formula comprises of each and every odd number, from unit endlessly, that is not an element of the endless structure of odd numbers displayed in on table 4 above. Consequently, we can, with the above table, determine the solution set for the FHPPs' formula as being as follows; SSFHPPF = {1, 3, 5, 7, 9, 13, 15, 17, 19, 23, 27, 29, 33, 35, 37, 43, 45, 49, 55, 57, 59, 63, 65,...}

Note that elements of this set are so systematically laid out that when each and every element is substituted for the variable in the formula, one after another, from the first to each and every one of them in their endless chain of succession, there is not a single FHPP on the entire sequence of natural numbers that will not be found.

It is also self evident from the above table that any odd number whose last digit is 1 (unit), other than 1 itself, cannot be substituted for the variable in this particular formula, because all values to be obtained are composites, all of whose initial divisor is prime 5.

The solution set for the FHPPs formula can also be determined by knowledge of the distribution, on the sequence of odd numbers, of numerical positions of non-substitutes, for the variable in the formula. Table 8 below reveals this distribution.

TABLE 8. THE DISTRIBUTION OF NUMERICAL POSITIONS, ON THE SEQUENCE OF ODD NUMBERS, OF NON-SUBSTITUTES FOR THE VARIABLE IN THE FIRST HALF PAIR PRIMES FORMULA.

	7	13	19	25	31	37	43	49	55	61	67	73	79	85	•••
5	6	11	16	21	26	31	36	41	46	51	56	61	66	71	•••
11	13	24	35	46	57	68	79	90	101	112	123	134	145	156	•••
17	20	37	54	71	88	105	122	139	156	173	190	207	224	241	•••
23	27	50	73	96	119	142	165	188	211	234	257	280	303	326	•••
29	34	63	92	121	150	179	208	237	266	295	324	353	382	411	•••
35	41	76	111	146	181	216	251	286	321	356	391	426	461	496	•••
41	48	89	130	171	212	253	294	335	376	417	458	499	540	581	•••
47	55	102	149	196	243	290	337	384	431	478	525	572	619	666	•••
53	62	115	168	221	274	327	380	433	486	539	592	645	698	751	•••
59	69	128	187	246	305	364	423	482	541	600	659	718	777	836	•••
65	76	141	206	271	336	401	466	531	596	661	726	791	856	921	•••
71	83	154	225	296	367	438	509	580	651	722	793	864	935	1006	•••
77	90	167	244	321	398	475	552	629	706	783	860	937	1014	1091	•••
83	97	180	263	346	429	512	595	678	761	844	927	1010	1093	1176	•••
89	104	193	282	371	460	549	638	727	816	905	994	1083	1172	1261	•••
•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••

The endless structure of numbers displayed in table 8 above are counting numbers indicating the distribution of numerical positions of non-substitute elements, on the sequence of odd numbers. Note that since odd numbers are those numbers of the form (2N + 1) - 2, where N is any natural number, variable N in the expression X = (2N + 1) - 2, indicate the position of odd number 'X' on the sequence of odd numbers. For example, if we pick any odd number say 59, this numbers' numerical position on the sequence of odd numbers can be worked out by replacing X with 59 and solving the equation for variable N as follows; (2N + 1) - 2 = 59; 2N = (59 - 1)

1) + 2; N =  $[(59 - 1) + 2] \div 2$ ; N = 30. The value of the expression is 30, meaning that 59 is the  $30^{th}$  number on the sequence of odd numbers. In short, the numerical position of any odd number on the sequence of odd numbers is N =  $[(X + 2) - 1] \div 2$ , where X is that odd number.

With regard to table 8 above, elements of the solution set are those odd numbers whose numerical positions on the sequence, are not part of the endless structure of numbers displayed on the table. In other words, elements of the solution set for the FHPPs formula are those numbers of the form (2N+1)-2, where variable N is any natural number and which

numbers are not elements of the above endless table of natural distribution of numerical positions for the variable's non-substitutes, on the sequence of odd numbers.

If any number, other than the headers in the first row and column, is picked from the table, and substituted for variable N in the expression (2N + 1)- 2, the value of the expression will be a nonsubstitute element, which when substituted for variable no in the FHPPs formula will result in a composite odd number divisible by its two odd number factors indicated as headers of the column and row under which that number falls on the table. On the other hand, if any numerical position, in terms of ordinary counting numbers from 1 endlessly, is not part of the structure of the endless numbers indicated by the table, it can substitute variable N in the expression (2N + 1) - 2, and the value of the expression will be an element of a solution set for the FHPPs formula.

From table 8 above, it can be seen that counting numbers that are not part of the structure of the table, and which, can therefore, be used to pick elements of the solution set from the sequence of odd numbers, include numbers less than 6, and each and every number greater than 6 not falling within the structure of the table. Part of the set of such numbers, as shown by the table is as follows;

If any of the above counting numbers is substituted for N in the expression (2N + 1) - 2, the value of the expression will be an element of a solution set for the FHPPs formula.

The Second half pair primes (SHPPs) formula has two complementary tables that show the distribution of its variable's non-substitutes on the sequence of odd numbers. These tables are as follows:

Table 9 (a) : The first table of natural distribution of odd numbers that must not be substituted for variable  $n_{so}$  in the formula for finding second half pair primes ( $P_2 = 3n_{so} - 2$ ).

G.12	G.14	G.26	G.38	G.50	G.62	G.74	G.86	G.98	G.110	G.122	G.134	
G.14	17	45	59	73	87	101	115	129	143	157	171	
G.26	31	57	83	109	135	161	187	213	239	265	291	
G.38	45	83	121	159	197	235	273	311	349	387	425	
G.50	59	109	159	209	259	309	359	409	459	509	559	
G.62	73	135	197	259	321	383	445	507	569	631	693	
G.74	87	161	235	309	383	457	531	605	679	753	827	
G.86	101	187	273	359	445	531	617	703	789	875	961	
G.98	115	213	311	409	507	605	703	801	899	997	1095	
G.110	129	239	349	459	569	679	789	899	1009	1119	1229	
G.122	143	265	387	509	631	753	875	997	1119	1241	1363	
G.134	157	291	425	559	693	827	961	1095	1229	1363	1497	

TABLE **9(b):** THE SECOND TABLE OF NATURAL DISTRIBUTION OF ODD NUMBERS THAT MUST NOT BE SUBSTITUTED FOR VARIABLE  $n_{so}$  in the formula for finding second half pair primes  $(P_2 = 3n_{so} - 2)$ .

G.12	G.10	G.22	G.34	G.46	G.58	G.70	G.82	G.94	G.106	G.118	G.130	•••
G.10	9	19	29	39	49	59	69	79	89	99	109	•••
G.22	19	41	63	85	107	129	151	173	195	217	239	•••
G.34	29	63	97	131	165	199	233	267	301	335	369	•••
G.46	39	85	131	177	223	269	315	361	407	453	499	
G.58	49	107	165	223	281	339	397	455	513	571	629	•••
G.70	59	129	199	269	339	409	479	549	619	689	759	
G.82	69	151	233	315	397	479	561	643	725	807	889	•••
G.94	79	173	267	361	455	549	643	737	831	925	1019	•••
G.106	89	195	301	407	513	619	725	831	937	1043	1149	•••
G.118	99	217	335	453	571	689	807	925	1043	1161	1279	•••
G.130	109	239	369	499	629	759	889	1019	1149	1279	1409	•••
•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••

Tables 9(a) and 9(b) above, can help us determine the solution set for the SHPPs formula because, they in combination, show which numbers on the sequence of odd numbers, are not elements of the solution set for the formula. In other words, the solution set for the formula comprises of each and every odd number, from 3 endlessly, which is not an element of, or is missing from a combination of the above two endless structures of odd numbers. With the help of the two tables, we can determine the missing odd numbers or the solution set for the formula as being as follows;

SSSHPPF = { 3, 5, 7, 11, 13, 15, 21, 23, 25, 27, 33, 35, 37, 43, 47, 51, 53, 55, 61, 65,...}

Elements of this set begin from 3 and continue endlessly at an ascending order of hidden perfect regularity. Here too, These elements are so systematically laid out that when each and every one of them is substituted for the variable, one after another, from the first element to each and every one of them in their endless chain of succession, there is not a single SHPP on the entire sequence of natural numbers that will not be found.

It is also evident from the complementary tables above that any odd number whose last digit is 9, including 9 itself, cannot be substituted for the variable in this particular formula because all values to be obtained are composites, all of whose initial divisor is prime 5.

The solution set for the SHPPs formula can equally be determined by knowledge of the distribution, on the sequence of odd numbers, of numerical positions of non-substitutes, for the variable in the formula. Tables 12 (a) and 12 (b) below show this distribution.

TABLE 12 (a). THE FIRST TABLE OF THE DISTRIBUTION OF NUMERICAL POSITIONS OF NON-SUBSTITUTES, ON THE SEQUENCE OF ODD NUMBERS, FOR THE VARIABLE IN THE SECOND HALF PAIR PRIMES FORMULA.

	7	13	19	25	31	37	43	49	55	61	67	73	79	85	91	
7	9	16	23	30	37	44	51	58	65	72	79	86	93	100	107	
13	16	29	42	55	68	81	94	107	120	133	146	159	172	185	198	
19	23	42	61	80	99	118	137	156	175	194	213	232	251	270	289	
25	30	55	80	105	130	155	180	205	230	255	280	305	330	355	380	
31	37	68	99	130	161	192	223	254	285	316	347	378	409	440	471	
37	44	81	118	155	192	229	266	303	340	377	414	451	488	525	562	
43	51	94	137	180	223	266	309	352	395	438	481	524	567	610	653	•••
49	58	107	156	205	254	303	352	401	450	499	548	597	646	695	744	•••
55	65	120	175	230	285	340	395	450	505	560	615	670	725	780	835	•••
61	72	133	194	255	316	377	438	499	560	621	682	743	804	865	926	
67	79	146	213	280	347	414	481	548	615	682	749	816	883	950	1017	•••
73	86	159	232	305	378	451	524	597	670	743	816	889	962	1035	1108	•••
79	93	172	251	330	409	488	567	646	725	804	883	962	1041	1120	1199	•••
85	100	185	270	355	440	525	610	695	780	865	950	1035	1120	1205	1290	•••
91	107	198	289	380	471	562	653	744	835	926	1017	1108	1199	1290	1381	•••
•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••

TABLE 12 (b). THE SECOND TABLE OF THE DISTRIBUTION OF NUMERICAL POSITIONS, ON THE SEQUENCE OF ODD NUMBERS, OF NON-SUBSTITUTES, FOR THE VARIABLE IN THE SECOND HALF PAIR PRIMES FORMULA.

	5	11	17	23	29	35	41	47	53	59	65	71	77	83	89	
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	
11	10	21	32	43	54	65	76	87	98	109	120	131	142	153	164	
17	15	32	49	66	83	100	117	134	151	168	185	202	219	236	253	•••
23	20	43	66	89	112	135	158	181	204	227	250	273	296	319	342	•••
29	25	54	83	112	141	170	199	228	257	286	315	344	373	402	431	•••
35	30	65	100	135	170	205	240	275	310	345	380	415	450	485	520	•••
41	35	76	117	158	199	240	281	322	363	404	445	486	527	568	609	
47	40	87	134	181	228	275	322	369	416	463	510	557	604	651	698	
53	45	98	151	204	257	310	363	416	469	522	575	628	681	734	787	
59	50	109	168	227	286	345	404	463	522	581	640	699	758	817	876	•••
65	55	120	185	250	315	380	445	510	575	640	705	770	835	900	965	•••
71	60	131	202	273	344	415	486	557	628	699	770	841	912	983	1054	•••
77	65	142	219	296	373	450	527	604	681	758	835	912	989	1066	1143	•••
83	70	153	236	319	402	485	568	651	734	817	900	983	1066	1149	1232	•••
89	75	164	253	342	431	520	609	698	787	876	965	1054	1143	1232	1321	•••
			•••	•••	•••	•••	•••	•••	•••	•••	•••	•••		•••		

Tables 12 (a) and 12(b) above, in combination, show the distribution of numerical positions, on the sequence of odd numbers, of non-substitutes, for the variable in the SHPPs formula. Table 12 (a) indicates the numerical positions of non-substitutes relating to multiples of visible divisors of second half pair odd numbers, while table 12 (b) shows the numerical positions of non substitutes relating to multiples of their invisible divisors.

The two tables can help us determine the solution set for the SHPPs formula because they indicate numerical positions of non-substitute elements on the sequence of odd numbers. In essence, they help us to separate, on the sequence of odd numbers, nonsubstitute elements from elements of the solution set. In short, any odd number on the sequence, whose numerical position is not part of the structure of numbers on either table is an element of the solution set. This means that if any number, other than the headers in the first row and column of either table is picked and substituted for variable N in the expression (2N + 1) - 2, the value of the expression will be a non-substitute element, which when substituted for variable no in the SHPPs formula, will result in a composite odd number divisible by its two odd number factors indicated as headers of the column and row under which that number falls.

On the other hand, if any numerical position, in terms of counting numbers, is not part of the structure of the endless numbers indicated by both tables, it can substitute variable N in the expression (2N+1)-2, and the value of the expression will be an element of a solution set for the second half pair primes formula.

From the twin tables above, it can be seen that counting numbers that are not part of the structure of either table and which, can therefore, be used to pick elements of the solution set from the sequence of odd numbers, include numbers less than 5 and 9, other than unit and each and every number greater than 5 and 9 not falling within the structure of either table. Part of the set of such numbers, as evidenced by both tables, is as follows:

{2,3,4,6,7,8,11,12,13,14,17,18,19,22,24,26,27,28,31, 33,34, 36,38,39,41,46,..}

Note that if any of the above counting numbers is substituted for N in the expression (2N + 1) - 2, the value of the expression will be an element of a solution set for the SHPPs formula.

#### I. INTRODUCTION

This article, introduces a new standard formula which can find prime numbers without having to stumble upon composite numbers. The article gives background information on the formula and presents tables and formulas for determining its solution set. The formula is standard because it reveals the natural location of prime numbers on the sequence of natural numbers. It is standard because there is no prime other than 2 and 3 on the endless sequence of natural numbers that it can skip or fail to locate. It is also standard because it provides a basis upon which other formulas for locating primes can be found.

Wikipedia, the Free Encyclopedia has noted that there is no known useful formula that yields only primes and no composites

(http://en.wikipedia.org/wiki/Prime\_number 29.9.2010) (7/18/2012 11: 07). True indeed, the hitherto prime search history has been of frustrations, disappointments and uncertainties with regard to how far identified formulae will keep on yielding primes without having to stumble upon composites and crashing to a halt.

There are specific example of Pierre de Fermat who conjectured that all numbers of the form 2 raised to the power of  $(2^n) + 1$ , were n is an integer equal to or greater than 1, are primes. Fermat is said to have been able to verify his claim only up to 2 raised to the power  $(2^4) + 1$ , the shortened version of which is  $2^{16}$ + 1. This claim is said to have been invalidated by Euler who proved that the very next Fermat number  $(2^{32} + 1 = 4294967297)$  was a composite with 641 as one of its factors (divisors). The same fate befalls other primes such as the Sophie Germaine Primes of the form 2P + 1 = P and the Primorial or Primorial Factorial Primes of the form  $P=n^*+1$  or  $P=n^*-1$ . Last but not least, there is uncertainty with regard to the extent of the spread of Mersenne Primes whose form is  $P=2^{q}-1$ , where q is a prime number. Just like many others, even this most popular formula does not always yield primes. For instance, when prime 11 is substituted for q in the Mersenne formula the difference is 2047 which is a composite number whose initial divisor is 23.

#### A. The Mersenne Formula and its Limitations.

The first major limitation of the Mersenne formula is that it relies on primes to find other primes. The weakness with this approach is that primes to be substituted for the variable in the formula may not be readily available and more especially that the substitution of any known prime for the variable in the formula does not necessarily yield another prime in order for it to have its own reservoir of primes to use in finding further primes. The second major weakness is that the formula only yields selective primes and cannot be used to find all primes on the sequence of natural numbers. Furthermore, no one knows for sure if these primes are endless. So far, there are only 46 known Mersenne Primes (http://www.mersenne.org/default.php 25/08/2011).

#### B. The Sieve of Eratosthenes and its Limitation

The Sieve of Eratosthenes (ca 240) is undoubtedly impeccable and precise in establishing any prime number on the sequence of natural numbers. In my view, its major contribution to the theory of prime numbers is its underlining assumption that composite numbers are but multiples of primes. However, apart from the known limitation that it is only suitable for smaller identifying primes (http://primes.utm.edu/prove2\_1.html 25.08/2011), there is another limitation. This other limitation is that in order to use it to find primes we first have to have knowledge of some initial finite set of primes. To identify primes falling within any given range of numbers on the number line, we first have to generate multiples of all primes up to the square root of the maximum number up to which we want to establish primes and thereafter, strike off all these multiples from a complete list of natural numbers up to our set maximum number. If we wish to limit ourselves to multiples of odd primes only, we first generate all odd numbers up to a certain limit and then strike off all multiples of odd primes up to that limit (Chris http://primes.utm.edu/prove2 1.html Caldwell. 25.08/2011).

The weakness with this method is that it presupposes that we already have a method other than this method that will establish for us the initial prime/ set of primes that we will use to generate the multiples we will require to find further primes. Granted, as Caldwell demonstrates, identifies 3 as the initial odd prime, which will then lead us to the next prime and thereafter, another prime and then to another in that order going on without end. However, in the absence of such a systematic lead to other primes without any breaks in continuity, it would not be that easy to determine a set of primes that we may use to find further primes from ranges of odd numbers commencing with odd numbers other than 3 itself. For instance if we wish to establish primes from say 2,221 to 3, 2221. We have to have a prior knowledge of odd primes whose multiples fall within this specific range of numbers so that we may strike those multiples off that range of odd numbers to leave only primes.

In my opinion, the most successful method of generating primes must not involve primes to find other primes, because we are then starting with an assumption that initial primes are readily available for use. A better approach is to find primes regardless of themselves. I submit further that any successful formula for finding primes must be able to locate each and every prime on the sequence of natural numbers and that such a formula must never stumble upon any composite.

The rest of this article provides background information on the new formula and presents tables and formulas for determining its solution set. Such tables include tables of natural distribution of the formula's non-substitute elements on the sequence of odd numbers and those that reveal their numerical positions on the sequence. The significance of such tables is that they enable the isolation of elements of the formula's solution set from non-solution set elements on the sequence of odd numbers.

#### II. BACKGROUND TO THE NEW FORMULA

There are certain facts about primes upon which the new formula for locating them is based. These include; the existence of specific sets of odd numbers within which primes are found, the determination of actual divisors of multiples falling within those specific prime bearing odd numbers and the perfect regularity in the distribution of those multiples on the sequence of those prime bearing odd numbers.

#### A. Paired Odd Numbers

All primes other than 2 and 3, do not occur anyhow on the sequence of natural numbers. The overall structure of their spread is such that they only occur within a set of paired odd numbers (SPON) with a difference of 2 between them. This is a fact whose main proof is the presence of twin primes on the sequence. Paired odd numbers (PON) are located in between each and every pair of odd multiples of 3 on the sequence. In this text the expression for these odd multiples of 3 is  $3n_0$  where the variable  $n_0$  represents any odd number on the sequence. The depressed  $n_0$  below the baseline of the variable in the expression, even though not conventional, is meant to identify this particular variable with odd numbers as its only values.

In between each and every pair of odd multiples of

3, there is a total of five other numbers. Three of these in between numbers are even and only two are odd, and therefore the only candidates for primes. The spread of these in between numbers is such that the first number after the first odd multiple of 3, is always an even number. The even number is then followed by the first in between odd number, which is followed by another even number, which apparently is also divisible by 3, after that there is another odd number, followed by an even number after which comes the second odd multiple of 3 to close that particular section and open another section ahead with an even number. The following pattern shows how the numbers are spread out;

 $3n_o$ , even, <u>odd</u>, even, <u>odd</u>, even,  $3n_o$ , even, <u>odd</u>, even, <u>odd</u>, even,  $3n_o$ ...

In the above illustration, positions of odd numbers other than odd multiples of 3 have been underlined. These are the numbers that are being referred to as PONs because they are not only odd but exist in pairs as well. They are the only candidates for primes because all primes other than 2 and 3, be it Mersenne, Fermant, Sophie Germain, Premorial or Prime Factorial, are all elements of either of the two sets of numbers. which numbers are located in between pairs of odd multiples of 3. Prime 5 is a sum of 3 + 2, Prime 7, is the difference of 9 - 2. Prime 11 is the sum of 9 + 2. Prime 13, is the difference of 15 - 2, it is like that for each and every prime endlessly. In this text, the set of these paired odd numbers is presented as follows:

SPON = {5,7, 11,13, 17,19, 23,25, 35,37, 41,43, 47,49, ...}

Elements of this set begin with pair 5,7 and proceed endlessly in ascending order at a uniform gap of 4. Isolating the set of paired odd numbers from the sequence of natural numbers implies eliminating from the number line all numbers positioned in places not underlined in the above illustration. These places appear in bold in the following illustration;

 $3n_o$ , even, odd, even, dd, even, dd, even, dd, even, odd, even, dn...

Excluded from the number line, are essentially two sets of numbers. These are 2 and its endless multiples (the set of even numbers) and 3 and its endless multiples

#### The Six Column Table of Natural Numbers

To illustrate further the natural spread of paired odd numbers on the sequence of natural numbers, all natural numbers other than 1, 2 and 3, can be arranged into six columns as follows;

TABLE 1: THE SIX COLUMN TABLE SHOWING THE DISTRIBUTION OF PAIRED ODD NUMBERS, ON THE SEQUENCE OF NATURAL NUMBERS GREATER THAN 3 (This table must be read horizontally).

	4	5	6	7	8
9	10	11	12	13	14
15	16	17	18	19	20
21	22	23	24	25	26
27	28	29	30	31	32
33	34	35	36	37	38
39	40	41	42	43	44
45	46	47	48	49	50
51	52	53	54	55	56
57	58	59	60	61	62
63	64	65	66	67	68
69	70	71	72	73	74
75	76	77	78	79	80
81	82	83	84	85	86

In table 1, above, paired odd numbers are in columns 3 and 5, both of which are in bold font. As indicated already these are the only two columns on the endless sequence of natural numbers where prime numbers of either set can be located. Numbers in columns 2, 4, and 6 are ultimately elements of the set of even numbers which set is of the form (2N+4)-2 where N is any natural number. Numbers in column 1 are ultimately odd multiples of 3, all of whose ultimate form is (6N+9)-6 where N is any natural number.

Note that table 1 above exclude natural numbers less than 4 because of their distortion effect on it. For instance, it has been indicated that primes 2 and 3 are the only primes on the entire sequence of natural numbers which do not belong to either set of paired odd numbers. Instead, 3 belongs to column 1 where it is the only prime in that endless sequence of numbers, and 2 belong to column 6 where it is the only prime on that endless sequence of numbers

# B. Splitting the Set of Paired Odd Numbers into Two

The set of paired odd numbers is split into two different sets, these are; the set of first half pair odd numbers (SFHPON) and the set of second half pair odd numbers (SSHPON).

In table 1 above, the first set is indicated by column 3. Its elements are as follows; SFHPON =

 $\{5, 11, 17, 23, 29, 35, ...\}$ . These elements are of the form (6N + 5) - 6 were N is any natural number. The second set is indicated by column 5, Its elements are as follows; SSHPON =  $\{7, 13, 19, 25, 31, 37, 43, 49, ...\}$ . Elements of this second set are of the form 6N + 7 - 6 were N is any natural number.

There are a three main reasons for splitting the set of paired odd numbers into two. Firstly, it enables the systematic location of primes. Secondly, elements of the two sets are positioned differently from each other on the sequence of natural numbers. Table 1 above partly confirms this fact. Last but not least, multiples within the two sets do not share common sets of divisors in exactly the same way. Putting it differently, even though essentially the two sets have the same sets of divisors, their respective multiples are not the same. For example, whereas multiples of 5 in the set of first half pair odd numbers are of the form (30N + 35) - 30 where N is any natural number , multiples of 5 in the set of second half pair odd numbers are of the form (30N + 25) - 30, where N is any natural number. These different sets of multiples are as follows:

 $30N + 35 - 30 = \{35, 65, 95, 125, 155, 185, 215, 245, 275 ...\}$  $30N + 25 - 30 = \{25, 55, 85, 115, 145, 175, 205, 235, 265...\}$ 

#### C. Splitting the Set of Primes into Two

Just as it is necessary to split the set of paired odd numbers into two, it is equally necessary to split the set of primes into two; the set of first half pair primes (FHPP)] and the set of second half pair primes (SHPP)]. The split is owed to the fact that the unified set of primes is a subset of the unified set of paired odd numbers. FHPPs are a subset of the set of FHPONs and SHPPs a subset of SHPONs.

Elements of the two sets of primes are different from each other in terms of their location on the sequence of natural numbers. The Position of FHPPs on the sequence is indicated by the form  $3n_o+2$  where the variable is an element of the solution set for the first formula. The position of SHPPs is of the

form  $3n_o - 2$  where the variable is an element of the solution set for this second formula. With regard to their locations on the sequence of paired odd numbers, FHPPs primes are entirely located on the sequence of first half pair odd numbers (FHPONs) which numbers are of the form (6N + 5) - 6, where N is any natural number. The position of such primes on this particular sequence is of the form  $N = [(P_1 + 6) - 5] \div 3$ , where  $P_1$  is a confirmed prime. N in this regard is the numerical position of that confirmed prime on the sequence of FHPONs.

On the other hand, SHPPs are entirely located on the sequence of second half pair odd numbers (SHPONs) which numbers are of the form (6N+7)-6, where N is any natural number. The position of such primes on that sequence is of the form  $N = [(P_2 + 6) - 7] \div 3$ , where  $P_2$  is a confirmed prime. Variable N in this respect is the numerical position of that confirmed prime on the sequence of SHPONs.

In short, Table 1 above illustrates this point further. FHPPs are only located in column 3 of the table, while SHPPs will only be found in column 5 of the table.

Proof

We can determine both the location of any prime on the sequence of natural numbers and on the sequence of paired odd numbers by picking any known primes at random, say 83267, 76697, and proceeding as follows;

To determine the form to which each of the above primes belong, and to be able to determine their exact position on the sequence of natural numbers, we can test for the first form by using the expression  $n_0 = (P_1 - 2) \div 3$  and for the second form by the expression  $n_0 = (P_2 + 2) \div 3$ . In other words, for the first form, we subtract 2 from a known prime and divide the difference by 3 to determine if the difference is an odd multiple of 3. For the second form, we add 2 to a known prime and divide the sum by 3 to determine if the difference is an odd multiple of 3. We should also take note of the fact that if one form fails, then the prime in question belong to the

other form.

#### Example 1: Set membership of Prime 83267

 $n_o = (P_1 - 2) \div 3$ ;  $n_o = (83267 - 2) \div 3$ ;  $n_o = 27755$ . Since the value of the expression is a whole number, it means that the difference of 83267 - 2 is an odd multiple of 3. Therefore, It is confirmed that prime 83267, is a FHPP ( $P_1 = 3n_o + 2$ ). It also means that the exact location of prime 83267 on the number line is two scale marks ahead of, or to the right of 83265 which we have established to be an odd composite number divisible by 3.

We can also locate the actual positions of the above randomly picked prime on the sequence of FHPONs (column 3 of table 1), as follows;  $N = [(P_1 + 6) - 5] \div 6$  where  $P_1$  is a confirmed prime;  $N = [(83267 + 6) - 5] \div 6$ ; N = 13878

The value of the expression is 13878, which is a whole number. This confirms that prime 83267 is indeed an element of column 3 of the six column table and that its numerical position on the sequence of first half pair old numbers (column 3 of table 1) is 13878. Note that if the value of the expression turns out to be a mixed number, then the prime in question has no numerical position among elements in column 3, meaning that it is not an element of the set of first half pair odd numbers.

### Example 2: Set Membership for Prime 76697,

We test the prime's membership by using either of the two following forms;

$$n_o = (P_1 - 2) \div 3$$
;  $n_o = (P_2 + 2) \div 3$ 

 $n_o=(P_1-2)\div 3;\, n_o=(76697-2)\div 3;\, n_o=25565;$  therefore prime 76697 is also a FHPP. Its specific location on the number line is two scale marks ahead of 76695 which is an odd composite number divisible by 3.

The location of this prime on column 3 of the six column table can be confirmed as follows;

$$N = \{ |(P_1 + 6)| - 5\} \div 6 ; N = \{ |(76697 + 6)| - 5\} \div 6;$$

$$N = 12.783$$

This confirms that this prime is element number 12783 of the set of first half pair odd numbers and that its actual location on the six column table is

column 3 of row number 12783. Note that if any confirmed prime fails the test for membership of the first set, it will definitely pass the test for membership of the second set, and vice versa.

# Set Membership for Mersenne Primes.

As already stated, any prime other than 2 and 3, must be an element of either of the two sets. To prove this point, we can identify the set membership for the second, third and fourth Mersenne primes corresponding to P = (3,5,7) in the Mersenne formula. This set of primes is as follows;  $\{7,31,127\}$ . Set membership for each of these primes can be established as follows;

Test for either of the following 2 forms;

$$n_o = (P_1 - 2) \div 3$$
;  $n_o = (P_2 + 2) \div 3$ 

### Set Membership for Primes 7, 31 and 127

We pick the second form because it is evident from the text above that all the above three primes are elements of the set of second half pair primes. We can confirm this as follows; Prime 7;  $n_o = (P_2 + 2) \div 3$ ;  $n_o = (7 + 2) \div 3$ ;  $n_o = 3$ . Note that the specific location of prime 7 on the sequence of natural numbers is two scale marks on the left of 9, (7 + 2 above ) which is an odd multiple of 3 as the above expression confirms

Prime 31; 
$$n_o = (P_2 + 2) \div 3$$
;  $n_o = (31 + 2) \div 3$ ;  $n_o = 11$ 

Note that the specific location of prime 31 on the sequence of natural numbers is two scale marks on the left of 33, (31 + 2 above) which is also an odd multiple of 3 as the above expression confirms.

Prime 127; 
$$n_o = (P_2 + 2) \div 3$$
;  $n_o = (127 + 2) \div 3$ ;  $n_o = 43$ .

Note that the specific location of prime 127 on the sequence of natural numbers is two scale marks on the left of 129, (127 + 2 above) which is yet another odd multiple of 3 as the above expression confirms.

Since the values of all the three expressions are whole numbers it is confirmed that all the three Mersenne primes are elements of the set of SHPPs. Furthermore, Note that if any of these examples of the Mersenne primes were tested for membership of FHPPs the values of all the three expressions will be mixed numbers.

The location of Mersenne Primes 7, 31 and 127 on

the sequence of second half pair odd numbers (column 5 of the six column table) can also be confirmed as follows;

$$N = \{[(P_1 + 6)] - 7\} \div 6; \ N = \{[(7 + 6)] - 7\} \div 6; \ ; \ N = 1$$

$$N = \{[(P_1 + 6)] - 7\} \div 6; \ N = \{[(31 + 6)] - 7\} \div 6; \ N = 5$$

$$N = \{[(P_1 + 6)] - 7\} \div 6; \ N = \{[(127 + 6)] - 7\} \div 6; \ N = 21$$

The above expressions confirm that all the three Mersenne primes indicated above are elements of the set of second half pair odd numbers (SHPONs) which set is represented by column 5 of the six column table. The above expressions show that prime 7 is the first element of this set. Its actual position on the six column table is column 5, row number 1. Prime 31 is the 5<sup>th</sup> element of this set. Its location is column 5 row number 5. Prime 127 is element number 21 in this set. Its actual position on the six column table is column 5, row number 21.

It is likely, (subject to further investigation) that all Mersenne primes are SHPPs. It is also probable that Mersenne composites are elements of the set of SHPONs. One example is Mersenne composite 2047 generated by the form  $P = 2^q - 1$  where q is prime 11.

In the expression  $2047 = 3n_o - 2$ , variable  $n_o$  is 683 which is a whole number. On the other hand, in the expression  $2047 = 3n_o + 2$ , variable  $n_o$  is a mixed number meaning that composite number 2047 is not an element of the set of first half pair odd numbers (FHPONs) but of the second set. We can also confirm the location of this Mersenne composite on the six column table as follows;

$$N = \{[(P_1 + 6)] - 7\} \div 6; N = \{[(2047 + 6)] - 7\} \div 6; N$$
= 341

This confirms that Mersenne composite number 2047 is element number 341 in the set of SHPONs. Its actual location on the six column table is column 5, row number 341.

# Set Membership for Pierre de Fermat Prime and Composite

The Pierre de Fermat prime of the form  $2^{16} + 1$  can be tested for membership of the first set of primes as follows;  $n_o = (P_1 - 2) \div 3$ ;  $n_o = (65536 - 2) \div 3$ ;  $n_o = 21844.67$ ; and for membership of the second set as

follows;  $n_o = (P_2 + 2) \div 3$ ;  $n_o = (65536 + 2) \div 3$ ; = 21846.

Equally, the Fermat composite of the form  $2^{32} + 1$  can be tested for membership of the set of first half pair odd numbers as follows;  $N = [(X + 6) - 5] \div 6$ ;  $N = [(4294967297 + 6) - 5] \div 6$  N = 715827883 and for the second set as follows;  $N = [(X + 6) - 7] \div 6$ ;  $N = [(4294967297 + 6) - 7] \div 6$ ; N = 715827882.667.

The above expressions has established that the Fermat prime is a first half pair prime (FHPP), while the Fermat composite is an element of the set of first half pair odd numbers (FHPONs). With regard to the location of these two numbers on the six column table, it has been established that both the prime and the composite are located in column 3 but on different rows. The location of the former is row 21846 while that of the latter is row 715827883

# D. Divisors of Natural Numbers

Each and every number on the sequence of natural numbers is an even divisor of numbers ahead of it and which numbers are located at intervals equal to its absolute value. The first natural number, which is 1, divides each and every number ahead of it. 2 divides every second number ahead of it. 3 divides every third number. It is like this for each and every succeeding number endlessly. However, whereas each and every number on the sequence is an even divisor of succeeding numbers located at intervals equal to its absolute value, there are some numbers whose location on the sequence is not equivalent to any of its preceding numbers' absolute values or division intervals. Such are the prime numbers which only 1 divides evenly because it is the only number on the sequence whose division interval being 1, skips no number on the sequence.

#### **Most Appropriate Divisors**

There are some divisors that are themselves divisible by divisors that precede them. These are divisors that are located at intervals equal to the absolute values of preceding divisors. Multiples of such divisors are merely subsets of sets of multiples of their initial divisors which are themselves indivisible. In essence therefore, only primes numbers are the ultimate or most appropriate divisors of each and every divisible number on the sequence

of natural numbers. The implication of this is that the most appropriate set of divisors to use in testing the prime status of any odd number is not just a set of any odd numbers less than the square root of that number but only prime numbers up to the square root of that number.

#### E. Divisors of Paired Odd Numbers

In this text, the attention is not on each and every divisor on the sequence of natural numbers but only on divisors of paired odd numbers (PONs) because as has been indicated already this is the only set of numbers in which all primes other than 2 and 3, are located. As mentioned earlier, there are two sets of PONs; These are; the set of first half pair odd numbers (FHPONs) within which first half pair primes (FHPPs) are found, and the set of second half pair odd numbers (SHPONs) which is the location for second half pair primes (SHPPs). The two sets have each, two sets of divisors unique to itself.

The set of FHPONs has two sets of divisors unique to itself. These are the set of visible divisors and the set of invisible divisors.

The set of visible divisors, is a set of elements of the form (6N + 5) - 6 where N is any natural number. This set is as follows; {5, 11, 17, 23, 29, 35, 41....} Elements of this set begin with 5 and proceed endlessly in ascending order at a uniform gap of 6. In this text, this set has been called 'the set of visible divisors of first half pair odd numbers' because being the same numbers on the sequence of first half pair odd numbers they are self evident divisors of multiples among those numbers. This is so because each and every number appearing on the sequence has an endless chain of multiples ahead of it, which multiples are situated at intervals equal to its absolute value. The first divisor 5, divides every fifth number on the sequence. The second divisor 11, divides every eleventh number, the third number which is 13 divides, every thirteenth number, and so on in that order endlessly. However, unlike divisors on the sequence of natural numbers whose absolute values not only equal their division intervals but also the actual difference between each and every one of their endless multiples, the absolute value of visible divisors of first half pair odd numbers do not equal the actual differences between each and every one of their endless multiples on that particular sequence. Instead, actual differences between each and every multiple (actual division interval) of any given first half pair odd number divisor is of the form 7N-N where N is any such divisor and where 7N is that divisor's first multiple on the sequence of first half pair odd numbers.

#### **Example**

Pick any divisor of the form 6N + 5 - 6 where N is any natural number, let us say 641, and establish its first multiple on the sequence of first half pair odd numbers and the difference between each and every one of its multiples on the sequence. To establish this number's first multiple on the sequence, we substitute 641 for N in the formula X = 7N and evaluate the expression as follows;  $X = 7 \times 641$ : X =4,487. The value of the expression 4,4387 is divisor 641's first multiple on the sequence. To establish the difference between each and every one of divisor 641's multiples on the sequence, we substitute 641 for N in the formula X = 7N - N; and evaluate the expression as follows;  $X = (7 \times 641) - 641$ ; X =3846. The value of the expression is 3846. This means that even if divisor 641 evenly divides every 641st number on the sequence of first half pair odd numbers, the actual difference between each and every one of its endless multiples is not its absolute value but 3846 which figure is six times its absolute value.

The second set of divisors of first half pair odd numbers which has been called 'the set of invisible divisors of first half pair odd numbers' is a set of elements of the form (6N + 7) - 6 where N is any natural number. This set is as follows  $\{7, 13, 19, 25, 31, 37, 43, 49 ...\}$  Elements of this set begin with 7 and proceed endlessly in ascending order at a uniform gap of 6. This set has been named 'the set of invisible divisors of first half pair odd numbers' because none of its elements are elements of the set of first half pair odd numbers and yet each and every element of this set of divisors has endless multiples within the set of first half pair odd numbers. In short, the invisibility is owed to the fact that despite having

multiples within the set of first half pair odd numbers, actual elements of this second set divisors are not part of the set of first half pair odd numbers. Invisible divisors' first multiples on the sequence of first half pair odd numbers are of the form 5N where N is any such divisor. The differences between each and every multiple of any of these divisors is of the form 5N + N where N is any such divisor.

#### **Example**

Pick any invisible divisor of the form (6N + 7) - 6where N is any natural number, let us say 113, and establish its first multiple on the sequence of first half pair odd numbers and the difference between each and every one of its multiples on the sequence. To establish this number's first multiple on the sequence, we substitute 113 for N in the formula X = 5N and evaluate the expression as follows;  $X = 5 \times 113$ : X =565. The value of the expression 565 is invisible divisor 113's first multiple on the sequence. To establish the difference between each and every one of invisible divisor 113's multiples on the sequence, we substitute 113 for N in the formula X = 5N + N; and evaluate the expression as follows;  $X = (5 \times 113)$ + 113; X = 678. The value of the expression is 678. This means that invisible divisor 113's multiples on the sequence of first half pair odd numbers have a difference of 678 in between each other. It also follows that the divisor's second multiple is its first multiple plus 678, which is 565 + 678 = 1243. The third multiple which is 1243 + 678 = 1921. The rest of its multiples continue endlessly in the order.

Knowledge of these two separate sets of divisors is key to the identification of FHPPs on the sequence of FHPONs because primes on this particular sequence are those numbers that are not multiples of any element of either of the two sets of divisors. In other words, it is only the multiples of elements of the two sets of divisors above that constitute the only composite numbers on the sequence of FHPONs.

The table below shows the actual distribution of FHPPs on the sequence of FHPONs;

TABLE 2 (a): THE DISTRIBUTION OF FIRST HALF PAIR PRIMES ON THE SEQUENCE OF FIRST HALF PAIR ODD NUMBERS

36		30	66	102	138	174	210	246	282	318	354	390	426	462	
	6	5	11	17	23	29	35	41	47	53	59	65	71	77	•••
42	7	35	77	119	161	203	245	287	329	371	413	455	497	539	
78	13	65	143	221	299	377	455	533	611	689	767	845	923	1001	
114	19	95	209	323	437	551	665	779	893	1007	1121	1235	1349	1463	•••
150	25	125	275	425	575	725	875	1025	1175	1325	1475	1625	1775	1925	•••
186	31	155	341	527	713	899	1085	1271	1457	1643	1829	2015	2201	2387	•••
222	37	185	407	629	851	1073	1295	1517	1739	1961	2183	2405	2627	2849	
258	43	215	473	731	989	1247	1505	1763	2021	2279	2537	2795	3053	3311	•••
294	49	245	539	833	1127	1421	1715	2009	2303	2597	2891	3185	3479	3773	•••
330	55	275	605	935	1265	1595	1925	2255	2585	2915	3245	3575	3905	4235	•••
366	61	305	671	1037	1403	1769	2135	2501	2867	3233	3599	3965	4331	4697	
402	67	335	737	1139	1541	1943	2345	2747	3149	3551	3953	4355	4757	5159	
438	73	365	803	1241	1679	2117	2555	2993	3431	3869	4307	4745	5183	5621	
474	79	395	869	1343	1817	2291	2765	3239	3713	4187	4661	5135	5609	6083	
510	85	425	935	1445	1955	2465	2975	3485	3995	4505	5015	5525	6035	6545	•••
546	91	455	1001	1547	2093	2639	3185	3731	4277	4823	5369	5915	6461	7007	
582	97	485	1067	1649	2231	2813	3395	3977	4559	5141	5723	6305	6887	7469	•••
618	103	515	1133	1751	2369	2987	3605	4223	4841	5459	6077	6695	7313	7931	•••
654	109	545	1199	1853	2507	3161	3815	4469	5123	5777	6431	7085	7739	8393	•••
690	115	575	1265	1955	2645	3335	4025	4715	5405	6095	6785	7475	8165	8855	•••
726	121	605	1331	3057	2783	3509	4235	4961	5687	6413	7139	7865	8591	9317	
	•••		•••	•••	•••	•••	•••	•••	•••		•••	•••	•••	•••	

On table 2 (a) above, the first row is the sequence of FHPONs which comprises of both FHPPs and first half pair composite odd numbers. The rest of the rows are entirely first half pair composite odd numbers extracted from the first row to leave only FHPPs on that row. This extraction is such that the table indicates each and every divisor's endless multiples stretching ahead of it on the sequence of FHPONs. Multiples of visible divisors are indicated by columns while those of invisible divisors are indicated by rows. In order to identify primes from the first row, we simply extend it to any desirable stretch and eliminate from it any numbers appearing in the rest of the rows, making sure that each and every such row or column is also extended up to the limit chosen for the first row. On table 2 (a) above, the first row has been extended to 77 and the numbers in black font are the only numbers indicated as composites by the rows and columns up to that limit. On table 2 (b) below, the first row has been extended to 2591 and elements of rows and columns up to that limit eliminated to leave only first half pair primes.

TABLE **2 (b):** THE DISTRIBUTION OF FIRST HALF PAIR PRIMES ON THE SEQUENCE OF FIRST HALF PAIR ODD NUMBERS UP TO THE LIMIT OF 2,591 (This table must be read horizontally).

5	11	17	23	29		41	47	53	59		71
	83	89		101	107	113			131	137	
149			167	173	179		191	197			
	227	233	239		251	257	263	269		281	
293			311	317					347	353	359
			383	389		401			419		431
	443	449		461	467				491	497	503
509		521						557	563	569	
	587	593	599			617				641	647
653	659			677	683			701			719
			743			761		773			
797		809		821	827		839			857	863
		881	887				911			929	
941	947	953			971	977	983				
1013	1019		1031			1049		1061			
	1091	1097	1103	1109							1151
	1163			1181	1187	1193				1217	1223
1229					1259			1277	1283	1289	
1301	1307		1319							1361	1367
1373						1409			1427	1433	1439
	1451					1481	1487	1493	1499		1511
	1523					1553	1559		1571		1583
		1601	1607	1613	1619			1637			
	1667					1697		1709		1721	
1733									1787		
	1811		1823				1847				1871
1877		1889		1901	1907	1913			1931		
1949				1973	1979			1997	2003		
	2027		2039				2063	2069		2081	2087
	2099		2111			2129		2141		2153	
							2207	2213			
2237	2243				2267	2273				2297	
2309				2333	2339		2351	2357			
2381		2393	2399		2411	2417	2423			2441	2447
	2459			2477							
	2531		2545	2549					2579		2591

# F. Most Appropriate Divisors of First Half Pair Odd Numbers

The two sets of divisors of first half pair odd numbers comprises of divisors that are indivisible by any of the preceding divisors and those that are divisible by some divisors preceding them. Since multiples of divisible divisors are complete subsets of sets of multiples of the divisible divisors' initial divisors, the most appropriate set of divisors for first half pair odd numbers is a combined set of first and second half pair primes. This implies that the prime status of any first half pair odd number, is more efficiently determined by the sort of trial division that involves only primes other than 2 and 3, less than the square root of that number. On table 2(b) above the eliminated composite numbers are first half pair odd

number multiples of primes other than 2 and 3, less than the square root of 2,591, all of which multiples are less than the 2591 limit. This set of primes is as follows; { 5,7, 11, 13,17,19,23,29,31,37,41, 43}

# G. Divisors of Second Half Pair Odd Numbers

The set of second half pair odd numbers within which second half pair primes are located have two sets of divisors also. These are the set of visible divisors and the set of invisible divisors. The first set has elements of the form (6N + 7) - 6 where N is any natural number. The set is as follows;  $\{7, 13, 19, 25, 31, 37, 43 \ldots\}$ . Its elements begin with 7 and proceed endlessly in ascending order at a uniform gap of 6. The set has been called a set of visible divisors of SHPONs because its elements are identical to elements of the set of SHPONs and are

therefore their self evident divisors. First multiples of each and every one of these divisors on the sequence of second half pair odd numbers are of the form 7N where N is any such divisor. The differences between each and every multiple of any of these divisors is of the form 7N - N, where N is any such divisor.

#### **Example**

Pick any divisor of the form 6N + 7 - 6 where N is any natural number, let us say 113 yet again and establish its first multiple on the sequence of second half pair odd numbers and the difference between each and every one of its multiples on the sequence.

To establish this divisor's first multiple on the sequence, we substitute 113 for N in the formula X =7N and evaluate the expression as follows; X = 7 x113: X = 791. The value of the expression 791 is visible divisor 113's first multiple on the sequence. To establish the difference between each and every one of visible divisor 113's multiples on the sequence, we substitute 113 for N in the formula X = 7N - N; and evaluate the expression as follows; X = $(7 \times 113) - 113$ ; X = 678. The value of the expression is 678. This means that invisible divisor's multiples on the sequence of second half pair odd numbers have a difference of 678 in between each other. It also follows that the divisor's second multiple is its first multiple plus 678, which is 791 + 678 = 1469. The third multiple is 1469 + 678 =2147. The rest of its multiples continue in that order endlessly.

The second set, the set of invisible of divisors of second half pair odd numbers have elements of the form (6N + 5) - 6 where N is any natural number. The set is as follows {5, 11, 17, 23, 29, 35, 41, 47 ...}. Its elements begin with 5 and proceed endlessly in ascending order at a uniform gap of 6. This set has been called a set of invisible divisors of second half pair odd numbers because even if none of its elements are elements of the set of first half pair odd numbers, each and every one of its elements have endless multiples on that sequence of numbers. Such divisors' first multiples on the sequence of second half pair odd numbers are of the form 5N where N is any such divisor. The difference between each and every one of their multiples are of the form 5N + N where N is any such divisor

#### **Example**

Pick any divisor of the form (6N + 5) - 6 where N is any natural number, let us say 641 even this time, and establish its first multiple on the sequence of second half pair odd numbers and the difference between each and every one of its multiples on the sequence.

To establish this invisible divisor's first multiple on the sequence, we substitute the divisor for N in the formula X = 5N and evaluate the expression as follows;  $X = 5 \times 641$ ; X = 3205. The value of the expression, 3205, is invisible divisor 641's first multiple on the sequence of second half pair odd numbers. To establish the difference between each and every one of its multiples on the sequence, We substitute this same divisor for N in the formula X = 5N + N and evaluate it as follows;  $X = (5 \times 641) +$ 641; X = 3846. The value of the expression, 3846, is the difference between each and every multiple of this particular invisible divisor on the sequence of second half pair odd number. This also implies that the divisor's second multiple on this particular sequence is 3205 + 3846 = 7051. Its third multiple is 7051 + 3846 = 10897, and so on in that order endlessly. Knowledge of the two separate sets of divisors is key to the identification of Second half pair primes on the sequence of second half pair odd numbers because primes on this particular sequence are those numbers that are not multiples of any element of either of the two sets of divisors. In other words, it is only the multiples of elements of the two sets of divisors above that constitute the only composite numbers on the sequence of second half pair odd numbers.

TABLE **3(a):** THE DISTRIBUTION OF SECOND HALF PAIR PRIMES ON THE SEQUENCE OF SECOND HALF PAIR ODD NUMBERS

36	30		66		102		138		174		210		246		282	
36		42		78		114		150		186		222		258		
6	5		11		17		23		29		35		41		47	
6		7		13		19		25		31		37		43		•••
	25	49	55	91	85	133	115	175	145	217	175	259	205	301	235	
	55	91	121	169	187	247	253	325	319	403	385	481	451	559	517	•••
	85	133	187	247	289	361	391	575	493	589	595	703	697	817	799	•••
	115	175	253	325	391	475	529	625	667	775	805	925	943	1075	1081	•••
	145	217	319	403	493	589	667	775	841	961	1015	1147	1189	1333	1363	•••
	175	259	385	481	595	703	805	925	1015	1147	1225	1369	1435	1591	1645	•••
	205	301	451	559	697	817	943	1075	1189	1333	1435	1591	1681	1849	1927	•••
	235	343	517	637	799	931	1081	1225	1363	1519	1645	1813	1927	2107	2209	
	265	385	583	715	901	1045	1219	1375	1537	1705	1855	2035	2173	2365	2491	•••
	295	427	649	793	1003	1159	1357	1525	1711	1891	2065	2257	2419	2623	2773	
	325	469	715	871	1105	1273	1495	1675	1885	2077	2275	2479	2665	2881	3055	•••
	355	511	781	949	1207	1387	1633	1825	2059	2263	2485	2701	2911	3139	3337	•••
	385	553	847	1027	1309	1501	1771	1975	2233	2449	2695	2923	3157	3397	3619	•••

On table 3 (a) above, the first three rows comprises of headers. The third row comprises of invisible divisors of second half pair odd numbers. The first amongst them indicates gaps between multiples of each and every visible divisor. The second row indicates gaps in between multiples of invisible divisors. The third row indicates the set of invisible divisors.

On the main text of the table, the first row indicates the sequence of second half pair odd numbers which comprises of both second half pair primes and second half pair composite odd numbers. The rest of the rows are entirely second half pair composite odd numbers extracted from the first row to leave only second half pair primes on that row. This extraction is such that the table indicates each and every divisor's endless multiples stretching ahead of it on

the sequence of second half pair odd numbers. Columns in black font indicate multiples of invisible divisors. Those in bold font indicate multiples of visible divisors.

In order to identify second half pair primes from the first row, we simply extend it to any desirable stretch and eliminate from it any numbers appearing in the rest of the rows, making sure that each and every such row is also extended up to the limit chosen for the first row. On table 3 (a) above, the first row has been extended to 43 and the only number in black font (25) is the only number indicated as composite by the columns up to that limit. On table 3 (b) below, the first row has been extended to 2593 and elements of columns up to that limit eliminated to leave only second half pair primes.

TABLE 3 (b): THE DISTRIBUTION OF SECOND HALF PAIR PRIMES ON THE SEQUENCE OF SECOND HALF PAIR ODD NUMBERS UP TO THE LIMIT OF 2,593. (This table must be read horizontally)

7	13	19		31	37	43			61	67	73
79			97	103	109			127		139	
151	157	163			181		193	199		211	
223	229		241					271	277	283	
		307	313			331	337		349		
367	373	379			397		409		421		433
439			457	463				487		499	
		523			541	547				571	577
			601	607	613	619		631		643	
	661		673			691			709		
727	733	739		751	757		769			787	
		811		823	829				853	859	
	877	883				907		919			937
				967				991	997		1009
	1021		1033	1039		1051		1063	1069		

1087	1093				1117	1123	1129				1153
		1171					1201		1213		
1231	1237		1249					1279		1291	1297
1303			1321	1327							
	1381			1399				1423	1429		
1447	1453	1459		1471		1483	1489				
		1531		1543	1549			1567		1579	
	1597		1609		1621						1657
1663	1669				1693	1699				1723	
	1741	1747	1753	1759			1777	1783	1789		1801
				1831					1861	1867	1873
1879									1933		
1951						1987	1993	1999		2011	2017
	2029				2053					2083	2089
			2113			2131	2137	2143			2161
		2179				2203			2221		
2239		2251			2269		2281	2287	2293		
2311					2341	2347				2371	2377
2383	2389								2437		
		2467	2473					2503			2521
		2539		2551	2557						2593

# H. Most Appropriate Divisors of Second Half Pair Odd Numbers

The two sets of divisors of second half pair odd numbers comprises of divisors that are indivisible by any of the preceding divisors and those that are divisible by some divisors preceding them. Since multiples of divisible divisors are complete subsets of sets of multiples of the divisible divisors' initial divisors, the most appropriate set of divisors for second half pair odd numbers is a combined set of first and second half pair primes. Therefore, the prime status of any second half pair odd number, is more efficiently determined by the sort of trial division that involves only primes other than 2 and 3, less than the square root of that number. On table 3(b) above the eliminated composite numbers are second half pair odd number multiples of primes other than 2 and 3, less than the square root of 2,593, all of which multiples are less than the 2593 limit. This set of primes is as follows; {5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43}

#### **Conclusion**

In concluding this section, it is worth noting that the fact that the sequence of natural numbers comprises of only primes and their multiples, which multiples are systematically distributed on the sequence, implies that primes are not randomly distributed on that sequence. Rather their perfect regularity on that sequence is a function of the self evident perfect

regularity in the distribution of their multiples. Similarly, the fact that the sequence of paired odd numbers comprises of only primes and their and which multiples multiples, are also systematically located on that sequence, implies that primes are not randomly distributed on that sequence either. The systematic distribution of primes on either sequence, is such that each and every divisor indicates its own endless multiples stretched out ahead of it and in the process, revealing those numbers, which no single divisor, among a totality of divisors for any given limit of numbers on the sequence, is able to indicate or divide. This is precisely the reason why valid formulas for locating primes can be found.

#### III. THE NEW STANDARD FORMULA

Having given background information this section now presents the new standard formula as follows;

 $P = 3n_{so} \pm 2$ , where  $n_{so}$  is any special odd number equal to or greater than 1, which numbers belong to appropriate solution sets for the formula.

In the formula, the variable  $n_{so}$  represent specific odd numbers that must replace the variable if the formula has to yield only prime numbers. The depressed 'so' at the baseline of the variable is meant to indicate the fact that it is not any natural number that can be used as a substitute for the variable, but only specific odd numbers that are elements of the

formula's appropriate solution sets. For example, even numbers cannot replace the variable. At the same time, it is not each and every odd number that can replace it. If appropriate solution sets are identified and their elements used to replace the variable, each and every value to be obtained will be a prime. If elements of these solution sets are systematically substituted for the variable one after another, the formula will yield each and every succeeding prime beginning with prime 5 and going on without end.

#### A. Splitting the Formula into Two

To use this formula effectively, it is necessary to split it into two complementary ones. These two separate but complementary formulas are as follows;

 $P_1 = 3n_{so} + 2$  where  $n_{so}$  is any specific odd number equal to or greater than 1, which numbers belong to a set of appropriate solutions for this particular formula,

 $P_2 = 3n_{so}$ - 2 where  $n_{so}$  is any specific odd number equal to or greater than 3, which numbers belong to a set of appropriate solutions for this particular formula,

The two formulas above are complementary because they complement each other, or take turns in locating each and every prime on the sequences of natural numbers. The reason for having these two formulas is that even though the current practice is to treat all primes as belonging to one single set, primes belong to two different sets. These are the set of FHPPs and the set of SHPPs.

#### B. The First Complementary Formula

The first complementary formula;  $P_1 = 3n_{so} + 2$ , is the standard formula for finding all FHPPs. Such primes extend from 5 and continue endlessly with a hidden perfect regularity. This hidden perfect regularity is implied by a perfect regularity in the even divisibility of natural numbers created by a systematic spread of natural numbers other than unit, as even divisors of their endless multiples ahead of them, which multiples are located at intervals equal to their absolute values, on the sequence of natural numbers. This set of primes is as follows;

SFHPP = {5, 11, 17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101,...}.

The solution set for the formula for finding first half pair primes is as follows;

SSFHPPF = {1, 3, 5, 7, 9, 13, 15, 17, 19, 23, 27, 29, 33, 35, 37, 43, 45, 49, 55, 57, 59, 63 ...}

Elements of this set of solutions begin from unit and continue endlessly in hidden perfect regularity, This hidden perfect regularity is indicated by the perfect regularity in the distribution of non-substitutes on the sequence of odd numbers. Elements of this set are specific odd numbers each of which, when substituted for the variable in the standard formula for finding FHPPs, will make the formula yield such a prime. In addition, elements of this set are so systematically laid out that when each and every element is substituted for the variable, one after another, from the first element to each and every one of them in their endless chain of succession, there is not a single FHPP on the entire sequence of natural numbers that will not be found

#### C. The Second Complementary Formula

The second complementary formula;  $P_2 = 3n_{so} - 2$ , is a standard formula for SHPPs. Such primes extend from 7 and continue endlessly with a hidden perfect regularity. This regularity is confirmed by a perfect regularity in the even divisibility of natural numbers determined by a systematic spread of natural numbers other than unit as even divisors of their endless multiples ahead of them, which multiples are located at intervals equal to their absolute values, on the sequence of natural numbers. This set of primes is as follows;

SSHPP= {7, 13, 19, 31, 37, 43, 61, 67, 73, 79, 97, 103, 109....}.

The solution set for the formula for finding SHPPs is as follows;

SSSHPPF = { 3, 5, 7, 11, 13, 15, 21, 23, 25, 27, 33, 35, 37, 43, 47, 51, 53, 55, 61, 65,...}

Elements of this set begin from 3 and continue endlessly at an ascending order of hidden perfect regularity. The regularity is established by the systematic distribution of the formula's nonsubstitutes on the sequence of odd numbers. Elements of this solution set are specific odd numbers, each one of which, when substituted for the variable in the standard formula for SHPPs will yield such a prime. These elements are also so systematically spread out that when each and every one of them replaces the variable in their endless chain of succession, there is not a single prime of this type on the entire sequence of natural numbers that will be missed.

### The Infinitude of Primes of Either Set

Each of the two solution sets has endless elements because as has been long established, primes are endless on the sequence of natural numbers. The oldest prove is said to be Euclid's Theorem who around 300 BC submitted that the primes are endless on the sequence, even though their density within natural numbers is ZERO. To prove the infinitude of primes, Euclid had argued that if elements of any finite set of primes were multiplied together, and unit is added to the product, the sum obtained will be indivisible by any of the elements of the finite set of primes. His argument was that the sum obtained will either be another prime not in the initial finite set, or a composite whose prime factors are not in the original set as well. He reasoned that whether the resulting sum is a prime or not, there is at least one more prime that was not in the initial finite set of primes. He thus had concluded that there are more primes than any given finite number (Euclid, Elements; Book IX, Prospects 20). Even though Euclid was right about primes being endless, and his proof valid, his statement that their intensity among natural numbers is ZERO contradicted his correct position on their infinitude on the sequence.

Do primes completely fed away on the sequence of natural numbers? It is very easy to believe that they do. Consider the fact that each and every natural number on the sequence is an even divider of each and every number located ahead of it at intervals equal to its absolute value. 2 divides every second number ahead of it on the sequence, meaning that 50 percent of numbers on the entire endless sequence are composites divisible by 2. In short, 2 subtracts 50 percent of the numbers from the entire sequence to leave only 50 percent. The number 3, divides every third number ahead of it on the sequence, 4, divides every forth number. Five divide every fifth number

and so on in that order endlessly. This gives the impression that as more and more divisors join the queue, a definite stage will be reached when all indivisible numbers fed away completely. However, the actual situation is far from this assumption. Other than 2 which picks out 50 percent of divisible numbers from the sequence, no other number takes as much as its absolute value suggests. For example even if 3 divides every third number on the sequence, it will not subtract a third from the 50 percent of numbers on the sequence that 2 cannot divide, because 50 percent of its multiples are also multiples of 2. In addition, divisible divisors do not lessen the intensity of primes on natural numbers ahead of them because their multiples are entire subsets of sets of multiples of their initial divisors.

In short, it is only primes that lessen the intensity of indivisible numbers among natural numbers ahead of them on the sequence. But would the pile up of primes as even divisors of their multiples located ahead of them result in a total elimination of indivisible numbers at some definite point? Again the answer is no, because the bigger the prime, the more multiple it shares with preceding divisors, and the lesser the numbers it is able to divide amongst numbers that are indivisible by all numbers that precede it. To be specific, when for instance a prime number enters a queue of divisors on the sequence of first or second half pair odd numbers, its first set of multiples are also multiples of primes that precede it on that sequence. Thereafter, its subsequent multiples are such that each and every multiple located at intervals equal to the absolute value of each and every preceding prime are already multiples of such preceding primes. It is precisely this factor that makes it impossible for indivisible numbers to fed away completely on the sequence. In short, the 'subtraction' of 'remaining' indivisible numbers by succeeding indivisible divisors joining the endless queue of divisors will not result in ZERO because there is in actual fact no subtraction taking place. Instead there is the materialization of more and more indivisibles at larger and larger distances on the sequence

In order to relate the nature and character of natural numbers to the endless spread of primes at wider and wider distances on the sequence, we can also consider natural numbers as varying from each other in terms of their variations in length. The varying length can be determined by a number of units each and every number contains. In this regard, the number 1 could be said to be one unit long. 2 is two units long, 3 is three units long, 4 is four units long, and so on in that order for each and every succeeding number endlessly. With this conception, the process of determining which number is a prime and which one is a composite, becomes a question of trying to achieve the exact length of a larger number by first arranging smaller numbers of equal lengths into even groups reflecting any of the preceding numbers and then joining them together and matching their sum with the actual length of the bigger number being tested. If the actual length of the number being tested cannot be achieved by a total length of any such even groups, then that number is a prime. In other words, the divisibility of any natural number depends on how many units that number has. If the totality of such units cannot be arranged in even groups of any natural number or numbers greater than unit, preceding it, without having to leave out any unit or units out of such groupings, then such a number is indivisible by any of its preceding numbers and therefore, a prime. For example, 4 is divisible by 2 (its preceding number, because it has four units which can be arranged in two groups of 2 units each. On the other hand, 5 is not divisible by any of its preceding numbers (2,3, or 4) because its five units cannot be arranged in groups of any of these three numbers without having to leave a unit or units out of such groupings.

Primes do not diminish but merely space out more and more on the sequence, as the constant pile up of units progresses to infinity. In short primes are never counted down but materialize, one after another, at definite stages in the constant accumulation of units when such a definite totality of units cannot be arranged in even groups of any of the preceding totalities of units without having to leave any unit or units out of such even groupings. In short, the lessening intensity of primes on the sequence of natural numbers merely implies increases in permutations of units required to reach more and

more numbers indivisible by any of their preceding numbers greater than unit.

#### IV. DETERMINING THE SOLUTION SETS

The most important aspect of the new standard formula is the determination of elements of the two solution sets that satisfy it. The solution sets and their elements are determined by using tables and formulas. The tables in particular are such that they enable the isolation of elements of the formula's solution sets from the sequence of odd numbers. There are two main types of such tables, those that show the actual distribution of non-substitute elements on the sequence of odd numbers and those that show their numerical positions on the sequence. Since the formula has been split into two complementary ones, each of the two formulas has its own tables and formulas for determining its own solution set

# A. Solution Set for the First Half Pair Primes Formula $(P_1 = 3n_{so} + 2)$ .

The solution for the FHPPs formula ( $P_1 = 3n_{so} + 2$ ), is determined by two types of tables. The first type show the distribution of non substitute elements on the sequence of odd numbers while the second type indicate the numerical positions on it.

This article has identified three tables of the first type and one table of the second type. The three comprises of the table that show an endless structure of odd numbers, none of which are elements of the solution set, the one that show the linear progression of such elements on the sequence of odd numbers and the one that shows the actual spread of solution set elements on any complete section of the endless sequence of odd numbers. The single table of the second type is an endless structure of natural numbers indicating numerical positions of non-substitute elements on the sequence of odd numbers. The following are the tables;

G.12	G.10	G.22	G.34	G.46	G.58	G.70	G.82	G.94	G.106	G.118	G.130	
G.14	11	25	39	53	67	81	95	109	123	137	151	
G.26	21	47	73	99	125	151	177	203	229	255	281	
G.38	31	69	107	145	183	221	259	297	335	373	411	
G.50	41	91	141	191	241	291	341	391	441	491	541	
G.62	51	113	175	237	299	361	423	485	547	609	671	
G.74	61	135	209	283	357	431	505	579	653	727	801	
G.86	71	157	243	329	415	501	587	673	759	845	931	
G.98	81	179	277	375	473	571	669	767	865	963	1061	
G.110	91	201	311	421	531	641	751	861	971	1081	1191	
G.122	101	223	345	467	589	711	833	955	1077	1199	1321	
G.134	111	245	379	513	647	781	915	1049	1183	1317	1451	•••
G.146	121	267	413	559	705	851	997	1143	1289	1435	1581	

Table 4: The table of natural distribution of odd numbers that must not be substituted for  $n_{so}$  in the standard formula for finding first half pair primes  $(P_1 = 3n_{so} + 2)$ .

Table 4, above is a display of an easy to appreciate pattern of endless rows and columns of odd numbers, inclusive of primes and composites, none of which must be substituted for variable  $n_{\rm so}$  in the standard formula for finding FHPPs. The table is 'natural' distribution of odd numbers., because as indicated by headers, the numbers in all columns and rows are not randomly compiled, but proceed endlessly in an ascending order of perfect regularity.

The table can be used to determine the solution set for the formula because its elements are those odd numbers that are not part of the systematic structure of the table above. Such missing odd numbers can easily be indentified and listed. The more the table is extended, the more the elements of the solution set will show up.

## Columns and Rows as Sets of Non-Substitutes

The table comprises of two groups of sets of nonsubstitutes. The first group is represented by columns while the second is represented by rows. The two groups have each a set of divisors to which it relates. Columns relate to visible divisors of FHPONs, while rows relate to their invisible divisors.

## Columns as First Sets of Non-Substitutes

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Columns relate to visible divisors in the sense that if any of their respective elements replaces valuable  $n_o$  in the FHPPs formula, the result will be a composite odd number divisible by a visible divisor to which that particular element's set relate. These particular sets are known by their first elements. For example,

the first set is set 11, because the first element in this set is 11. The second set is set is 25 because the first element in this set is 25. Therefore, these sets, as indicated by their first elements, are sets of the form (14N + 11) - 14, where N is any natural number, meaning that they start with set 11 and continues endlessly, in ascending order, at a uniform gap of 14. A visible divisor to which any given set relate can be established by first substituting the first element of that set for X in the in the equation; N = [(X + 14) -11] ÷ 14 and evaluating it. To find such a divisor, the value of this first expression can then be substituted for variable N in the equation; X = 6N + 5 - 6, and solve it for X. Gaps in between elements in each and every set are of the form X = 2N where N is a visible divisor to which that set relate.

#### Rows as Second Sets of Non-Substitutes

Rows relate to invisible divisors in the sense if any of their respective elements is substituted for valuable  $n_0$  in the FHPPs formula, the result will be a composite odd number divisible by an invisible divisor to which that particular element's set relate. The sets are known by their first elements. For example, the first set is also 11, because the first element in this set is 11. The second set is set is 21 because the first element in this set is 21. Therefore, these sets, as indicated by their first elements, are sets of the form 10N + 11 - 10, where N is any natural number, meaning that they start with set 11 and continues endlessly, in ascending order, at a uniform

gap of 10. An invisible divisor to which any given set relate can be established by first substituting the first element of that set for X in the in the equation;  $N = [(X+10)-11] \div 10$  and evaluating it. To find such a divisor, the value of this first expression can then be substituted for variable N in the equation; X = 6N + 7 - 6, and solve it for X. Gaps in between elements in each and every set also of the form X = 2N where N is a visible divisor to which that set relate.

# Dual Membership for Elements of Sets of Non-Substitutes

While as we have seen above, there are two different sets of non-substitute elements, each and every individual element of either set has dual membership in the sense that any such element can be considered to belong to either a row or a column. Consequently, each and every composite odd number to be obtained by substituting any such number for the variable in the FHPPs formula, will be divisible by both a visible divisor related to its row and an invisible divisor related to its column. In either case the resulting quotient is the odd composite's corresponding divisor of the other set.

#### Linear Progression of Non-Substitutes

We can also move away from the systematic cluster of non-substitutes above by establishing the linear progression of such non-substitutes on the sequence of odd numbers. Table 5 below establishes this single line progression of non-substitutes;

Table 5: The linear progression of non substitutes on the sequence of odd numbers, for the first half pair primes' formula  $(P_1 = 3n_{so} + 2)$ .

	22	34	46	58	70	82	94	106	118	130	142	154	166	178	190	202	214	226	•••
11																			11
21																			21
	25																		25
31																			31
		39																	39
41																			41
	47																		47
51																			51
			53																53
61																			61
				67															67
	69																		69
71																			71
		73																	73
81					81														81
91	91																		91
						95													95
			99																99
101																			101
		107																	107
							109												109
111	440	ļ			ļ	1													111
	113																		113
121		ļ			ļ	1		100											121
				105				123											123
101				125															125
131	125																		131
	135								127										135
1.4.1		1.4.1							137										137
141		141	145						-										141
151		1	145		151	1				151									145 151
151	157				151				-	151									151
161	13/								-										
161											165								161 165
171									-		103								
171		175																	171 175
		1/3				177													177
	179					1//						179							177
	1/9		I	1	1		I	1		l	l	1/9	ĺ	l	1	1	1	1	1/9

181																	181
				183													183
191			191														191
											193						193
201	201																201
							203										203
												207					207
		209															209
211																	211
221					221								221				221
	223																223
								229									229
231																	231
														235			235
			237														237
241				241													241
		243															243
	245																245
															249		249
251																	251
									255								255
	ļ		ļ			259	ļ										259
261	ļ		ļ			ļ	ļ										261
	1					1	1									263	263

Table 5, above shows how elements of the first group of sets, identified as columns in table 4, combine into a union set of non-substitutes (the progression of non-substitute elements in a single line). This union set is imposed on the seemingly last column of the table.

The manner in which all sets combine into a union set of non-substitutes is such that there is an initial set into which each and every succeeding set feeds. This initial set is column 1 of table 4, which is now column 1 in table 5. This initial set has numbers of the form 10N + 11 - 10 where N is any natural number, meaning that its elements begin from 11 and proceed endlessly in ascending order, at a uniform gap of 10. This uniform gap implies that there is a total of four odd numbers missing in between each and every element of this set. The feeding into this set by each and every other succeeding set implies elements of such sets either occupying their appropriate locations in any of the four odd number gaps in the initial set, or intersecting with identical elements in that set.

The first set to feed into the initial set is the second column of table 4, which is now the second column of table 5. This set has numbers of the form 22N + 25 - 22 where N is any natural number. It extends from 25 and continues endlessly in ascending order at a

uniform gap of 22. One after another, elements of this set either fill up gaps of their natural location or intersect with their identical elements in the initial set. The second set to feed into the initial set is column 3 of table 4 which is now column 3 of table 5. This set begin with 39 and continues endlessly in ascending order at a uniform gap of 34. Elements of this set either fill up gaps of their natural location within the initial set or intersect with their identical elements in either of the two preceding sets. In short elements of each and every succeeding set feed into the initial set by filling up the gaps of their designated locations or intersecting with identical elements in sets that precede them. In table 5 above, intersecting elements are presented in red font.

A union set of non substitutes for any given range of odd numbers is complete at a point where the first element of a succeeding set is the last odd number in that given range. In table 5 above, the union set of non-substitutes is complete only up to 263 because it is the first element of the very last set within this range.

#### The Actual Solution Set

In the range of odd numbers covered by table 5 above (1 to 263), the actual solution set for the FHPPs' formula comprises of all odd numbers within this range which are missing from the indicated union

set of non- substitutes.

#### The Seven Column Odd Numbers Table

The last of the three first tables is the seven column odd numbers table constructed on the combined logic of tables 4 and 5 above. It has been noted above that there is an initial set which has regular gaps, some of which are taken up by elements of each and every succeeding set in a union set of non substitutes. This initial set has been identified as being column number 1 of table 4, and subsequent sets as being each and every succeeding columns of the same table. It has also been stated that the uniform gaps between elements of this initial set represent a total of four missing odd numbers/gaps in between each and every element. Since elements of the actual solution set are those odd numbers that are represented by gaps that cannot be taken up by any elements of any of the endless number of sets of non substitutes, such elements can be located on the sequence of odd numbers by identifying and eliminating all non-substitutes amongst them.

It has also been mentioned that rows on table 4 represent the second group of sets of non-substitutes related to invisible divisors of first half pair odd numbers. In this particular case, the initial set amongst them is the first row. The set has numbers of the form (14N + 11) - 14 where N is any natural number, meaning that its elements begin with 11 and proceed endlessly in an ascending order at a uniform gap of 14. This uniform gap implies that there is a total of six odd numbers missing in between each and every element of this set. Into this initial set feeds elements of all other succeeding sets presented as columns. One after another, elements of the succeeding sets either occupy their appropriate locations in any of the six odd number gaps in the initial set or intersect with identical elements in that set. Since elements of the actual solution set are those

odd numbers that are represented by gaps that cannot be taken up by any elements of any of the endless number of sets of non-substitutes, such elements can be located on the sequence of odd numbers by indentifying and eliminating all non-substitutes among them.

Table 6(a) below, shows how the entire sequence of odd numbers can be arranged in seven columns, for easy identification of elements of the solution set among odd numbers falling in between elements of the set of the form 14N + 11 - 14 where N is any natural numbers.

TABLE **6(a)**: THE SEVEN COLUMN ODD NUMBER TABLE FOR LOCATING SUBSTITUTES AND NON-SUBSTITUTES IN THE FORMULA FOR FIRST HALF PAIR PRIMES. (This table must be read horizontally).

	1	3	5	7	9	11
13	15	17	19	21	23	25
27	29	31	33	35	37	39
41	43	45	47	49	51	53
55	57	59	61	63	65	67
69	71	73	75	77	79	81
83	85	87	89	91	93	95
97	99	101	103	105	107	109
111	113	115	117	119	121	123
125	127	129	131	133	135	137
139	141	143	145	147	149	151
153	155	157	159	161	163	165
167	169	171	173	175	177	179
•••	•••	•••	•••	•••	•••	•••

Table 6(a) above, is an arrangement of the entire sequence of odd numbers in seven columns only. For the purpose of demonstrating the location solution set elements on the seven column table, the table has been reproduce below and extended to 100 rows comprising of the first 669 odd numbers. In this range, the first odd number is 1 in the second column of row 1, while the last number is 1397 in the seventh column of row 100;

TABLE 6 (b): THE SEVEN COLUMN ODD NUMBERS' TABLE EXTENDED TO 100<sup>TH</sup> ROW.

S.N	C1	C2	C3	C4	C5	<i>C</i> 6	<i>C</i> 7	S.N	C1	C2	C3	C4	C5	C6	C7
1		1	3	5	7	9	11	51	699	701	703	705	707	709	711
2	13	15	17	19	21	23	25	52	713	715	717	719	721	723	725
3	27	29	31	33	35	37	39	53	727	729	731	733	735	737	739
4	41	43	45	47	49	51	53	54	741	743	745	747	749	751	753
5	55	57	59	61	63	65	67	55	755	757	759	761	763	765	767
6	69	71	73	75	77	79	81	56	769	771	773	775	777	779	781
7	83	85	87	89	91	93	95	57	783	785	787	789	791	793	795
8	97	99	101	103	105	107	109	58	797	799	801	803	805	807	809
9	111	113	115	117	119	121	123	59	811	813	815	817	819	821	823
10	125	127	129	131	133	135	137	60	825	827	829	831	833	835	837
11	139	141	143	145	147	149	151	61	839	841	843	845	847	849	851
12	153	155	157	159	161	163	165	62	853	855	857	859	861	863	865
13	167	169	171	173	175	177	179	63	867	869	871	873	875	877	879
14	181	183	185	187	189	191	193	64	881	883	885	887	889	891	893
15	195	197	199	201	203	205	207	65	895	897	899	901	903	905	907
16	209	211	213	215	217	219	221	66	909	911	913	915	917	919	921
17	223	225	227	229	231	233	235	67	923	925	927	929	931	933	935
18	237	239	241	243	245	247	249	68	937	939	941	943	945	947	949
19	251	253	255	257	259	261	263	69	951	953	955	957	959	961	963
20	265	267	269	271	273	275	277	70	965	967	969	971	973	975	977
21	275	281	283	285	287	289	291	71	979	981	983	985	987	989	991
22	293	295	297	299	301	303	305	72	993	995	997	999	1001	1003	1005
23	307	309	311	313	315	317	319	73	1007	1009	1011	1013	1015	1017	1019
24	321	323	325	327	329	331	333	74	1021	1023	1025	1027	1029	1031	1033
25	335	337	339	341	343	345	347	75	1035	1037	1039	1041	1043	1045	1047
26	349	351	353	355	357	359	361	76	1049	1051	1053	1055	1057	1059	1061
27	363	365	367	369	371	373	375	77	1063	1065	1067	1069	1071	1073	1075
28	377	379	381	383	385	387	389	78	1077	1079	1081	1083	1085	1087	1089
29	391	393	395	397	399	401	403	79	1091	1093	1095	1097	1099	1101	1103
30	405	407	409	411	413	415	417	80	1105	1107	1109	1111	1113	1115	1117
31	419	421	423	425	427	429	431	81	1119	1121	1123	1125	1127	1129	1131
32	433	435	437	439	441	443	445	82	1133	1135	1137	1139	1141	1143	1145
33	447	449	451	453	455	457	459	83	1147	1149	1151	1153	1155	1157	1159
34	461	463	465	467	469	471	473	84	1161	1163	1165	1167	1169	1171	1173
35	475	477	479	481	483	485	487	85	1175	1177	1179	1181	1183	1185	1187
36	489	491	493	495	497	499	501	86	1189	1191	1193	1195	1197	1199	1201
37	503	505	507	509	511	513	515	87	1203	1205	1207	1209	1211	1213	1215
38	517	519	521	523	525	527	529	88	1217	1219	1221	1223	1225	1227	1229
39	531	533	535	537	539	541	543	89	1231	1233	1235	1237	1239	1241	1243
40	545	547	549	551	553	555	557	90	1245	1247	1249	1251	1253	1255	1257
41	559	561	563	565	567	569	571	91	1259	1261	1263	1265	1267	1269	1271
42	573	575	577	579	581	583	585	92	1273	1275	1277	1279	1281	1283	1285
43	587	589	591	593	595	597	599	93	1287	1289	1291	1293	1295	1297	1299
44	601	603	605	607	609	611	613	94	1301	1303	1305	1307	1309	1311	1313
45	615	617	619	621	623	625	627	95	1315	1317	1319	1321	1323	1325	1327
46	629 643	631 645	633 647	635 649	637 651	639 653	641 655	96	1329 1343	1331 1345	1333 1347	1335 1349	1337 1351	1339 1353	1341 1355
47	657	659						98	1343	1345		1363		1367	1369
48	671	673	661 675	663 677	665 679	667 681	669	98	1357	1373	1361 1375	1363	1365 1379	1367	
50	685	687	689	691	693	695	683 697	100	1371	1373	1375	1377	1379	1395	1383 1397
30	000	007	009	091	093	093	097		1303		1309	1391	1393		
			<u> </u>	<u> </u>	L	<u> </u>		•••		•••	l	l	l		•••

On table 6(b) above, column 7 represents the initial set of non-substitutes relating to invisible divisors of first half pair odd numbers. There is, therefore, no single element of the solution set in this particular set of endless elements. Columns 1 to 6 represent odd numbers in between elements of the initial set of non-

substitutes. Therefore, it is only from these columns that elements of the solution set for FHPPs' formula can be located.

In the process of locating elements of the solution set from the seven column table, a choice can be made between using the first group or second group of sets of non-substitutes. What is important is that it does not matter what group of sets is used because the results are the same for either group, for it is self evident that the table can be read in terms of either rows or columns without having to miss any numbers.

It is evident from row 1 of table 4 that column 7 of the seven column table, constitute first elements of each and every one of the endless sets in the second group of non-substitutes. Therefore, generating elements of these sets and striking them off from the seven column table will leave only substitutes on the table.

Each and every range of odd numbers has a definite number of sets whose elements have to be identified and struck off from the table. It has been started above that a union set of non substitutes for any given range of odd numbers is complete at a point where the first element of a succeeding set is the last number in that given range. From this statement we can deduce that the number of sets whose elements should be produced, each up to that range, is the rank or number of the set whose first element is equal to or just less than the upper limit of the range of odd numbers within which we want to establish elements of the solution set. In table 6.(b) above, our upper limit number, 1397 is indicated as the first element of set number 100.

Where the table is not available, we can still determine the number of appropriate sets by a formula method. Since according to table 2, all first elements of this second group of sets are of the form (14N+11)-14, we can use the guess and check method to substitute valuable N in the following inequality, for a number that will satisfy it. If we use 100 which is reflecting on the table, this inequality will be satisfied as follows;  $(14N+11)-14 \le 1397$ ;  $[(14 \times 100)+11]-14 \le 1397$ ;  $1397 \le 1397$ . We have thus determined that there is a total of 100 sets, each of whose elements we have to identify up to the limit of 1397, and eliminate from table 6(b) above.

# Discarding Unnecessary Sets From the List of Appropriate Sets

It is not each and every one of those 100 sets that need to be generated. We can discard those sets that are unnecessary to reproduce. Such sets include the first set 11 (the first set), and all those that are subsets of sets that precede them. Elements of the first set are unnecessary to produce because being numbers whose last digit is 1, they are self evident and can be eliminated on sight from the seven column table. Discarding the first set also means discarding all those sets whose second element is less than any selected upper limit (1397 in our case) because only their first elements which are already removable on sight, will fall within any such selected range of numbers. Sets that are subsets of sets that precede them are also unnecessary to produce because all their elements intersect with some elements of sets that precede them, meaning that they are already elements of sets in which they have an original location.

Since all second elements are of the form (26N+21)-26, where N is any number, we can discard all those sets whose second element is less than our upper limit of 1397, by using the guess and check method to substitute valuable N in the following inequality, for a number that will satisfy it;  $26N+21-26 \le 1397$ ; If we try 54, the value of the expression will be as follows;  $(26 \times 54) + 21 - 26 \le 1397$ ;  $1399 \le 1397$ ; If we try 53 the value of the expression will be as follows;  $26 \times 53 + 21 - 21 \le 1397$ ;  $1373 \le 1397$ . This means that the number that will satisfy the expression is 53, This further implies that the number we have obtained which is 1373 is the second element of the last appropriate set to cover our specified range of odd numbers

Next is to look for the first element of the set to which 1373 is the second element. Since according to table 2, all first elements are numbers of the form, (14N+11)-14, we can establish the first element of this set by substituting N with the same number 53, in the following equation and solving the equation for X, that is; X = (14N+11)-14;  $X = (14 \times 53)+11-14$ ; X = 739. The value of the expression is 739, meaning that the first element of the set in which 1373 is the second element is 739. This also means that the sets of non-substitutes whose elements we are supposed to produce up to our set limit of 1397 are (53-1) in total. They begin from the second set whose first element is 25(column 2, table 4) and end with the set whose first element is 739. These

sets as indicated by their first elements are as follows; 25, 39, 53, 67, 81, 95, 109, 123, 137, 151, 165, 179, 193, 207, 221, 235, 249, 263, 277, 291, 305, 319, 333, 347, 361, 375, 389, 403, 417, 431, 445, 459, 473, 487, 501, 515, 529, 543, 557, 571, 585, 599, 613, 627, 641, 655, 669, 683, 697, 711, 725, 739,

From the above sets, we can now remove those that are subsets of sets that precede them, for the

simple reason that their elements are a repetition of some elements of sets of that precede them. These subsets are such that each and every set on the sequence has an endless number of subsets stretched out ahead of it at uniform gaps specific to itself. Table 7 below shows the distribution of subsets of preceding sets of non-substitutes on the sequence.

TABLE 7: THE DISTRIBUTION OF SUBSETS OF PRECEDING SETS OF NON-SUBSTITUTES ON THE SEQUENCE OF SETS OF NON-SUBSTITUTES

SN	1	2	3	4	5	6	7	8	9	10	11	
SET	11	25	39	53	67	81	95	109	123	137	151	
G84	G70	G154	G238	G322	G406	G490	G574	G658	G742	G826	G910	
G98	81	179	277	375	473	571	669	767	865	963	1061	
G182	151	333	515	697	879	1061	1243	1425	1607	1789	1971	
G266	221	487	753	1019	1285	1551	1817	2083	2349	2615	2881	
G350	291	641	991	1341	1691	2041	2391	2741	3091	3441	3791	
G434	361	795	1229	1663	2097	2531	2965	3399	8333	4267	4701	
G518	431	949	1467	1985	2503	3021	3539	4057	4575	5093	5611	
G602	501	1103	1705	2307	2909	3511	4113	4715	5317	5919	6521	

Table7 above indicates the endless subsets each and every set of non-substitutes has on the sequence of sets of non-substitutes. The first three rows are headers. The first amongst them indicate serial numbers for the sets. The second row shows the actual sets as represented by their first elements. The third row indicate uniform gaps between each and every subset stretched ahead of each and every set on the sequence. Columns on table ( fourth row downwards) indicate subsets that are stretched in front of each and every indicated set of non substitutes on the sequence. For example, nonsubstitute sets that are subsets of set 11 and which subsets are stretched out ahead of it on the sequence, are those that comprise of column one of the main text of the table and which subsets have a uniform gap of 70 in between them. Similarly, sets that are subsets of 25 are those that comprises of column two of the table (table 7) and which subsets have a uniform gap of 154 in between them.

We can use table 7 above to identify and remove all sets that are subsets of their preceding sets within our selected range by extending the second row to 739 and deleting from that row any set that is an element of any column or columns from the fourth row downwards.

Alternatively, we can identify and remove such subsets by using a formula that establishes gaps between subsets of each and every set on the sequence. This formula is X = (7N - N) + 4, where N is any such element. The subset elements up to our chosen limit of 739 can be identified and removed

by firstly establishing gaps between subsets of each set and using such gaps to work out actual subsets for each set. For example if we want to establish sets that are subsets of set 11, we should first determine the uniform gap between its subsets stretched out ahead of it on the sequence as follows;

$$X = (7N - N) + 4$$
;  $X = (7x11 - 11) + 4$ ;  $X = 70$ ;

With this uniform gap between subset elements of 11 established, we can now to work out its subsets as follows;

70N + 11 - 70 ={81, 151, 221, 291, 361, 431, 501, 571, 641, 711 781, ...}

Note that these are the same subsets indicated for set 11 in the first column of the above table.

Using the formula, we can thus identify and remove such subsets from our selected range as follows; Set 11, Gap between subset elements : X = (7N - N) + 4; X = (7x11 - 11) + 4; X = 70; Subsets up to 739 ; 70N + 11 - N; 70N + 11 - 70  $\leq$  739 = {81, 151, 221, 291, 361, 431, 501, 571, 641, 711.}

We can then workout the rest of the subsets within our given range as follows;

Set 25, These are sets whose elements are of the form (154N+179)-154 where N is any natural number. Within our selected range, these sets are as follows;  $(1543N+179)-154 \le 739 = \{179, 333, 487, 641, \}$ 

Set 39, These are sets whose elements are of the form (238N + 277) - 238 where N is any natural number. Within our selected range, there are two such sets which are as follows; (238N + 277) - 238

```
\leq 739 = {277, 515}
Set 53: (322N + 375) - 322\leq 739 = {375, 697};
Subset of 67: (406N + 473) - 406\leq 739 = {473,};
Set 81set: (490N + 571) - 490\leq 739 = {571};
Set 95; set which is the last within our chosen range.
This set is as follows; (574N + 669) - 574 \leq 739 = {669};
When all such subsets are discarded, the remaining
```

When all such subsets are discarded, the remaining most appropriate sets within our chosen upper limit will be only 33 out of a total of 100. These sets are as follows:

{25, 39, 53, 67, 95, 109, 123, 137, 165, 193, 207, 235, 249, 263, 305, 319, 347, 389, 403, 417, 445, 459, 529, 543, 557, 585, 599, 613, 627, 655, 683, 725, 739}

The next stage is to establish uniform gaps between elements of each and every one of the above sets. The formula for working out such gaps is  $X = [(7N - N) + 4] \div 7$  where N is any such set (or where N is the first element of any such set)

When this formula is applied, the uniform gaps between each and every one of these elements will be established as follows;

```
25, 39, 53, 67, 95, 109, 123, 137, 165, 193, 207, 235, 249, 263, 305, 319, 347, 389, 403, 417, 22, 34, 46, 58, 82, 94, 106, 118, 142, 166, 178, 202, 214, 226, 262, 274, 298, 334, 346, 358,
```

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445, 459, 529, 543, 557, 585, 599, 613, 627, 655, 683, 725, 739 382, 394, 454, 466, 478, 502, 514, 526, 538, 562, 586, 622, 634
```

Elements of the above remaining sets can then be generated, for each and every set, up to our upper limit of 1397. The totality of such elements will be as follows;

25, 47, 69, 91, 113, 135, 157, 179, 201, 223, 245, 267, 289, 311, 333, 355, 377, 399, 421, 443, 465, 487, 509, 531, 553, 575, 597, 619, 641, 663, 685, 707, 729, 751, 773, 795, 817, 839, 861, 883, 905, 927, 949, 971, 993, 1015, 1037, 1059, 1081, 1103, 1125, 1147, 1169, 1191, 1213, 1235, 1257, 1279, 1301, 1323, 1345, 1367, 1389, 39, 73, 107, 141, 175, 209, 243, 277, 311, 345, 379, 413, 447, 481, 515, 549, 583, 617, 651, 685, 719, 753, 787, 821, 855, 889, 923, 957, 991, 1025, 1059, 1093, 1127, 1161, 1195, 1229, 1263, 1297, 1331, 1365,53, 99, 145, 191, 237, 283, 329, 375, 421, 467, 513, 559, 605, 651, 697, 743, 789, 835, 881, 927, 973, 1019, 1065, 1111, 1157, 1203, 1249, 1295, 1341, 1387, 67, 125, 183, 241, 299, 357, 415, 473, 531, 589, 647, 705, 763, 821, 879, 937, 995, 1053, 1111, 1169, 1227, 1285, 1343, 95, 177, 259, 341, 423, 505, 587, 669, 751, 833, 915, 997, 1079, 1161, 1243, 1325, 109, 203, 297, 391, 485, 579, 673, 767, 861, 955, 1049, 1143, 1237, 1331, 123, 229, 335, 441, 547, 653, 759, 865, 971, 1077, 1183, 1289, 1395, 137, 255, 373, 491, 609, 727, 845, 963, 1081, 1199, 1317,165, 307, 449, 591, 733, 875, 1017,1159, 1301,193, 359, 525, 691, 857, 1023, 1189, 1355,207, 385, 563, 741, 919, 1097, 1275,235, 437, 639, 841, 1043, 1245,249, 463, 677, 891, 1105, 1319,263, 489, 715, 941, 1167, 1393,305, 577, 829, 1091, 1353,319, 593, 867, 1141,347, 645, 943, 1241,389, 723, 1057, 1391,403, 749, 1095,417, 775, 1133,445, 827, 1209,459, 853, 1247, 529, 983, 543, 1009, 557, 1035, 585, 1087, 599, 1113, 613, 1139, 627, 1165, 655, 1217, 683, 1269, 725, 1347, 739, 1373, 753,

Each and every one of these elements can then be deleted from the seven column table, in addition to the deletion of the entire column seven and each and every number in columns 1 to 6 whose last digit is 1.

When this is done the seven column table will remain only with the following elements of the solution set, each on its specific position on the seven column table.

TABLE 6 (C): ELEMENTS OF THE SOLUTION SET FOR THE FIRST HALF PAIR PRIMES FORMULA UP TO THE  $100^{TH}$  ROW OF THE SEVEN COLUMN TABLE. (This table must be read horizontally).

S.N	C1	C2	<i>C3</i>	C4	C5	C6	<i>C</i> 7	S.N	C1	C2	C3	C4	C5	C6	C7
1	01	1	3	5	7	9	<i>-</i>	51	699		703	01		709	
2	13	15	17	19		23		52	713		717			, 0,	
3	27	29		33	35	37		53	, 10		, , ,		735	737	
4		43	45		49	-		54			745	747	,,,,		1
5	55	57	59		63	65		55	755	757				765	1
6				75	77	79		56	769				777	779	
7	83	85	87	89		93		57	783	785				793	
8	97			103	105			58	797	799		803	805	807	
9			115	117	119			59		813	815		819		
10		127	129		133			60	825						
11	139		143		147	149		61			843		847	849	
12	153	155		159		163		62				859		863	
13	167	169		173				63		869		873		877	
14			185	187	189			64			885	887			
15	195	197	199			205		65	895	897	899		903		
16			213	215	217	219		66	909		913		917		
17	223	225	227					67		925		929		933	
18		239				247		68		939			945	947	
19		253		257				69		953			959		
20	265		269		273	275		70	965	967	969			975	
21	275			285	287			71	979			985	987	989	
22	293	295				303		72				999		1003	
23		309		313	315	317		73	1007			1013			
24		323	325	327				74				1027	1029		
25		337	339		343			75			1039			1045	
26	349		353					76				1055			
27	363	365	367	369				77	1063		1067	1069		1073	
28				383		387		78				1083	1085		
29		393	395	397				79					1099		
30	405	407	409	12.5	425	120		80	4440	1107	1109			1115	
31	419	10.5		425	427	429		81	1119	1107	1123			1129	
32	433	435		439	455	455		82		1135	1137	1150	1155		
33				453	455	457		83		1149		1153	1155		-
34	17.5	477	470		469			84	1175	1163	1170			1105	
35	475	477	479	495	483 497	499		85	1175	1177	1179		1197	1185	
<i>36 37</i>	502		493 507	493	497	499		86 87		1205	1193 1207		119/	<del>                                     </del>	<b>-</b>
38	503 517	519	307	523		527		88		1205	1207	1223	1225	-	$\vdash$
39	J11	533	535	537	539	341		89		1219		1223	1223	-	$\vdash$
40	545	555	555	331	333	555		90		1433			1253	1255	
41	5+3		563	565	567	569		91	1259			1265	1267	1433	$\vdash$
42	573		303	303	301	307	<del>                                     </del>	92	1273		1277	1203	1207	1283	
43	313				595			93	1287		12//	1293		1203	$\vdash$
44		603		607				94	1207	1303	1305	1307	1309		
45	615	000		557	623	625		95	1315	1000	1000	1007	1207		
46	629		633	635	637			96	1329		1333	1335	1337	1339	
47	643		000	649	557			97	1027		1000	1349	100,	1337	
48	657	659			665	667		98	1357	1359		1363			
49			675		679			99			1375	1377	1379		
50		687	689		693	695		100	1385						

Note that when each and every one of the above elements are used as a substitute, one after another, for variable  $n_{so}$  in the formula for FHPPs, no such primes existing within this specific range of numbers will be skipped. Table 6.(c) above shows that there altogether 290 elements of the solution set for the

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FHPPs formula in the first 699 numbers on the sequence of odd numbers. These are all odd numbers from 1 to 1397. These elements represent a total of 290 FHPPs on the first 4,158 natural numbers from unit to 4,157. In this range of numbers, the FHPP is established by the seven column table as follows;

 $P_1=3n_o+2$ ;  $P_1=(3 \times 1)+2$ ;  $P_1=3+2$ ;  $P_1=5$  The last first half pair prime in the range is as follows;

$$P_1 = 3n_o + 2$$
;  $P_1 = (3 \times 1385) + 2$ ;  $P_1 = 4155 + 2$ ;  $P_1 = 4157$ .

# Numerical Positions of Non-Substitutes on the Sequence of Odd Numbers

The solution set for the FHPPs formula can also be determined by the identification of numerical positions of non-substitutes on the sequence of odd numbers. In this particular context, a numerical position refers to that odd number's actual position on the sequence in terms of counting numbers. For example the numerical position of 1 is 1 because it is the first odd number on the sequence. The numerical position of 3 is 2 because it is the second odd number on the sequence.

Since odd numbers are those numbers of the form

(2N+1)-2, where N is any natural number , variable N in the expression X=(2N+1)-2, indicate the position of odd number 'X' on the sequence of odd numbers. For example, if we pick any odd number say 59, this numbers' numerical position on the sequence of odd numbers can be worked out by replacing X with 59 and solving the equation for variable N as follows;

(2N+1)-2=59; 2N=(59-1)+2; N=[(59-1)+2]+2; N=30. The value of the expression is 30, meaning that 59 is the  $30^{th}$  number on the sequence of odd numbers. In short the algebraic expression for numerical positions of odd numbers on the sequence of odd numbers is N=[(X-1)+2]+2, where X is that odd number. Table 8 below is a display of numerical positions of non-substitutes on the sequence of odd numbers.

TABLE 8: DISTRIBUTION OF NUMERICAL POSITIONS, ON THE SEQUENCE OF ODD NUMBERS, OF NON-SUBSTITUTES FOR THE VARIABLE IN THE FIRST HALF PAIR PRIMES FORMULA.

	7	13	19	25	31	37	43	49	55	61	67	73	79	85	•••
5	6	11	16	21	26	31	36	41	46	51	56	61	66	71	•••
11	13	24	35	46	57	68	79	90	101	112	123	134	145	156	•••
17	20	37	54	71	88	105	122	139	156	173	190	207	224	241	
23	27	50	73	96	119	142	165	188	211	234	257	280	303	326	
29	34	63	92	121	150	179	208	237	266	295	324	353	382	411	•••
35	41	76	111	146	181	216	251	286	321	356	391	426	461	496	•••
41	48	89	130	171	212	253	294	335	376	417	458	499	540	581	
47	55	102	149	196	243	290	337	384	431	478	525	572	619	666	
53	62	115	168	221	274	327	380	433	486	539	592	645	698	751	
59	69	128	187	246	305	364	423	482	541	600	659	718	777	836	
65	76	141	206	271	336	401	466	531	596	661	726	791	856	921	
71	83	154	225	296	367	438	509	580	651	722	793	864	935	1006	•••
77	90	167	244	321	398	475	552	629	706	783	860	937	1014	1091	
83	97	180	263	346	429	512	595	678	761	844	927	1010	1093	1176	
89	104	193	282	371	460	549	638	727	816	905	994	1083	1172	1261	
•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	

The importance of table 6 above is that it separates non-substitutes from elements of the solution set on the sequence of odd numbers. This is so because, elements of the solution set for the FHPPs formula are those odd numbers whose numerical positions on the sequence, are not part of the endless structure of numbers displayed by the table. In other words, elements of the solution set are those numbers of the form (2N+1)-2, where variable N is any natural number, and which numbers are not elements of the above endless table of natural distribution of numerical positions for the variable's non-substitutes on the sequence.

If any number other than the headers in the first row and column is picked from the table, and substituted for variable N in the expression (2N+1)-2, the value of the expression will be a nonsubstitute element, which when substituted for variable  $n_{\rm o}$  in the FHPPs formula will result in a composite odd number divisible by its two odd number factors indicated as headers of the column and row under which that number falls on the table.

For example, if any number is picked from the above table, say 69, and substituted for variable N in the expression (2N+1)-2, the value of the expression will be a non-substitute element, which when substituted for variable  $n_{\text{o}}$  in the first half pair

primes formula will result in a composite odd number divisible by its two odd number factors indicated as headers of the column and roll under which that number falls on the table;

 $|(2\ x\ 69)+1|-2=137;\ P_1=(3\ x\ 137)+2;\, P_1\neq 413;$ 

Note that the end value 413 is not a prime but a composite odd number divisible by two odd numbers, these are 7 which is a header for a column in which 69 falls and 59 which is a header for the row in which 69 falls. We can prove this by dividing 413 by both 7 and 59 as follows;

$$413 \div 7 = 59$$
;  $413 \div 59 = 7$ 

Giving yet another example, let us take another number indicated on the table, say 111 which falls under the column whose header is 19 and a row whose header is 35. And proceed as follows

$$[(2 \times 111) + 1] - 2 = 221; P_1 = (3 \times 221) + 2; P_{1 \neq}$$
 665:

The value of the expression (665), is not a prime because the table indicates that it is divisible by 19 and 35. Thus;  $665 \div 19 = 35$ ;  $665 \div 35 = 19$ 

On the other hand, any numerical position, in terms of ordinary counting numbers from 1 endlessly, which is not part of the structure of the endless numbers indicated by the table, can substitute variable N in the expression (2N+1)-2, and the value of the expression will be an element of a solution set for the FHPPs formula.

From table 2 above, it can be seen that counting numbers that are not part of the structure of the table, and which, can, be used to pick elements of the solution set from the sequence of odd numbers, include natural numbers less than 6, and each and every number greater than 6 not falling within the structure of the table. Part of the set of such numbers, as shown by the table is as follows:

If any of the above counting numbers is substituted for N in the expression (2N+1)-2, the value of the expression will be an element of a solution set for the first half pair primes formula. To prove this, we can pick any of these numbers, say 17, and proceed as follows;

$$[(2 \times 17) + 1] - 2 = 33; P_1 = (3 \times 33) + 2; P_1 = 101;$$

We can prove that 101 is a first half pair prime because as both tables 4 and 6 (c) above confirm, 33 is not an element of any of the countless sets of non-substitutes for the variable in the first half pair primes formula.

# B. Solution Set for the Second Half Pair Primes Formula $(P_2 = 3n_{so} - 2)$

Having shown how to determine the solution set for the first half pair primes formula, we now can turn to the determination of the solution set for the second complementary formula.

The solution for the second half pair primes formula  $(P_2 = 3n_{so} - 2)$  is also determined by two types of tables. That is the type that show the distribution of non substitute elements on the sequence of odd numbers and the type that indicate their numerical positions on it.

This article has identified four tables of the first type and one table of the second type. Among the first type are two complementary tables that show two complementary endless structures of odd numbers, none of which are elements of the solution set. The third table shows the linear progression of such elements on the sequence of odd numbers while the fourth one shows the distribution of elements of the solution set on the seven column odd number table. The single table of the second type is an endless structure of natural numbers indicating numerical positions of non-substitute elements on the sequence of odd numbers. We now turn to these tables.

Tables 9 (a) and 9 (b) below show the systematic distribution of odd numbers that must not be substituted for the variable. Table 9 (a) relate to the first set of divisors of second half pair odd numbers (visible divisors) while table 9 (b) relate to the second set of divisors of such numbers (invisible divisors).

TABLE 9 (a): NATURAL DISTRIBUTION OF ODD NUMBERS RELATED TO THE FIRST SET OF DIVISORS OF SECOND HALF PAIR ODD NUMBERS, THAT MUST NOT BE SUBSTITUTED FOR  $N_{SO}$  IN THE STANDARD FORMULA FOR FINDING SECOND HALF PAIR PRIMES.

G.12	G.14	G.26	G.38	G.50	G.62	G.74	G.86	G.98	G.110	G.122	G.134	
G.14	17	31	45	59	73	87	101	115	129	143	157	
G.26	31	57	83	109	135	161	187	213	239	265	291	
G.38	45	83	121	159	197	235	273	311	349	387	425	
G.50	59	109	159	209	259	309	359	409	459	509	559	
G.62	73	135	197	259	321	383	445	507	569	631	693	
G.74	87	161	235	309	383	457	531	605	679	753	827	
G.86	101	187	273	359	445	531	617	703	789	875	961	
G.98	115	213	311	409	507	605	703	801	899	997	1095	
G.110	129	239	349	459	569	679	789	899	1009	1119	1229	
G.122	143	265	387	509	631	753	875	997	1119	1241	1363	
G.134	157	291	425	559	693	827	961	1095	1229	1363	1497	

TABLE 9 (b): NATURAL DISTRIBUTION OF SPECIAL ODD NUMBERS RELATED TO THE INVISIBLE DIVISORS OF SECOND HALF PAIR ODD NUMBERS, THAT MUST NOT BE SUBSTITUTED FOR  $N_{SO}$  IN THE FORMULA FOR FINDING SECOND HALF PAIR PRIMES.

G.12	G.10	G.22	G.34	G.46	G.58	G.70	G.82	G.94	G.106	G.118	G.130	
G.10	9	19	29	39	49	59	69	79	89	99	109	•••
G.22	19	41	63	85	107	129	151	173	195	217	239	•••
G.34	29	63	97	131	165	199	233	267	301	335	369	•••
G.46	39	85	131	177	223	269	315	361	407	453	499	•••
G.58	49	107	165	223	281	339	397	455	513	571	629	•••
G.70	59	129	199	269	339	409	479	549	619	689	759	•••
G.82	69	151	233	315	397	479	561	643	725	807	889	
G.94	79	173	267	361	455	549	643	737	831	925	1019	
G.106	89	195	301	407	513	619	725	831	937	1043	1149	•••
G.118	99	217	335	453	571	689	807	925	1043	1161	1279	•••
G.130	109	239	369	499	629	759	889	1019	1149	1279	1409	
•••		•••	:	•••	•••	•••	•••	•••	•••	•••	•••	

Both tables comprises of sets of non-substitutes because replacing the variable in the second half pair primes formula for any of their elements will result in a composite odd number divisible by the divisor to which that particular element's set relate.

Table 9 (a) is a display of the first group of sets of non-substitutes for the variable in the SHPPs formula. These odd numbers extend from 17 and continue without end in both rows and columns, at uniform gaps indicated by headers. Both columns and rows are related to divisors of the form 6N + 7 - 6, where N is a natural number. This set of divisors is as follows; VDSHPON ={7, 13, 19, 25, 31, 37, 43, ...}. This is the set of divisors that has been identified as a set of visible divisors of second half pair odd numbers. Each and every succeeding column and row on the table is related to each and every one of these divisors in their endless succession. In other words, column 1 and row 1 are both related to the first divisor which is 7, column 2 and row 2 are both

related the second divisor which is 13 and so in that order endlessly. Each and every column shares a common divisor with each and every row because as can be seen from the table, each and every column is a mirror image of each and every row. Elements on this endless table are non-substitutes because if any of them replaces variable  $n_{\rm o}$  in the SHPPs formula, the result will be a composite number divisible by a divisor to which that element's row and column relate.

In this first group of sets the first set is column 1 or row 1 which is related to divisor 7 the second set is column 2 or row 2 which is related to the second divisor 13. The third set is column 3 or row three which is related to the third divisor 19, and so on in that order endlessly. There are three formulas to be used in determining endless elements of each and every one of the never ending sets of non-substitutes. The first formula is for determining a divisor to which a given endless set will relate. This formula is

as follow; X = (6N + 7) - 6, where N is any natural number equal to or greater than 1. The second formula is for determining the first element of a set relating to that specific divisor. This formula is as follows; X = (14N + 17) - 14, where N is any natural number substituted for N in the first formula. The last formula is for determining the gap between each and every element in that particular set. This formula is as follows; X = (12N + 14) - 12, where N is the same natural number substituted for N in the first and second formula. With these three formulas, we can determine any such endless set as follows;

#### Example 1

Select any divisor from the first set of divisors by picking any natural number equal to or greater than 1 and substitute it for N in the first formula; (X = 6N)+7-6). For easy demonstration, let us pick a natural number which we know will give us set of numbers appearing in table 9 (a) above. Let us say 7, and work out the divisor by substituting variable N with 7 in the first formula and evaluating the expression as follows;  $X = [(6 \times 7) + 7] - 6$ ; (42 + 6)7) -6; 49 - 6; X = 43. The value of the expression is 43, meaning that our chosen divisor is 43. We can then determine the first element of a set related to divisor 43 by substituting 7 yet again for variable N in the second formula and evaluate the expression as follows;  $X = [(14 \times 7) + 17] - 14; (98 + 17) - 14$ ;115 - 14 I = 101. The value of the expression, which is 101, is the first element of a set related to divisor 43. We can then work out the gap between elements in this particular set by substituting 7 yet again for variable N in the last formula and evaluate the expression as follows;  $X = [(12 \times 7) + 14] - 12$ ; (84 + 14) - 12; 98 - 12; X = 86. The value of the expression which is 86 is the gap between elements in this particular endless set whose initial element is related to divisor 43 and whose first element is 101. With this gap determined, we can then produce this endless set of non-substitutes whose first element is 101, by simply adding 86 to 101 and then repeating the addition at each and every stage as follows {101, 187, 273, 359, 445, 531, 617, 703, 789, 875, 961, ...} Note that this is the same set of non-substitutes appearing as column 7 or row 7 in table 9 (a) above.

Example 2

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For further proof, we can also pick a natural number which would give us a divisor whose related set is not covered by table 9(a) above, let us say 27, and then proceed with evaluating each of the three different expressions as follows; X = 6N + 7 - 6; =  $[(6 \times 27) + 7] - 6$ ; = (162 + 7) - 6; = 169 - 6; X = 163; X = 14N + 17 - 14; =  $[(14 \times 27) + 17] - 14$ ; = (378 + 17) - 14; 395 - 14; X = 381; X = 12N + 14 - 12; =  $[(12 \times 27) + 14] - 12$ ; = (324 + 14) - 12; = 338 - 12; X = 326

Having picked a divisor from the first set of divisors, and having known both the initial number of its related endless series of numbers and the gap between elements in that particular endless set, we can then work out the endless set of these numbers as follows; VD163 SSON = {381, 707, 1033, 1359, 1685, 2011, 2337, ...} Note that even if this set is has not been reached by table 9 (a) above, extending it will show this set to be column 27 and row 27.

Table 9 (b) above comprises of a second group of sets of non-substitutes for the variable in the SHPPs formula. The sets presented as both columns and rows relate to divisors of the form 6N + 5 - 6 where N is any natural number. These divisors are as follows; IDSHPON = {5, 11, 17, 23, 29, 35, 41, 47, 53, 59, 65, 71, 77, 83, ...}. It is a set of invisible divisors of second half pair odd numbers'. The first set, column 1 or row 1, is related to the first divisor which is 5, the second set, column 2 or row 2, is related to the second divisor which is 11, the third set column 3 or roll 3 is related to the third divisor which is 23, and so on in that order endlessly. Each and every column shares a common divisor with each and every row because here too, each and every column is a mirror image of each and every row. Elements on this endless table are non-substitutes because if any of them replaces variable no in the SHPPs formula, the result will be a composite number divisible by a divisor to which that element's row and column relate.

There are three formulas to be used in determining each of the subsequent sets of non-substitutes in this second group of sets. The first formula is for determining an invisible divisor to which a given endless set relate. This formula is as follows; X = (6N + 5) - 6, were N is any natural number equal to or greater than 1. The second formula is for determining the first element of the set related to the identified divisor. This formula is as follows; X = (6N + 5) + (6N + 5)

(10N + 9) - 10, were N is any natural number substituted for N in the first formula. The last formula is for determining the gap between elements in this particular set, this formula is as follows; X = (12N + 10) - 12, were N is the same natural number substituted for N in the first and second formula. With these three formulas, we can determine any such endless set of non-substitutes as follows;

#### Example 1

Select any invisible divisor by picking any natural number equal to or greater than 1 and substitute it for N in the first formula; X = (6N + 5) - 6. For easy demonstration, let us pick a natural number which we know will give us set of numbers covered by table 9 (b) above, let us say 7, and work out the invisible divisor by substituting variable N with 7 in the first formula and evaluating the expression as follows;  $X = [(6 \times 7) + 5] - 6$ ; (42 + 5) - 6; 47 - 6; X = 41. The value of the expression is 41, meaning that our chosen divisor is invisible divisor 41.

We can then determine the first element of the set related to invisible divisor 41 by substituting 7 yet again for variable N in the second formula and evaluate the expression as follows;  $X = [(10 \times 7) + 9] - 10$ ; (70 + 9) - 10; 79 - 10; X = 69. The value of the expression, which is 69, is the initial number related to invisible divisor 41. We can then work out the gap between elements in this particular endless set whose first element is 69, by substituting 7 yet again for variable N in the last formula and evaluate the expression as follows;  $X = [(12 \times 7) + 10] - 12$ ; (84 + 10) - 12; 94 - 12 X = 82. The value of the expression which is 82 is the progression interval of

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an endless series of numbers extending from 69. With this uniform gap between elements determined, we can then produce an endless set of numbers extending from 69 by simply adding 82 to 69 and then repeating the addition at each and every stage as follows {69, 151, 233, 315, 397, 561, 643, 725, 807, 889 ...} Note that this is the same set of non-substitutes indicated as column 7 or row 7 in table 9(b) above.

#### Example 2

For further proof, we can also pick a natural number which would give us a divisor whose related set has not been reached by table 9(b) above, let us say 27, and then proceed with evaluating each of the three different expressions as follows; X = 6N + 5 - 6; [(6  $\times$  27) + 5] - 6; (162 + 5) - 6; 167 - 6; X = 161;  $X = [(10 \times 27) + 9] - 10$ ; (270 + 9) - 10; 279 - 10; X = 269.  $X = [(12 \times 27) + 10] - 12$ ; (324 + 10) - 12; 334 - 12; X = 322

Having picked a divisor from the first set of divisors and having known both the initial number of its related endless series of numbers and the uniform gap between elements in that particular set, we can then easily work out the elements of this endless set as follows; ID161 SSON = {269, 591, 913, 1235, 1557, ...} Note that if table 9(b) above is extended to include this set, this particular set will be revealed as being column or row 27.

#### Combining Tables 9(a) and 9(b)

In order to to identify the solution set for the SHPPs formula, tables 9(a) and 9(b) need to be combined. This combined table is as follows;

TABLE 9 (c), COMBINED TABLE OF ODD NUMBERS THAT MUST NEVER BE SUBSTITUTED FOR N <sub>SO</sub> IN THE FORMULA FOR SECOND HALF PA	R
PRIMES.	

G12	G10		G22		G34		G46		G58		G70		G82		G94		
G12		G14		G26		G38		G50		G62		G74		G86		G98	•••
G10	9		19		29		39		49		59		69		79		
G14		17		31		45		59		73		87		101		115	•••
	19	31	41	57	63	83	85	109	107	135	129	161	151	187	173	213	
	29	45	63	83	97	121	131	159	165	197	199	235	233	273	287	311	
	39	59	85	109	131	159	177	209	223	259	269	309	315	359	361	409	
	49	73	107	135	165	197	223	259	281	321	339	383	397	445	455	507	
	59	87	129	161	199	239	269	309	339	383	409	457	479	531	549	605	
	69	101	151	187	233	273	315	359	397	445	479	531	561	617	643	703	
	79	115	173	213	267	311	361	409	455	507	549	605	643	703	737	801	
	89	129	195	239	301	349	407	459	513	569	619	679	725	789	831	899	
	99	143	217	265	335	387	453	509	571	631	689	753	807	879	925	997	
	109	157	239	291	369	425	499	559	629	693	759	827	889	961	1019	1095	
	119	171	261	317	403	463	545	609	687	755	829	901	971	1047	1113	1193	
		•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	•••	

The above table combines the two groups of endless sets of non-substitutes. The columns presented in bold font constitute sets of non-substitutes related to invisible divisors of second half pair odd numbers while the columns presented in black font constitute sets of non-substitutes related to visible divisors of second half pair odd numbers.

The table is of endless continuity. The rows presented in bold font, increase in number horizontally at a uniform gap of 10, in terms of first elements in each and every succeeding set. At the same time, there is indicated for each and every row, uniform gaps between elements in each and every set, which gaps increases by 12, from the first set to each and every succeeding set. Similarly, the rows presented in black font also increase in number horizontally at a different a uniform gap of 14 in terms of first elements in each and every succeeding set. Uniform gaps between elements in each and every column/set, have also been indicated, which gaps also increase by 12 from the first set to each and every succeeding set.

From this combined table, we can identify the solution set for the SHPPs formula. We can do so by simply picking out all missing odd numbers whose range, in terms of number of digits, is complete on the table. On table 9(c) above, the odd numbers whose range is complete are only two range of numbers. These are, single digit odd numbers and two digit odd numbers. Note that the more the table is extended the more other range of numbers will be complete.

### Solution Set Containing Single Digit Numbers

To pick out a solution set containing single digit numbers, simply generate the set of all single digit odd numbers other than unit as follows; { 3, 5, 7, 9}. The next thing is to strike off from this set all single digit numbers appearing on the table. Since in the table has only one single digit odd number, which is 9, only 9 will be removed from this set meaning that the single digit number solution set for the SHPPs formula is as follows; { 3, 5, 7}

### Solution Set Containing Two Digit Numbers

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To pick out a solution set containing two digit odd numbers, generate a set of all two digit odd numbers as follows; {11, 13, 15, 17, 19, 21, 23, 25, 27, 29 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99}

Pick out all two digit odd numbers from the table as follows; {19, 29, 39, 49, 59, 69,79, 89, 99, 41, 63, 85, 97, 17, 31, 45, 59, 73, 87, 57, 83}

Strike out these numbers from the first set of two digit odd numbers and remain with the following two digit odd number solution set for the SHPPs formula; {11, 13, 15, 21, 23, 25, 27, 33, 35, 37, 43, 47, 51, 53, 55, 61, 65, 67, 71, 75, 77, 81, 91, 93, 95,}

# Solution Sets Containing Odd Numbers of Other Digits

The more table 9(c) is extended, the more solution sets containing odd numbers of other digits will be determined.

### Linear Progression of Non-Substitutes

Here too, we can also move away from the systematic cluster of non-substitutes above by establishing the linear progression of such non-substitute on the sequence of odd numbers. Table 10 below shows this linear progression;

Table 10. Linear progression of non-substitutes on the sequence of odd numbers, for the second half pair primes formula ( $P_2 = 3n_{so} - 2$ ).

G10		G22		G34		G46		G58		G70		G82		G94		G106		
	G14		G26		G38		G50		G62		G74		G86		G98		G110	
9																		9
	17																	17
19		19																19
29				29														29
	31		31															31 39
39						39												39
		41																41
	45				45													45
49								49										49
			57															57
59	59						59			59								59
		63		63														63
69												69						69
	73								73									73
79														79				79 83
		0.5	83		83	0.5												83
	07	85				85					07							85
-00	87										87					00		87 89
89				97												89		97
99				91														99
99	101												101					101
	101	107						107					101					107
109		107	109				109	107										109
137	115		107				107								115			115
119	110														110			119
					121													121
129	129	129								129							129	129

The table 10 above shows how sets of non-substitutes in the second half pair primes formula combine into a union set of non-substitutes (linear progression of non-substitute elements). The union set is shown in the last column of the table. The three dots underneath the last indicated figure of 129 shows that the set continues without end in that particular order. In the same way, the three dots indicated at the bottom of each and every column shows endless continuity of each and every such column

In addition, the three dots each, at the 'end' of the first and second row of the table, comprising of headers, are indicators for an endless uniform continuity of columns on the table and the changing uniform gaps in between each and every succeeding column's elements. This also implies that the seemingly last column showing the union set of non-substitutes is only an imposition on the table.

### The Unification of Sets of Non-Substitutes

The manner in which sets of non-substitute are combining into a union set is such that there is an initial set into which each and every succeeding subset is feeding into. The initial set is column 1. It is

a set whose elements are of the form 10N+9-10, where N is any natural number. The uniform gap of 10 implies that there is a total of 4 odd numbers missing in between each and every element of this set. The feeding into this set by elements of each and every other succeeding set implies elements of such sets either taking their actual locations in any of the four missing odd number locations or intersecting with identical elements of the initial set.

The first set to feed into the initial set is the second column, whose elements are of the form (14N+17)-14, where N is any natural number. One after another, elements of this set either fill up gaps of their actual location or intersect with their identical elements in the initial set. Other sets follow, one after another, with their elements either occupying gaps of their natural location in the initial set or intersecting with identical elements in the initial set and in other sets that precede them. On the table above, intersecting elements are presented in red font.

A union set of non-substitutes for any given range of odd numbers is complete at a point at which the first element of a succeeding set is the last odd number in that given range. In the table above, the union set of non-substitutes for the variable in the second half pair primes formula is complete only up to 129 because it is the first element of the last set within that particular range.

In the range of odd numbers covered by the table 7 above, the solution set for the SHPPs formula comprises of all odd numbers within this range that are missing from the indicated union set of non-substitutes.

### The Seven Column Odd Numbers' Table

Arising from the combined table of natural distribution of sets of non-substitutes in the formula for SHPPs, and as read with the two separate tables prior to it, a seven column table has been constructed. The purpose of the table is to expose elements of the solution set for the formula and show their natural distribution on the sequence of odd numbers. This table is as follows:

TABLE 11 (a). THE SEVEN COLUMN ODD NUMBERS' TABLE FOR LOCATING SUBSTITUTES AND NON-SUBSTITUTES IN THE FORMULA FOR SECOND HALF PAIR PRIMES.

							App.
5	7	9	11	13	15	17	9
19	21	23	25	27	29	31	19
33	35	37	39	41	43	45	29
47	49	51	53	55	57	59	39
61	63	65	67	69	71	73	49
75	77	79	81	83	85	87	59
89	91	93	95	97	99	101	69
103	105	107	109	111	113	115	79
117	119	121	123	125	127	129	89
131	133	135	137	139	141	143	99
145	147	145	151	153	155	157	109
159	161	163	165	167	169	171	119
173	175	177	179	181	183	185	129
187	189	191	193	195	197	199	139
						•••	•••

Other than the first two odd numbers 1 and 3, which have been omitted for being inconsistent with the logic of the table, and the last column which has been added as an appendage, Table 11(a) above indicates, the entire sequence of odd numbers arranged in seven columns only. This can be seen by reading the table horizontally within the confines of the seven columns.

# Locating the Solution Set Within the Seven Column Table

Column 7, represents the initial set of non-

substitutes related to visible divisors of second half pair odd numbers. It is a set whose elements are of the form (14N+17)-14, where N is any natural number. It is related to visible divisor 7. There is therefore no single element of the solution set in this particular set of endless elements. Most importantly, the column is also an endless set of first elements of the first sets of non-substitutes, each of whose endless elements are systematically spread out within columns 1 to 6 of the table.

Similarly, the last column (appendage) represent the initial set of non-substitutes related to invisible divisors of second half pair odd numbers. It is a set whose elements are of the form (10N + 9) - 10, where N is any natural number. It is a set that is related to invisible divisor 5. This appended column is an endless set of first elements of the second group of sets of non-substitutes in the SHPPs formula, each of which endless elements are also systematically spread within the confines of the seven columns of the table.

# Deleting Sets of Non- Substitutes From the Seven Column Table

As indicated already, columns 7 and 8 of the seven column table above, respectively constitute first elements of each and every one of the endless sets of non-substitutes. Therefore, generating elements of these sets and striking them off from columns 1 to 6 of the seven column table will leave only substitutes on the table.

For easy demonstration, we can extend our seven column table to 100 rows comprising of the first 702 odd numbers (including 1 and 3 not included on the table). In this range, the first odd number is 1, in column 6 of the discarded first row, The second is 3 in column 7 of the same discarded first row. But because this first row has been discarded for its distortion effect on the logic of the table, there are 700 odd numbers in this range, starting with 5 in column 1 of roll 1, and ending with 1403 in column 7 of the 100<sup>th</sup> row.

Table 8 (b): The seven column table extended to the  $100^{TH}$  row.

S.N	C1	C2	C3	C4	C5	<i>C6</i>	<i>C</i> 7	App.	S.N	CI	C2	<i>C3</i>	C4	C5	C6	<i>C</i> 7	App.
1	5	7	9	11	13	15	17	9	51	705	707	709	911	713	715	717	509
2	19	21	23	25	27	29	31	19	52	719	721	723	725	727	729	731	519
3	33	35	37	39	41	43	45	29	53	733	735	737	739	741	743	745	529
4	47	49	51	53	55	57	59	39	54	747	749	751	753	755	757	759	539
5	61	63	65	67	69	71	73	49	55	761	763	765	767	769	771	773	549
6	75	77	79	81	83	85	87	59	56	775	777	779	781	783	785	787	559
7	89	91	93	95	97	99	101	69	57	789	791	793	795	797	799	801	569
8	103	105	107	109	111	113	115	79	58	803	805	807	809	811	813	815	579
9	117	119	121	123	125	127	129	89	59	817	819	821	823	825	827	829	589
10	131	133	135	137	139	141	143	99	60	831	833	835	837	839	841	843	599
11	145	147	149	151	153	155	157	109	61	845	847	849	851	853	855	857	609
12	159	161	163	165	167	169	171	119	62	859	861	863	865	867	869	871	619
13	173	175	177	179	181	183	185	129	63	873	875	877	879	881	883	885	629
14	187	189	191	193	195	197	199	139	64	887	889	891	893	895	897	899	639
15	201	203	205	207	209	211	213	149	65	901	903	905	907	909	911	913	649
16	215	217	219	221	223	225	227	159	66	915	917	919	921	923	925	927	659
17	229	231	233	235	237	239	241	169	67	929	931	933	935	937	939	941	669
18	243	245	247	249	251	253	255	179	68	943	945	947	949	951	953	955	679
19	257	259	261	263	265	267	269	189	69	957	959	961	963	965	967	969	689
20	271	273	275	277	279	281	283	199	70	971	973	975	977	979	981	983	699
21	285	287	289	291	293	295	297	209	71	985	987	989	991	993	995	997	709
22	299	301	303	305	307	309	311	219	72	999	1001	1003	1005	1007	1009	1011	719
23	313	315	317	319	321	323	325	229	73	1013	1015	1017	1019	1021	1023	1025	729
24	327	329	331	333	335	337	339	239	74	1027	1029	1031	1033	1035	1037	1039	739
25	341	343	345	347	349	351	353	249	75	1041	1043	1045	1047	1049	1051	1053	749
26	355	357	359	361	363	365	367	259	76	1055	1057	1059	1061	1063	1065	1067	759
27	369	371	373	375	377	379	381	269	77	1069	1071	1073	1075	1077	1079	1081	769
28	383	385	387	389	391	393	395	279	78	1083	1085	1087	1089	1091	1093	1095	779
29	397	399	401	403	405	407	409	289	79	1097	1099	1101	1103	1105	1107	1109	789
30	411	413	415	417	419	421	423	299	80	1111	1113	1115	1117	1119	1121	1123	799
31	425	427	429	431	433	435	437	309	81	1125	1127	1129	1131	1133	1135	1137	809
32	439	441	443	445	447	449	451	319	82	1139	1141	1143	1145	1147	1149	1151	819
33	453	455	457	459	461	463	465	329	83	1153	1155	1157	1159	1161	1163	1165	829
34	467	469	471	473	475	477	479	339	84	1167	1169	1171	1173	1175	1177	1179	839
35	481	483	485	487	489	491	493	349	85	1181	1183	1185	1187	1189	1191	1193	849
36	495 509	497	499	501	503	505	507	359	86	1195	1197	1199	1201	1203	1205	1207	859
37 38	523	511 525	513 527	515 529	517 531	519 533	521 535	369 379	87	1209 1223	1211 1225	1213 1227	1215 1229	1217 1231	1219 1233	1221 1235	869 879
39	523	539	541	543	545	547	549	389	88 89	1223	1225	1241	1243	1231	1233	1235	889
40	551	553	555	557	559	561	563	399	90	1257	1259	1241	1243	1243	1247	1249	899
41	565	567	569	571	573	575	577	409	91	1265	1255	1269	1237	1239	1201	1203	909
42	579	581	583	585	587	589	591	419	92	1203	1281	1283	1285	1273	1289	1277	919
43	593	595	597	599	601	603	605	429	93	1293	1295	1297	1299	1301	1303	1305	929
44	607	609	611	613	615	617	619	439	94	1307	1309	1311	1313	1315	1317	1319	939
45	621	623	625	627	629	631	633	449	95	1321	1323	1325	1327	1329	1331	1333	949
46	635	637	639	641	643	645	647	459	96	1335	1337	1339	1341	1343	1345	1347	959
47	649	651	653	655	657	659	661	469	97	1349	1351	1353	1355	1357	1359	1361	969
48	663	665	667	669	671	673	675	479	98	1363	1365	1367	1369	1371	1373	1375	979
49	677	679	681	683	685	687	689	489	99	1377	1379	1381	1383	1385	1387	1389	989
50	691	693	695	697	699	701	703	499	100		1393	1395	1397	1399	1401	1403	999
-			-/-	-//					100			•••	•••	•••	•••		
<u></u>			l .	l .	l .	l .				1	•••	•••			•••	•••	•••

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## Deleting Sets of Non-Substitutes from the Seven Column Table

In deciding what sets whose elements we should generate in order to cover our specified range, we should first determine from both columns 7 and 8, the most necessary sets whose elements we ought to produce, and within those sets, what elements we ought to discard.

The first thing is to establish for each column, the maximum number of sets whose elements we should generate, to cover our specified range of odd numbers. We begin from column 8 which has the smallest first element 9, and pick from it, elements up to an element whose second significant element is equal to or less than our selected limit of 1403. In this particular context, the second significant element of any set of non-substitutes implies the element after its first element which is not intersected with elements of sets preceding it. The two column sets have each, a different formula for working out second significant elements of their sets

With regard to sets indicated by Column 8, we begin by selecting a first element of the set whose second significant element we wish to establish, let us say 29, and substitute this chosen number for X in the following equation and evaluate it as follows;  $N = |(X + 10) - 9| \div 10$ ;  $N = |(29 + 10) - 9| \div 10$ ; N = 3

Next is to work out the gap between each and every element of this set whose first element is 29, by substituting 3 for N in the following equation and evaluating it as follows;

$$X = (12N + 10) - 12;$$
  $X = |(12 \times 3) + 10| - 12;$   $X = 34$ 

Next is to work out the second significant element in this chosen set by substituting 3 for N yet again in the following equation and evaluating it as follows:

$$X = (34N + 29) - 34$$
;  $X = |(34 \times 3) + 29| - 34$ ;  $X = 97$ 

We have established that 97 is the second significant element in the set of non-substitutes whose first element is 29, because all other elements before it (including the first element) are intersected by preceding sets.

Using the above method, the search for a set whose second significant element is equal to or less than our limit of 1403 will establish this set as one whose first element is 99, which second significant element is 1161. This means that the sets required from column S2, for our selected range are 10. These sets, as indicated by their first elements, are as follows {9, 19, 29, 39, 49, 59, 69, 79, 89, 99}.

With regard to sets in column 7, we begin by selecting a first element of a set whose second significant element we wish to establish, Let us say 87, and substitute it for X in following equation and evaluate it as follows;

$$N = [(X + 14) - 17] \div 14; N = [(87 + 14) - 17] \div 14; N = 6$$

Next is to work out the gap between each and every elements of this set whose first element is 87, by substituting 6 for N in the following equation and evaluating it as follows;

$$X = (12N + 14) - 12; X = [(12 \times 6) + 14] - 12; X = 74$$

Next is to work out the second significant element in this chosen set by substituting 6 for N yet again in the following equation and evaluating it as follows:

$$X = (74N + 87) - 74$$
;  $[(74 \times 6) + 87] - 74$ ;  $X = 457$ 

We have established that 457 is the second significant element in the set of non-substitutes whose first element is 87, because all other elements before it including 87 intersect with some elements of preceding sets.

When we follow the above method, our search for the set whose second significant element is equal to or less than our limit of 1403, will be established to be the set whose first element is 143, which second significant element is 1241. This means that the sets required form column 7, for our selected range are also 10. These sets as indicated by their first elements are as follows; { 17, 31, 45, 59, 73, 87, 101, 115, 129, 143, }. With the two sets determined in accordance with our selected range of numbers, the next stage is to prune both sets by removing unnecessary sets from amongst them. These unnecessary sets are those that are subsets of sets preceding them.

With regard to sets listed in column 8, First elements of sets that are to be discarded are of the form (50N+9)-50 where N is any natural number. This set is as follows;  $\{59,109,159,209,259...\}$ . Such sets can be discarded because they are subsets of a preceding set whose elements are of the form (10N+9)-10 where N is any natural number. Within the range of numbers that we have selected, there is at least one set of this form, and that is the set whose first element is 59. The omission of this set leaves us with a total of 9 sets whose element we have to generate and remove from the table. This pruned set is as follows;  $\{9,19,29,39,49,69,79,89,99\}$ .

With regard to sets listed in column 7, first elements of sets that should be removed are of the form (70N + 59) - 70 where N is any natural

number. This set is as follows; {59, 129, 199, 269, 339, ...}. Within our selected range, there are two sets of this form. These are the set whose first element is 59 and the one whose first element is 129. These two sets can be discarded because they are subsets of a preceding set of the form (10N + 9) – 10 where N is any natural number. This preceding initial set is located in column 8 of the seven column table. The omission of these two sets leaves us with a total of 8 sets whose element we have to generate and remove from the table. The Pruned set is as follows; {17, 31, 45, 73, 87, 101, 115, 143}.

The other sets that are to be omitted are sets of the form 98N + 115 - 98 where N is any natural number. This set is as follows  $\{115, 213, 213, 311, 409, 507, 605 ...\}$ . Such sets are not necessary because all their endless elements are located within column 7. In other words, they are all subsets of the set of the form (14N + 17) - 14 where N is any natural number. In our chosen range of numbers there is one such set which set is indicated by its first element of 115. If this set is removed, our final set of sets of appropriate non-substitutes will be as follows;  $\{17, 31, 45, 73, 87, 101, 143\}$ .

In order to systematically remove elements of the identified sets from the table, we first combine the two pruned sets in the ascending order of their combined elements as follows;

{9, 17, 19, 29, 31, 39, 45, 49, 69, 73, 79, 87, 89, 99, 101, 143}.

We can now generate elements of these sets, each up to our maximum limit of 1403 and delete them all from the seven column table to leave only elements of the solution set for the formula. Note that, elements of the first set need not be generated because being elements of the form (10N + 9) - 10, where N is any natural number, they are each

and every number whose last digit is 9 including 9 itself. They are therefore identifiable and can be removed on sight from the entire seven column table. Note too that elements of the second set all of which are of the form (14N+17)-14, where N is any natural number need not be produced because they in actual fact constitute column 7 which is removable on sight. The other set of numbers that requires to be removed on sight is the entire column 8 because it is only an appendage to the table

It should be further noted that the determination of the second significant elements for each of the two groups of sets, already explained in this section, implies disregarding for each and every set, elements that intersect with preceding sets. Therefore, instead of generating elements from the very beginning of each and every selected set, all elements of succeeding sets that intersect with elements of preceding have to be discarded. However, for the purpose of maintaining the original identity of all sets, an indication of the original starting point (first element) should be made together with the column to which each of the sets belong. For example the form (82N + 69) - 82where N is any natural number, which has 69 as its first element and 561 as its second significant element should be presented as follows; Column 8, 69: (82N + 561) - 82 < 1403.

This implies that generating elements of the set whose first element is 69, should not begin with 69, but with the set's second significant element which is 561. Consequently, elements of this set will not be of the form (82N+69)-82, but of the form (82N+561)-82 where N is any natural number.

The verbally, the expression: Column 8, 69:  $(82N + 561) - 82 \le 1403$ , means the set in Column 8 whose first element is 69, and whose gap between each and every element is 82, comprising of elements from 561 to an element less than or equal to the limit of 1403.

With these clarifications we can now generate elements of the remaining 16 most appropriate sets as follows; and thereafter, delete them all from the seven column table.

- 3. Column 8, 19:  $(22N + 41) 22 \le 1403$
- {41, 63, 85, 107, 129, 151, 173, 195, 217, 239, 261, 283, 305, 327, 349, 371, 393, 415, 437, 459, 481, 503, 525, 547, 569, 591, 613, 635, 657, 679, 701, 723, 745, 767, 789, 811, 833, 855, 877, 899, 921, 943, 965, 987, 1009, 1031, 1053, 1075, 1097, 1119, 1141, 1163, 1185, 1207, 1229, 1251, 1273, 1295, 1317, 1339, 1361, 1383}
  - 4. Column 8, 29: (34N + 97) 34 < 1403
- [97, 131, 165, 199, 233, 267, 301, 335, 369, 403, 437, 471, 505, 539, 573, 607, 641, 675, 709, 743, 777, 811, 845, 879, 913, 947, 981, 1015, 1049, 1083, 1117, 1151, 1185, 1219, 1253, 1287, 1321, 1355, 1389}
  - 5. Column 7, 31: (26N + 57) 26 < 1403
- {57, 83, 109, 135, 161, 187, 213, 239, 265, 291, 317, 343, 369, 395, 421, 447, 473, 499, 525, 551,577, 603, 629, 655, 681, 707, 733, 759, 785, 811, 837, 863, 889, 915, 941, 967, 993, 1019, 1045, 1071, 1097, 1123, 1149, 1175, 1201, 1227, 1253, 1279, 1305, 1331, 1357, 1383}
  - 6. Column 8, 39: (46N + 177) 46 < 1403

{177, 223, 269, 315, 361, 407, 453, 499, 545, 591, 637, 683, 729, 775, 821, 867, 913, 959, 1005, 1051, 1097, 1143, 1189, 1235, 1281, 1327, 1373}

- 7. Column 8, 45:  $38N + 121 38 \le 1403$
- {121, 159, 197, 235, 273, 311, 349, 387, 429, 463, 501, 539, 577, 615, 653, 691, 729, 767, 805, 843, 881, 919, 957, 995, 1033, 1071, 1109, 1147, 1185, 1223, 1261, 1299, 1337, 1375}
  - 8. Column 8, 49:  $(58N + 281) 58 \le 1403$
- {281, 339, 397, 455, 513, 571, 629, 687, 745, 803, 861, 919, 977, 1035, 1093, 1151, 1209, 1267, 1325, 1383}
  - 9. Column 8, 69:  $(82N + 561) 82 \le 1403$
- $\{561, 643, 725, 807, 889, 971, 1053, 1135, 1217, 1299, 1381\}$ 
  - 10. Column 7, 73:  $(62N + 321) 62 \le 1403$ .
- {321, 383, 445, 507, 569, 631, 693, 755, 817, 879, 941, 1003, 1065, 1127, 1189, 1251, 1313, 1375,}
  - 11. Column 8, 79:  $(94N + 737) 94 \le 1403$ .
- {737, 831, 925, 1019, 1113, 1207, 1301, 1395}
  - 12. Column 7, 87:  $(74N + 457) 74 \le 1403$ .
- {457, 531, 605, 679, 753, 827, 901, 975, 1045, 1123, 1197, 1271, 1345}
  - 13. Column 8, 89:  $(106N + 937) 106 \le 1403$ .
- {937, 1043, 1149, 1255, 1361,}
  - 14. Column 8, 99:  $(118N + 1161) 118 \le 1403$ .
- {1161, 1279, 1397,}Column 7, 101: (86N + 617) 86 < 1403.
- {617, 703, 789, 875, 961, 1047, 1133, 1219, 1305, 1391}
  - 15 Column 7, 143:  $(122N + 1241) 122 \le 1403$ .  $\{1241, 1363\}$

### Elements of the Solution Set up to the 100th Row of the Seven Column Table

After deleting from the seven column table, all sets of non-substitutes indicated above, the seven column table will reveal the natural spread of elements of the solution set, on the sequence of odd numbers up to the 100<sup>th</sup> row of the table as follows;

TABLE 11 (c):THE NATURAL SPREAD OF ELEMENTS OF THE SOLUTION SET FOR THE SECOND HALF PAIR PRIMES FORMULA, ON THE SEQUENCE OF ODD NUMBERS, UP TO THE  $100^{TH}$  ROW OF THE SEVEN COLUMN TABLE (This table must be read horizontally).

S.N	Cl	C2	C3	C4	C5	C6	<i>C7</i>	App.	S.N	C1	C2	C3	C4	C5	C6	C7	App.
1	5	7		11	13	15			51	705			911	713	715		
2		21	23	25	27				52		721			727			
3	33	35	37			43			53		735			741			
4	47		51	53	55				54	747		751	753		757		
5	61		65	67		71			55	761	763	765			771		
6	75	77		81					56				781	783			
7		91	93	95					57		791	793	795	797			
8	103	105			111	113			58						813		
9	117			123	125	127			59				823	825			
10		133		137		141			60			835			841		
11	145	147			153	155			61		847		851	853			
12			163		167				62				865				
13		175			181	183			63	873					883		
14			191	193					64	887			893	895	897		
15	201	203	205	207		211			65		903	905	907		911		
16	215			221		225			66		917			923			
17		231			237				67		931	933	935				

18	243	245	247		251	253	68		945			951	953	
19	257			263			69				963			
20	271		275	277			70	971	973					
21	285	287			293	295	71	985			991			
22			303		307		72		1001			1007		
23	313					323	73	1013		1017		1021	1023	
24			331	333		337	74	1027					1037	
25	341		345	347		351	75	1041						
26	355	357			363	365	76	1055	1057		1061	1063		
27			373	375	377		77			1073		1077		
28		385			391		78		1085	1087		1091	1093	
29			401		405		79			1101	1103	1105	1107	
30	411	413		417			80	1111		1115			1121	
31	425	427		431	433	435	81	1125			1131			
32		441	443				82				1145			
33					461		83	1153	1155	1157				
34	467				475	477	84	1167		1171	1173		1177	
35		483	485	487		491	85	1181	1183		1187		1191	
36	495	497					86	1195				1203	1205	
37		511		515	517		87		1211	1213	1215			
38	523		527			533	88		1225			1231	1233	
39	537		541	543			89	1237			1243	1245	1247	
40		553	555	557			90				1257			
41	565	567				575	91	1265					1275	
42		581	583	585	587		92			1283	1285			
43	593	595	597		601		93	1293		1297			1303	
44			611				94	1307		1311		1315		
45	621	623	625	627			95		1323					
46						645	96	1335			1341	1343		
47		651					97		1351	1353				
48	663	665	667		671	673	98		1365	1367		1371		
49	677				685		99	1377				1385	1387	
50			695	697			100		1393				1401	
														 •••

There are altogether, 288 elements of the solution set for the SHPPs formula in the first 703 odd numbers on the sequence of odd numbers. These are all odd numbers from 1 to 1403. These elements represent a total of 288 second half pair primes on the first 4,202 natural numbers from unit to 4,201.

In this range of numbers, the first SHPP is as follows;  $P_2=3n_o-2$ ;  $P_2=3$  x 3-2;  $P_2=9$  -2;  $P_2=7$ .

The last SHPP in the range is as follows;  $P_2=3n_o-2$ ;  $P_2=(3\ x\ 1401)-2$ ;  $P_2=4203-2$ ;  $P_2=4201$ .

# Numerical Positions of Non Substitutes on the Sequence of Odd Numbers

Elements of the solution set for the SHPPs formula can also be identified by knowledge of the distribution of numerical positions of non-substitute elements on the sequence of odd numbers. The tables 12(a) and 12(b) below show such a distribution.

TABLE 12 (a): THE FIRST TABLE OF THE DISTRIBUTION OF NUMERICAL POSITIONS, ON THE SEQUENCE OF ODD NUMBERS, OF NON-SUBSTITUTES, FOR THE VARIABLE IN THE SECOND HALF PAIR PRIMES FORMULA.

	7	13	19	25	31	37	43	49	55	61	67	73	79	85	91	•••
7	9	16	23	30	37	44	51	58	65	72	79	86	93	100	107	
13	16	29	42	55	68	81	94	107	120	133	146	159	172	185	198	•••
19	23	42	61	80	99	118	137	156	175	194	213	232	251	270	289	•••
25	30	55	80	105	130	155	180	205	230	255	280	305	330	355	380	•••
31	37	68	99	130	161	192	223	254	285	316	347	378	409	440	471	•••
37	44	81	118	155	192	229	266	303	340	377	414	451	488	525	562	•••
43	51	94	137	180	223	266	309	352	395	438	481	524	567	610	653	•••
49	58	107	156	205	254	303	352	401	450	499	548	597	646	695	744	•••
55	65	120	175	230	285	340	395	450	505	560	615	670	725	780	835	
61	72	133	194	255	316	377	438	499	560	621	682	743	804	865	926	
67	79	146	213	280	347	414	481	548	615	682	749	816	883	950	1017	•••

73	86	159	232	305	378	451	524	597	670	743	816	889	962	1035	1108	
79	93	172	251	330	409	488	567	646	725	804	883	962	1041	1120	1199	•••
85	100	185	270	355	440	525	610	695	780	865	950	1035	1120	1205	1290	:
91	107	198	289	380	471	562	653	744	835	926	1017	1108	1199	1290	1381	

TABLE 12 (b): THE SECOND TABLE OF THE DISTRIBUTION OF NUMERICAL POSITIONS, ON THE SEQUENCE OF ODD NUMBERS, OF NON-SUBSTITUTES, FOR THE VARIABLE IN THE SECOND HALF PAIR PRIMES FORMULA.

	5	11	17	23	29	35	41	47	53	59	65	71	77	83	89	•••
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	•••
11	10	21	32	43	54	65	76	87	98	109	120	131	142	153	164	•••
17	15	32	49	66	83	100	117	134	151	168	185	202	219	236	253	•••
23	20	43	66	89	112	135	158	181	204	227	250	273	296	319	342	•••
29	25	54	83	112	141	170	199	228	257	286	315	344	373	402	431	•••
35	30	65	100	135	170	205	240	275	310	345	380	415	450	485	520	
41	35	76	117	158	199	240	281	322	363	404	445	486	527	568	609	•••
47	40	87	134	181	228	275	322	369	416	463	510	557	604	651	698	•••
53	45	98	151	204	257	310	363	416	469	522	575	628	681	734	787	•••
59	50	109	168	227	286	345	404	463	522	581	640	699	758	817	876	•••
65	55	120	185	250	315	380	445	510	575	640	705	770	835	900	965	•••
71	60	131	202	273	344	415	486	557	628	699	770	841	912	983	1054	•••
77	65	142	219	296	373	450	527	604	681	758	835	912	989	1066	1143	
83	70	153	236	319	402	485	568	651	734	817	900	983	1066	1149	1232	•••
89	75	164	253	342	431	520	609	698	787	876	965	1054	1143	1232	1321	•••
	•••	•••	•••		•••	•••	•••	•••								•••

Tables 12(a) and 12(b) above, in combination, show the distribution of numerical positions, on the sequence of odd numbers, of non-substitutes, for the variable in the SHPPs formula. Table 12(a) indicate the numerical positions of non-substitutes relating to multiples of visible divisors of second half pair odd numbers, while table 12(b) show the numerical positions of non substitutes relating to multiples of their invisible divisors.

The sequence of odd numbers is such that it comprises of both non-substitutes and elements of the solution set for the second half pair primes formula. The significance of the two tables above is that they serve to separate, non-substitute elements from elements of the solution set on the sequence. This is so because they indicate the numerical positions of only non-substitute elements on the sequence, meaning that any odd number on the sequence, whose numerical position is not part of the structure of numbers on either table is an element of the solution set.

If any number appearing in the actual text of either of the two tables is picked and substituted for variable N in the expression (2N+1)-2, the value of the expression will be a non-substitute element, which when substituted for variable  $n_o$  in the SHPPs formula, will result in a composite odd number divisible by its two odd number factors indicated as headers of the column and roll under which that number falls on that particular table.

As an example, lets pick any number which is part of the structure of either table, say 135 in table 4

(b), substitute it for variable N in the expression 2N + 1 - 2, and evaluate the expression as follows;  $\{[(2 \text{ x } 135)] + 1\} - 2 = 269$ . Next is to substitute 269 for variable  $n_o$  in the SHPPs formula and evaluate the expression as follows;  $P_2 = (3 \text{ x } 269) - 2$ ;  $P_2 \neq 805$ . Note that the value of the expression which is 805 is not a prime but a composite number divisible by 23 and 35, which are headers for the column and roll respectively under which 135 falls. This can be proved as follows;

 $805 \div 23 = 35$ ;  $805 \div 35 = 23$ 

On the other hand, If any natural number other than 1, which is not part of the endless structure of numbers of either table is substituted for variable N in the expression (2N+1)-2, the value of the expression will be an element of a solution set for the SHPPs formula. With reference to the two tables above, counting numbers that can be used to pick elements of the solution set from the sequence of odd numbers, include numbers less than 5 and 9, other than unit, and each and every number greater than 5 and 9 not falling within the endless structure of either of the two tables. Part of the set of such numbers, as shown by the two is as follows;

{2,3,4,6,7,8,11,12,13,14,17,18,19,22,24,26,27,28,3 1, 33,34, 36,38,39,41,46,..}

Note that if any of the above counting numbers is substituted for N in the expression (2N + 1) - 2, the value of the expression will be an element of a solution set for the SHPPs. For example, let us pick any of the above numbers, none of which is part of the structure of either table above, say 41, and substitute it for variable N in the expression (2N +

1) -2, and evaluate the expression as follows;  $\{[(2 \times 41)] + 1\} - 2 = 81$ .

Next is to substitute 81 for variable  $n_o$  in the SHPPs formula and evaluate the expression as follows;  $P_2 = (3 \times 81) - 2$ ;  $P_2 = 241$ . Note that the value of the expression which is 241 is a second half pair prime. The evidence is that 81 is not an element of either table, but an element of the solution set for the SHPPs formula as confirmed by table 11 (c) above.

#### Conclusion

To conclude this section, it has been demonstrated that there is indeed a solution set that satisfies the new standard formula for finding primes and which solution set can be established in various ways.

# V. COMPOSITE ODD NUMBER DISCOUNT TABLES

A composite odd number discount table is one that contains specific odd numbers of any chosen range among which missing odd numbers other than specified obvious non-substitutes within that range, are the only substitutes (solutions) for variable  $n_{\rm so}$  in the standard formula for finding primes. There are two such tables. The first table is for the identification of a solution set for the first half pair

primes formula while the second table helps to identify a solution set for the second half pair primes formula.

#### Basic Rules

The basic rules involved in the compilation of the tables are as follows: Trimming the sequence of natural numbers to the most essential sequence of odd numbers called 'Paired Odd Numbers'; Recognizing that primes belong to two different sets, that is the set of first half pair primes (SFHPPs) and the set of second half pair primes (SHPPs). Excluding from the set of appropriate divisors of paired odd numbers, the numbers 2 and 3 and their endless multiples. appropriate sets of divisors to appropriate sets of paired odd numbers; Recognizing the difference between visible and invisible divisors of paired odd numbers and the distinction between them with regard to their divisibility of composites among such numbers and recognizing that primes are the only ultimate and most appropriate divisors of divisible natural numbers.

### A. The First Half Pair Primes Formula Table

The first table which can be used to identify elements of the solution set for the first half pair primes formula is table 13 (a) below:

TABLE 13 (a): FIRST HALF PAIR PRIMES FORMULA'S ( $P_1 = 3n_{so} + 2$ ) NON-SUBSTITUTE ODD NUMBERS RANGING FROM 1 TO 3,399. (Pick any missing odd number, excluding any number whose last digit is 1, other than 1 itself, falling before or in between any of the following numbers and substitute it for  $n_{so}$  in the above formula, and the result will be a first half pair prime). (This table must be read horizontally)

25, 39, 47, 53, 67, 69, 73, 95, 99, 107, 109, 113, 123, 125, 135, 137, 145, 157, 165, 175, 177, 179, 183, 193, 203, 207, 209, 223 229, 235, 237, 243, 245, 249, 255, 259, 263, 267, 277, 283, 289, 297, 299, 305, 307, 319, 329, 333, 335, 345, 347, 355, 357, 359, 373, 375, 377, 379, 385, 389, 399, 403, 413, 415, 417, 423, 437, 443, 445, 447, 449, 459, 463, 465, 467, 473, 485, 487, 489, 505, 509, 513, 515, 525, 529, 543, 547, 549, 553, 557, 559, 563, 567, 575, 579, 583, 585, 587, 589, 593, 597, 599, 605, 609, 613, 617, 619, 627, 639, 645, 647, 653, 655, 663, 669, 673, 677, 683, 685, 697, 705, 707, 715, 719, 723, 725, 727, 729, 733, 739, 743, 749, 753, 759, 763, 767, 773, 775, 787, 795, 809, 817, 823, 827, 829, 833, 835, 837, 839, 845, 853, 855, 857, 865, 867, 875, 879, 883, 889, 893, 905, 907, 915, 919, 923, 927, 935, 937, 943, 945, 949, 955, 957, 963, 973, 977, 983, 993, 995, 997, 1005, 1009, 1015, 1017, 1019, 1023, 1025, 1033, 1035, 1037, 1043, 1047, 1049, 1053, 1057, 1059, 1065, 1075, 1077, 1079, 1087, 1089, 1093, 1095, 1097, 1103, 1105, 1113, 1117, 1125, 1127, 1133, 1139, 1043, 1145, 1147, 1157, 1159, 1165, 1167, 1169,  $1173, \ 1183, \ 1187, \ 1189, \ 1195, \ 1199, \ 1203, \ 1209, \ 1213, \ 1215, \ 1217, \ 1227, \ 1229, \ 1235, \ 1237, \ 1243, \ 1245, \ 1247, \ 1249, \ 1257, \ 1263, \ 1247$ 1269, 1275, 1279, 1285, 1289, 1295, 1297, 1299, 1313, 1317, 1319, 1323, 1325, 1327, 1343, 1345, 1347, 1353, 1355, 1365, 1367, 1369, 1373, 1383, 1387, 1389, 1393, 1395, 1397, 1399, 1407, 1415, 1425, 1433, 1435, 1437, 1439, 1443, 1447, 1453, 1455, 1459, 1467, 1475, 1477, 1479, 1489, 1495, 1499, 1503, 1509, 1513, 1517, 1519, 1523, 1525, 1529, 1533, 1535, 1537, 1539, 1543, 1553, 1555, 1565, 1569, 1575 1579, ,1585, 1587, 1589, 1593, 1603, 1607, 1609, 1613, 1615, 1617, 1619, 1627, 1633, 1635, 1637, 1649, 1653 1659, 1663, 1665, 1675, 1677, 1685, 1687, 1689, 1691, 1697, 1703, 1705, 1707, 1709, 1713, 1719, 1725, 1727, 1733, 1735, 1737, 1739, 1747, 1749, 1755, 1763, 1773, 1775, 1779, 1785, 1787, 1789, 1801, 1803, 1807, 1809, 1815, 1817, 1819, 1829, 1837, 1845, 1847, 1849, 1853, 1855, 1859, 1865, 1867, 1869, 1873, 1875, 1877, 1887, 1893, 1895, 1899, 1907, 1909, 1915, 1917, 1919, 1923, 1925, 1929, 1939, 1943, 1945, 1957, 1963, 1969, 1973, 1977, 1983, 1985, 1987, 1989, 1997, 1999, 2005, 2007, 2013, 2019, 2023, 2025, 2027, 2035, 2039, 2045, 2049, 2053, 2055, 2059, 2063, 2069, 2075, 2077, 2079, 2083, 2093, 2097, 2113, 2115, 2123, 2125, 2127, 2133, 2135, 2137, 2139, 2143 2145, 2147, 2153, 2155, 2159, 2165, 2167, 2169, 2175, 2177, 2179, 2185, 2195, 2197, 2203, 2005, 2207, 2209, 2213, 2215, 2223, 2225, 2227, 2235, 2237, 2243, 2247, 2249, 2255, 2257, 2265, 2269, 2273, 2279, 2283, 2293, 2295, 2297, 2307, 2309, 2313, 2317, 2329, 2335, 2237, 2343, 2345, 2349, 2353, 2355, 2357, 2363, 2365, 2377, 2379, 2385, 2387, 2389, 2393, 2399, 2405, 2407, 2413, 2419, 2423, 2425, 2429, 2433, 2437, 2439, 2445, 2447, 2453, 2455, 2457, 2459, 2463, 2465, 2467, 2469, 2473, 2475, 2479, 2487, 2489, 2497, 2503, 2517, 2523, 2533, 2537, 2539, 2543, 2545, 2553, 2555, 2559, 2565, 2569, 2571, 2573, 2577, 2579, 2581, 2583, 2587, 2589, 2591, 2593, 2595, 2599, 2603, 2615, 2619 2623, 2629, 2637, 2641, 2643, 2647, 2653, 2655, 2657, 2659, 2663, 2665, 2667, 2673, 2675, 2677, 2683, 2685, 2687, 2699, 2709, 2713, 2717, 2719, 2725, 2727, 2729, 2733, 2735, 2737, 2749, 2753, 2755, 2759, 2767, 2769, 2773, 2775, 2777, 2779, 2783, 2785, 2793, 2797, 2799, 2803, 2805, 2813, 2817, 2819, 2823, 2825, 2827, 2829, 2835, 2839, 2843, 2849, 2853, 2855, 2859, 2863, 2867, 2873, 2877, 2879, 2883, 2885, 2895, 2903, 2905, 2907, 2909, 2919, 2923, 2925, 2929, 2933, 2937, 2947, 2957, 2959, 2963, 2965, 2967, 2969, 2973, 2975, 2979, 2985, 2993, 2995, 2997, 3005, 3007, 3015, 3017, 3023, 3025, 3027, 3029, 3033, 3035, 3037, 3039, 3043, 3047, 3049, 3055, 3059, 3063, 3065, 3077, 3083, 3087, 3089, 3095, 3099, 3105, 3109, 3115, 3117, 3119, 3127, 3129, 3133, 3135, 3147, 3149, 3167, 3169, 3175, 3185, 3187, 3189, 3193, 3197, 3199, 3203, 3205, 3213, 3215, 3217, 3219, 3223, 3227, 3233, 3235, 3237, 3229, 3333, 3337, 3339, 3343, 3347, 3349, 3357, 3365, 3369, 3373, 3339, 3333, 3337, 3339, 3343, 3347, 3349, 3357, 3365, 3369, 3373, 3375, 3389, 3395, 3399.

## B. Composite Number Discount Table of Any Range of Odd Numbers

Since it is unrealistic to generate an entire range of non-substitute odd numbers from their very beginning to any range of our choice, we can chose any desirable range of numbers within which we want to establish primes, determine the number of most appropriate sets of none substitutes falling within that range and establish the stage at which elements of each one of the identified most appropriate sets of non-substitutes enter that specific range, and use their progression intervals to determine the entire list of those elements up to the element ending that range. It means therefore that we can also produce a table to cover any desirable range of odd numbers within which we want to locate primes of either form.

The first thing is to determine the range of paired odd numbers within which we want to pick a solution set for an appropriate formula. If we chose a formula for finding first half pair primes (FHPPs), we must ensure that the odd numbers we pick as our range of operation are elements of the set of first half pair odd numbers. Both the minimum and maximum number we chose must be confirmed as elements of the set of first half pair odd numbers.

For example, we can select the following range of numbers which can be confirmed as elements of the set of first half pair odd numbers; 41555 to 43001. We can then work out a finite set of most appropriate visible and invisible divisors of first half pair odd numbers up to the last most critical divisor for our selected maximum number 43001. Note that as indicated already, the most appropriate

set of divisors of multiples of either set of paired odd numbers are primes other than 2 and 3 less than the square root of any selected upper limit, which in this case is 43001. These are primes ranging from first half pair prime 5 to second half pair prime 199. This combines set of primes is as follows; {5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193 197, 199}. Note that second half pair primes are in bold print to distinguish them from first half pair primes that are in normal print.

We can then produce, for each of these indivisible divisors', finite sets of special odd numbers related to them, and falling within our specified range of numbers from 41555 to 43001, none of which are substitutes for the variable in the formula. The key formula in the production of any such table is as follows;  $\{[(2n_o \div a) - D] \times 2a\} + b$ ; where  $n_o$  is any paired odd number representing the minimum range of odd numbers from which we want to establish non-substitutes, where a is any appropriate divisor to which the special odd numbers relate, where D represent units to the right of a decimal point in the quotient of  $n_o \div 2a$  and where b is an initial odd number which begins the endless sequence of special odd numbers related to divisor 'a'. The formula gives us a specific divisor's first special odd number in a chosen range of numbers upon which number we can generate, by using that divisor's division intervals, subsequent odd numbers in that series up to our chosen maximum range

We can establish the stage at which a given

divisors' related specific odd numbers will enter a specified range of numbers by proceeding as follows;

First we have to distinguish between visible and invisible divisors of the set of paired odd numbers we are working with. For first half pair odd numbers visible divisors are of the form 6N + 5 - 6 where N is any natural number, In the above combined set of primes, visible divisors are presented in plain print. On the other hand, invisible divisors are of the form 6N + 7 - 6 where N is any natural number and are presented in bold print in the combined set of primes above. For second half pair odd numbers, visible divisors are of the form 6N + 7 - 6 where N is any natural number , whereas invisible divisors are of the form 6N + 5 - 6 where N is any natural number .

We can then start our operation by determining an initial element in the divisor's set of non-substitutes related to it. If that divisor is a visible divisor of first half pair odd numbers the formula we shall use is as follows;  $b = (7a - 2) \div 3$ ; where a is that divisor. If it is an invisible divisor of first half pair odd numbers the formula we shall use is as follows;  $b = (5a - 2) \div 3$  where a is that divisor. For example, for first half pair odd numbers, divisor 5 being visible, the initial element of the set of numbers related to it is worked out as follows;

$$b = (7a - 2) \div 3; [(7 \times 5) - 2] \div 3; (35 - 2) \div 3; 33$$
  
  $\div 3$ ;  $b = 11$ .

On the other hand, divisor 7 being invisible, the initial element of the set of numbers related to it will be worked out as follows;

$$b = (5a - 2) \div 3$$
;  $[(5 \times 7) - 2] \div 3$ ;  $(35 - 2) \div 3$ ;  $33 \div 3$ ;  $b = 11$ .

The next step is to work out the progression intervals of the numbers related to those divisors by using the following formula; P = 2a where a, is that divisor, whether visible or invisible. For example with regard to the two divisors above, we can work out their respective progression intervals as follows;  $2 \times 5 = 10$  for odd numbers related to visible divisor 5, and  $2 \times 7 = 14$  for odd numbers related to invisible divisor 7.

The next step is to work out the point at which the special odd numbers (non-substitute elements) related to our selected divisors will enter our specified range of operation ( $\geq$  41555 $\leq$ 43001). The formula to use in order to establish these entry

points is as follows;  $\{[(n_{o1} \div 2a) - d] \times 2a\} + b$  where  $n_{o1}$  is a number indicating any selected minimum range (41555 in this case), where a is any selected divisor, where d represents units to the right of a decimal point in the quotient  $n_{o1} \div 2a$ , where b is an initial element of the set of numbers related to divisor a . For example, we can work out the points at which numbers related to the two divisors we have picked will enter the range of numbers beginning from 41555 respectively as follows; Entry point for numbers related to visible divisor 5 into range  $\geq 41555 \leq 43001$ ;

$$\begin{split} &\{[(n_{o1} \div 2a) - d] \times 2a\} + b; [41555 \div (2 \times 5) - d] \times \\ &2 \times 5 + 11; \; \{[(41555 \div 10) - d \;] \times 10\} + 11; \\ &[(4155.5 - .5) \times 10] + 11; \; (4155 \times 10) + 11 \;; \\ &41550 + 11 = 41561. \end{split}$$

The value of the expression is 41561. Note that this number is the entry point of non-substitutes related to visible divisor 5 of first half pair odd numbers in the range of numbers beginning with 41555.

To prove that 41561 is indeed visible divisor 5's first related special odd number in the range of numbers from 41555 onwards, we can subtract these numbers' progression interval  $(2 \times 5)$ , to get a difference that is just less than the minimum range; that is, 41561 - 10 = 41551. As further proof we can keep on subtracting this progression interval (10) form each and every difference we obtain until we reach 11 which is the initial number for this series of non-substitutes.

Similarly we can work out the entry point for numbers related to invisible divisor 7 into range  $\geq$  41555 $\leq$ 43001 as follows;

$$\begin{split} \{[(n_{o1} \div 2a) - d] \times 2a\} + b; & [41555 \div (2 \times 7) - d] \times \\ 2 \times 7 + 11; & \{[(41555 \div 14) - d] \times 14\} + 11; \\ [(2968.21 - .21) \times 14] + 11; & (2968 \times 14) + 11; \\ 41552 + 11 = 41563. \end{split}$$

In the same way, we can confirm that this number is indeed the first in the range from 41555 by subtracting the progression interval for this series of numbers  $(2 \times 7)$  from 41563 and obtain a difference that is less than the minimum range of 41555. We can also repeatedly subtract this progression interval form (14) from each and every difference obtained until we reach 11 which is also the initial number for this particular series of numbers.

The last thing is to produce complete sets of non-

substitutes related to the two identified divisors as follows; For visible divisor 5, the first element is 41561 which we have worked out to be its related numbers' entry point into the range beginning with 41555.

To get the rest of the elements up to our maximum of 43001, we add the progression interval for this particular series of numbers  $(2 \times 5)$  to the first element and then keep on adding this interval to each and every sum we obtain until we get the number equal to, or just less than 43001. For invisible divisor 7, the first element is 41563 which we established to be its related numbers' entry point into the range of numbers starting with

41555.To obtain the other remaining elements of this finite set, we simply add the progression interval for this particular series of numbers  $(2 \times 7)$  to the first element and thereafter to each and every sum up to end with a number just less than or equal to 43001.

If we generate all sets of special odd numbers related to visible and invisible divisors of first half pair odd numbers falling within our range of numbers from 41561 to 43001, arrange all elements in their ascending order and eliminate all non-substitute elements whose last digit is 1, The result will be table 13 (b) below.

TABLE 13 (b): FIRST HALF PAIR PRIMES FORMULA'S NON-SUBSTITUTE ODD NUMBERS RANGING FROM 41,561 TO 43,001 ( $P = 3n_{so} + 2$ ). (Pick any missing odd number , excluding any number whose last digit is 1, in between any of the following numbers and substitute it for variable  $n_{so}$  in the first half pair primes formula and the result will be a first half pair prime). (This table must be read horizontally).

41561 41563, 41565, 41569, 41575, 41577, 41583, 41585, 41587, 41595, 41603, 41605, 41609, 41613, 41619, 41623, 41625, 41627, 41629, 41633, 41637, 41647, 41649, 41653, 41655, 41657, 41673, 41675, 41677, 41679, 41683, 41685, 41689, 41693, 41695,  $41699,\ 41703,\ 41709,\ 41715,\ 41717,\ 41723,\ 41725,\ 41729,\ 41737,\ 41745,\ 41751,\ 41755,\ 41757,\ 41759,\ 41765,\ 41769,\ 41773,\ 41775,\ 4177$  $41777, 41783, \quad 41785, \quad 41787, \quad 41793, \quad 41797, \quad 41803, \quad 41805, \quad 41815, \quad 41825, \quad 41827, \quad 41829, \quad 41839, \quad 41843, \quad 41845, \quad 41847, \quad 418$  $41855, \quad 41857, 41859, \quad 41867, \quad 41869, \quad 41875, \quad 41877, \quad 41885, \quad 41887, \quad 41899, \quad 41907, \quad 41909, \quad 41913, \quad 41919, \quad 41923, \quad 41927, \quad 41933, \quad 41919, \quad 419$ 41935, 41939, 41943, 41945, 41947, 41953, 41955, 41957, 41959, 41963, 41967, 41969, 41979, 41983, 41985, 41989, 41993, 41995, 41997, 41999, 42005, 42009, 42019, 42023, 42025, 42027, 42029, 42033, 42037, 42039, 42045, 42049, 42053, 42055, 42059, 42063,  $42065,\ 42067,\ 42069,\ 42073,\ 42083,\ 42089,\ 42093,\ 42095,\ 42097,\ 42099,\ 42119,\ 42117,\ 42123,\ 42125,\ 42127,\ 42133,\ 42135,\ 42127,\ 42123,\ 42123,\ 4212$  $42137, 42139, \ 42143, 42145, \ 42149, 42155, 42159, \ 42165, \ 42167, 42169, 42173, 42175, 42177, \ 42179, \ 42187, \ 42189, 42193, \ 42195, \ 42189, \ 4$ 42197, 42199, 42203, 42205, 42207, 42209, 42215, 42219, 42223, 42225, 42229, 42233, 42235, 42243, 42245, 42249, 42259, 42263, 42265, 42269, 42273, 42276, 42277, 42285, 42287, 42297, 42299, 42303, 42305, 42309, 42313, 42315, 42317, 42319, 42323, 42325, 42327, 42333, 42335, 42337, 42339, 42347, 42349, 42353, 42357, 42363, 42365, 42367, 42369, 42373, 42375, 42383, 42389, 42393, 42395, 42397, 42403, 42407, 42409, ,42417, 42419, 42427, 42435, 42437, 42439, 42445, 42449, 42453, 42455,  $42459,\ 42463,\ 42465,\ 42469,\ 42473,\ 42475,\ 42477,\ 42479,\ 42487,\ 42499,\ 42503,\ 42505,\ 42507,\ 42515,\ 42519,\ 42523,\ 42519,\ 4251$ 42525, 42529, 42537, 42539, 42543, 42553, 42555, 42557, 42573, 42583, 42585, 42589, 42591, 42595, 42597, 42599, 42601, 42603, 42607, 42609, 42611, 42613, 42615, 42617, 42623, 42627, 42629, 42633, 42635, 42639, 42645, 42647, 42649, 42655, 42663, 42667, 42669, 41673, 42675, 42679, 42683, 42685, 42687, 42689, 42695, 42697, 42705, 42707, 42709, 42713, 42723, 42725, 42727, 42735, 42739, 42743, 42749, 42753, 42755, 42759, ,42765, 42767, 42769, 42777, 42785, 42787, 42789, 42793, 42795, 42807, 42809, 42813, 42815, 42817, 42819, 42823, 42833, 42835, 42837, 42843, 42845, 42847, 42853, 42855, 42857 42859, 42865, 42869, 42879, 42883, 42893, 42895, 42899, 42903, 42907, 42909, 42913, 42917, 42923, 42925, 42927, 42929, 42933, 42935, 42947, 42949, 42955, 42963, 42965, 42969, 42973, 42977, 42985, 42987, 42999, 43001

### C. The Second Half Pair Primes Formula Table

The second table which is meant for the identification of elements of the solution set for the second half pair primes formula is table 14 below. Note that this table too can be produced for any chosen range of odd numbers

TABLE 14, SECOND HALF PAIR PRIMES FORMULA'S NON-SUBSTITUTE ODD NUMBERS RANGING FROM 1 TO 3,399 ( $P_2 = 3n_{so} - 2$ ). (Pick any missing odd number, excluding 1 and 9 and any number whose last digit is 9, before or in between any of the following numbers and substitute it for variable  $n_{so}$  in the second half pair primes formula, and the result will be a second half pair prime). (This table must be read horizontally).

17, 31, 41, 45, 57, 63, 73, 83, 85, 87, 97, 101, 107, 115, 121, 131, 135, 143, 151, 157, 161, 165, 171, 173, 177, 185, 187, 195, 197, 213, 217, 223, 227, 233, 235, 241, 249, 255, 261, 265, 267, 273, 281, 283, 291, 297, 301, 305, 311, 315, 317, 321, 325, 327, 335, 343, 353, 361, 367, 371, 381, 383, 387, 393, 395, 397, 403, 407, 415, 419, 421, 423, 425, 437, 445, 447, 451, 453, 455, 457, 463, 465, 471, 473, 481, 493, 501, 503, 505, 507, 513, 521, 525, 531, 535, 545, 547, 551, 563, 561, 563, 571, 573, 577, 591,

603, 605, 607, 613, 615, 617, 631, 633, 635, 637, 641, 643, 647, 653, 655, 657, 661, 693, 675, 681, 683, 687, 691, 693, 701, 703,  $707, \ 717, \ 723, 725, 731, \ 733, 737, \ 743, 745, \ 753, 755, 767, \ 773, 775, \ 777, 785, \ 787, \ 801, 803, \ 805, 807, \ 811, \ 815, \ 817, \ 821, \ 827, \ 821, \$ 831, 833, 837, 843, 845, 855, 857, 861, 863, 867, 871, 875, 877, 881, 885, 901, 913, 915, 921, 927, 925, 937, 941, 943, 947, 1047, 1051, 1053, 1055, 1065, 1067, 1071, 1075, 1081, 1083, 1185, 1093, 1095, 1097, 1113, 1117, 1123, 1127, 1133, 1135, 1137, 1141, 1143, 1147, 1151, 1161, 1163, 1165, 1175, 1185, 1193, 1197, 1201, 1207, 1217, 1221, 1223, 1227, 1235, 1241, 1251,  $1253, 1255, 1261, 1263, 1267, 1271, 1273, 1277, \\ 1281, 1287, 1291, 1295, 1301, 1305, 1313, 1317, 1321, 1325, 1327, 1331, 1333, 1347, 1341, 13$ 1337, 1345, 1347, 1355, 1357, 1361, 1363, 1373, 1375, 1381, 1383, 1391, 1395, 1397, 1403, 1405, 1413, 1417, 1423, 1427, 1431,  $1435\ ,\ 1437,\ 1441,\ 1445,\ 1451,\ 1457,\ 1461,\ 1463,\ 1465,\ 1467,\ 1471,\ 1473,\ 1477,\ 1485,\ 1487,\ 1491,\ 1493,\ 1497,\ 1501,\ 1511,\ 1513,$ 1515, 1525, 1527, 1537, 1543, 1545, 1557, 1561, 1563, 1565, 1567, 1571, 1573, 1581, 1583, 1585, 1591, 1593, 1603, 1607, 1613, 1615, 1617, 1623, 1625, 1627, 1631, 1633, 1641, 1643, 1647, 1655, 1661, 1673, 1677, 1681, 1683, 1685, 1691, 1695, 1697, 1711, 1713, 1715, 1717, 1721, 1725, 1731, 1735, 1741, 1747, 1751, 1753, 1755, 1757, 1763, 1765, 1767, 1771, 1773, 1777, 1781, 1771, 1772, 1772, 1773, 1774, 17751785, 1787, 1791, 1793, 1795, 1797, 1801, 1821, 1823, 1825, 1831, 1833, 1837, 1845, 1847, 1851, 1863, 1865, 1967, 1871, 1873,  $1875,\ 1877,\ 1891,\ 1893,\ 1903,\ 1905,\ 1907,\ 1911,\ 1921,\ 1923,\ 1925,\ 1933,\ 1935,\ 1937,\ 1945,\ 1955,\ 1963,\ 1965,\ 1967,\ 1971,\ 1973,\ 1971,\$  $1977, \ 1981, 1983, 1987, 1991, 1993, \ 1995, 1997, \ 2001, 2005, \ 2007, \ 2011, 2017, 2021, 2033, 2035, \ 2037, \ 2043, \ 2047, \ 2053, 2057, \ 2067, \ 20$ 2061, 2063, 2065, 2075, 2081, 2085, 2087, 2095, 2097, 2103, 2107, 2105, 2111, 2117, 2131, 2135, 2137, 2145, 2147, 2153, 2155, 2163, 2165, 2167, 2171, 2173, 2175, 2181, 2187, 2195, 2197, 2201, 2205, 2211, 2215, 2217, 2223, 2233, 2241, 2243, 2247, 2251, 2253, 2257, 2361, 2263, 2267, 2271, 2273, 2283, 2285, 2287, 2293, 2297, 2301, 2305, 2307, 2311 2313, 2315, 2325, 2327, 2335, 2337, 2341, 2345, 2351, 2355, 2361, 2363, 2365, 2367, 2371, 2373, 2375, 2381, 2383, 2385, 2391, 2395, 2397, 2401, 2411, 2417, 2421, 2423, 2425, 2427, 2431, 2435, 2443, 2447, 2453, 2455, 2461, 2463, 2467, 2475, 2477, 2481, 2483, 2485, 2491, 2495, 2501, 2505, 2507, 2511, 2515, 2523, 2527, 2533, 2537, 2543, 2545, 2551, 2553, 2555, 2565, 2571, 2577, 2583, 2591, 2593, 2595, 2601, 2603, 2605, 2607, 2611, 2613, 2615, 2617, 2621, 2631, 2633, 2635, 2637, 2641, 2647, 2653, 2657, 2661, 2663, 2667, 2675, 26672677, 2681, 2683, 2691, 2693, 2695, 2703, 2705, 2707, 2711, 2713, 2715, 2717, 2725, 2733, 2735, 2743, 2747, 2751, 2753, 2761, 2767, 2775, 2781, 2783, 2787, 2791, 2795, 2801, 2803, 2805, 2813, 2817, 2825, 2827, 2831, 2833, 2835, 2837, 2845, 2851, 2853, 2857, 2863, 2865, 2871, 2873, 2885, 2887, 2891, 2895, 2901, 2915, 2917, 2923, 2925, 2931, 2933, 2937, 2943, 2945, 2951, 2953, 2957, 2967, 2961, 2971, 2973, 2983, 2985, 2987, 2993, 2995, 2997, 3007, 3011, 3013, 3021, 3025, 3027, 3033, 3041, 3047, 3055,  $3057,\ 3065,\ 3071,\ 3073,\ 3075,\ 3077,\ 3083,\ 3085,\ 3087,\ 3091,\ 3097, \\ 3101,\ 3103,\ 3105,\ 3111,\ 3121,\ 3123,\ 3125,\ 3127,\ 3137,\ 3143,\ 3125,\ 3127,\$ 3151, 3153, 3157, 3161, 3163, 3165, 3167, 3173, 3175, 3177, 3181, 3185, 3187, 3191, 3193, 3195, 3197, 3203, 3213, 3229, 3223, 3225, 3231, 3235, 3237, 3243, 3251, 3253, 3255, 3265, 3267, 3275, 3281, 3283, 3285, 3293, 3297, 3305, 3307, 3313, 3315, 3321, 3327, 3331, 3333, 3335, 3341, 3343, 3345, 3351, 3353, 3355, 3361, 3363, 3373, 3375, 3377, 3383, 3385, 3391, 3395, 3397, 3399.

### VI. CONCLUSION

This article has introduced a new standard formula for finding each and every prime number on the sequence of natural numbers without having to stumble upon composite numbers.

The formula is based on the recognition that primes only occur within a set of paired odd numbers, which numbers are located in between odd multiples of 3 on the sequence of natural numbers. The set of paired odd numbers comprises of two different sets of numbers. The first set is the set of first half pair odd numbers while the second set is the set of second half pair odd numbers. Each of these two sets has its own endless set of primes. The first set contains first half pair primes while the second set contains second half pair primes.

Owing to this differentiation between the two sets

of primes, the new standard formula is split into two complementary formulas. The first is for finding first half pair primes while the second is for finding second half pair primes. Each of the two formulas is satisfied by its own solution set of endless elements. These elements are indentified as missing numbers in tables of natural distribution of non-substitutes for variables in each of the two formulas. The significance of such tables is that they not only confirm the validity of the new standard formula, but also provide a basis upon which to arrive at various formulas of locating elements of the two solution sets.

The significance of this article is that it has laid a foundation for more informed research in the speedy and efficient location of prime numbers on the sequence of natural numbers.

#### **BIBLIOGRAPHY**

- [1] Abdelnoor R.E, Jason, A mathematical Dictionary Wheaton A. Weaton and Company Limited A division of programme press, Hennockon Road, Exeter Ex28RP (1979)
- [2] Borowski E.J. and J.M. Borwein, The Dictionary of Mathematics, Collins London and Glasgow (1989)
- [3] Caldwell Chris K. Caldwell@utm.edu
- [4] Dictionary of Mathematics, Second Edition MacCraw-Hill Staff, MacCraw-Hill Professional Publishing (01/2003)
- [5] Kabwe Chisanga Tiyaonse (March 15, 2012), Revelations on Prime Numbers A Pre-publication, Tiyaonse Kabwe, Lusaka, Zambia
- [6] Larson, Kanold, Stiff (1998), Algebra 1, An integrated Approach, Health and Company, A division of Houghton Mifflin Company, Canada.
- [7] Margaret L. Lial and John Hornsby, Begging Algebra, Eighth Edition, Addisson Wesley Longman Inc, England (2000).
- [8] Random-definition of random @your dictionary.com
- [9] Scottish Mathematics Group, Modern Mathematics for schools, Second Edition. Blackie and Son limited
- Bishopbriggs. Glasgow G642NZ London WC2H 7BP (1972)
- [10] Scottish Mathematics Group, 1 Modern Mathematics for Schools, Second Edition, Blackie and Son Limited, London, 1971.
- [11] (http://en.wikipedia.org/wiki/prime\_number)
- [12] http://www.mersenne.org/default.php
- [13] http://primes.utm.edu/largest.html
- [14] http://primes.utm.edu/notes/proofs/infinite/
- $[15]\ http://primes.utmn.edu/notes/proofs/infinite/euclids.html$
- [16]

http://primes.utm.edu/glossary/page.php?sort=FundamentalTheorem

[17] http://primes.utm.edu/glossary/xpage/Divisor.html