Observer design of time-varying Singular Systems (Transistor Circuits) Using Adomian Decomposition Method

S. Sekar[#1], M. Nalini[*2]

[#]Assistant Professor, Department of Mathematics, Government Arts College (Autonomous), Salem – 636 007, Tamil Nadu, India.

[*]Assistant Professor, Department of Mathematics, J.K.K. Nataraja College of Arts and Science, Komarapalayam, Namakkal – 638 183, Tamil Nadu, India.

***Abstract* — An interesting and real world problem is discussed in this paper in the kind of observer design of time-varying singular systems (Transistor Circuits). The results (approximate solutions) obtained using the Adomian Decomposition Method (ADM) and Single-term Haar wavelet series (STHW) [12] methods are compared with the exact solutions of the time-varying singular systems. It is found that the solution obtained using ADM is closer to the exact solutions of the time-varying singular systems. The high accuracy and the wide applicability of ADM approach will be demonstrated with numerical examples. Finally error graphs for approximate and exact solutions are presented in a graphical form to show the accuracy of the ADM. This ADM can be easily implemented in a digital computer and the solution can be obtained for any length of time.*

Keywords — *Singular systems of time-varying cases, Ordinary differential equations, Adomian Decomposition Method, Single-term Haar wavelet series.*

I. Introduction

Singular systems theory is being applied to solve a variety of problems involved in various disciplines of Science and Engineering. It is applied to analyse neurological events and catastrophic behaviour and they also provide a convenient form for the dynamical equations of large-scale interconnected systems. Further, singular systems are found in many areas such as constrained mechanical systems, fluid dynamics, chemical reaction kinetics, simulation of electrical networks, electrical circuit theory, power systems, aerospace engineering, robotics, aircraft dynamics, neural networks, neural delay systems, network analysis, time series analysis, system modeling, social systems, economic systems, biological systems etc.[2]

Reviewing the work done in time-varying singular systems, one cannot miss the excellent study carried out by Campbell; especially the numerical solutions of higher index linear time-varying singular systems of differential equations [3-7]. Wang [17] has treated linear time-varying singular systems and derived a necessary and sufficient condition for the existence of linear state feedback to eliminate impulsive behaviour. Chen and Shih [8] have discussed the optimum control of time-varying linear systems via Walsh functions. Hsiao and Wang [9] have studied the time-varying singular bilinear systems via Haar Wavelets.

In this paper we developed numerical methods for addressing time-varying singular systems by an application of the Adomian Decomposition Method which was studied by Sekar and team of his researchers [10-11, 13-16]. Recently, Sekar *et al*. [12] discussed the observer design of singular systems using STHW. In this paper, the same observer design of singular systems problem was considered (discussed by Sekar *et al*. [12]) but present a different approach using the Adomian Decomposition Method with more accuracy for observer design of singular systems.

II. Adomian Decomposition Method

Suppose k is a positive integer and $f_1, f_2, \ldots, f_k$ are k real continuous functions defined on some domain G. To obtain k differentiable functions $y_1, y_2, \ldots, y_k$ defined on the interval I such that $(t, y_1(t), y_2(t), \ldots, y_k(t)) \in G$ for $t \in I$.

Let us consider the problems in the following system of ordinary differential equations:

$$\frac{dy_i(t)}{dt} = f_i(t, y_1(t), y_2(t), \ldots, y_k(t)) \,, \quad y_i(t)|_{t=0} = \beta_i \qquad (1)$$

where β_i is a specified constant vector, $y_i(t)$ is the solution vector for $i = 1, 2, \ldots, k$. In the decomposition method, (1) is approximated by the operators in the form: $Ly_i(t) = f_i(t, y_1(t), y_2(t), \ldots, y_k(t))$ where L is the first order operator defined by $L = d/dt$ and $i = 1, 2, \ldots, k$.

Assuming the inverse operator of L is L^{-1} which is invertible and denoted by $L^{-1}(.) = \int_{t_0}^{t} (.)\, dt$, then applying L^{-1} to $L\, y_i(t)$ yields

$$L^{-1}Ly_i(t) = L^{-1} f_i(t, y_1(t), y_2(t), \ldots, y_k(t))$$

where $i = 1, 2, \ldots, k$. Thus

$$y_i(t) = y_i(t_0) + L^{-1} f_i(t, y_1(t), y_2(t), \ldots, y_k(t)).$$

Hence the decomposition method consists of representing $y_i(t)$ in the decomposition series form given by

$$y_i(t) = \sum_{n=0}^{\infty} f_{i,n}(t, y_1(t), y_2(t), \ldots, y_k(t))$$

where the components $y_{i,n}$, $n \geq 1$ and $i = 1, 2, \ldots, k$ can be computed readily in a recursive manner. Then the series solution is obtained as

$$y_i(t) = y_{i,0}(t) + \sum_{n=1}^{\infty} \{L^{-1} f_{i,n}(t, y_1(t), y_2(t), \ldots, y_k(t))\}$$

.

For a detailed explanation of decomposition method and a general formula of Adomian polynomials, we refer reader to [Adomian 1].

III. LINEAR TIME-VARYING TRANSISTOR CIRCUITS

Consider the linear time-varying singular system

$$K(t)\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t)$$

where *y(t)* is the *p*-output vector (observer), and *K*, *A*, *B*, *C* are matrices of appropriate dimensions. *x(t)* is an *n*-state vector, *u(t)* is an *r*-input control vector and *K(t)* is singular.

Taking into consideration the actual problem encountered in equation [12] a hypothetical problem is envisioned here in order to study the time-varying singular system in the transistor circuit.

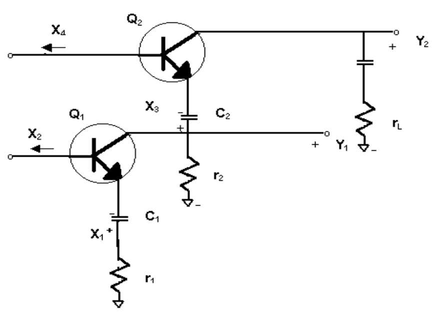

Fig.1 Transistor circuit

Assuming

$r_1 = r_2 = 1, r_L = \sin(t), \alpha_1 = 1 + t, \alpha_2 = 1 + e^{-t}, c_1 = c_2 = 1 + t.$

yields the singular system and the parameter values are selected in such a way that the system possesses an exact solution.

$$\left.\begin{aligned} &\begin{bmatrix} 1+t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1+t & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -(1+t) & 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u \\ &y(t) = \begin{bmatrix} 0 & 1+t & 0 & -1 \\ 0 & 0 & 0 & (1+e^{-t})\sin(t) \end{bmatrix} x(t) \end{aligned}\right\} \quad (2)$$

The need to remove the sparsity of the system matrices can be avoided by eliminating the algebraic variables to convert it into a state-space form. So the problem can be studied by observing the semi-state '*x*' using the output '*y*' in the singular system setting given in equation (2).

Let us take $u(t) = [1 + t + t^2/2 \quad 0]^T$, then the exact solutions of the observers of system (2) are

$$\left.\begin{aligned} y_1(t) &= -\left(1 + t + t^2 + \frac{t^3}{3}\right) + \frac{(1 + t + 3t^2/2 + t^3 + t^4/4)}{(1+t)} \\ y_2(t) &= -(1 + e^{-t})\sin(t)\left(\frac{(1 + t + 3t^2/2 + t^3 + t^4/4)}{(1+t)}\right) \end{aligned}\right\}$$

(3)

with $x(0) = [0 \; -1 \;\; 0 \; -1]^T$. The approximate solutions for the time-varying transistor circuit have been evaluated using the STHW methods developed by Sekar *et al.* [12] and the results are shown in Table 1-2 along with the solutions obtained using the STHW with step size $h = 0.5$ and are compared with the exact solutions of system (2) presented in equation (3). These ADM solutions are found to be more accurate when compared with the STHW methods.

IV. DISCRETE SOLUTIONS

Time t	$y_1(t)$	
	RK-Butcher Algorithm Error	ADM Error
0.00	1.00E-09	1.00E-11
0.50	1.00E-09	1.00E-11
1.00	1.00E-09	1.00E-11
1.50	1.00E-09	1.00E-11
2.00	1.00E-09	1.00E-11
2.50	1.00E-09	1.00E-11
3.00	1.00E-09	1.00E-11
3.50	1.00E-09	1.00E-11
4.00	1.00E-09	1.00E-11
4.50	1.00E-09	1.00E-11
5.00	1.00E-09	1.00E-11

Table 1 Errors in time-varying system

Time t	$y_2(t)$	
	RK-Butcher Algorithm Error	ADM Error
0.00	1.00E-09	1.00E-11
0.50	2.00E-09	2.00E-11
1.00	3.00E-09	3.00E-11
1.50	4.00E-09	4.00E-11
2.00	5.00E-09	5.00E-11
2.50	6.00E-09	6.00E-11
3.00	7.00E-09	7.00E-11
3.50	8.00E-09	8.00E-11
4.00	8.00E-09	8.00E-11
4.50	9.00E-09	9.00E-11
5.00	9.00E-09	9.00E-11

Table 2 Errors in time-varying system

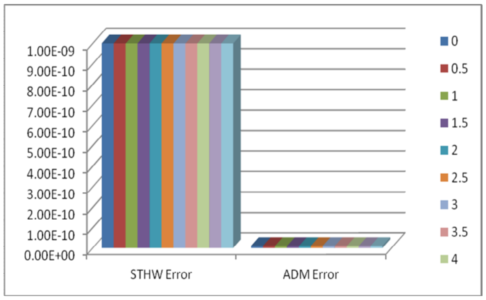

Fig.2 Error graph for " $y_1(t)$ " at various time intervals

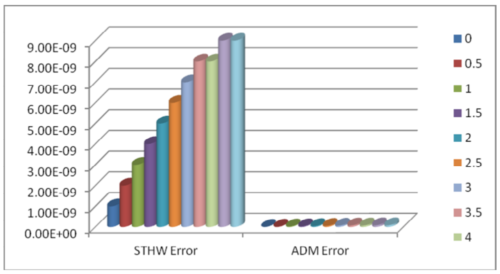

Fig.3 Error graph for " $y_2(t)$ " at various time intervals

Errors between the exact and the approximate solutions are also given in Table 1-2. To exhibit the efficiency of the discussed methods, an error graph is presented for the observer variable $y_1(t)$ and $y_2(t)$ in Fig. 2-3 at various time intervals up to the time t = 5.0 and from this it is observed that the ADM gives more accurate results when compared to the STHW method discussed by Sekar *et al.* [12].

V. Conclusions

The approximate solutions obtained using ADM give more accurate values when compared to STHW methods discussed by Sekar *et al.* [12]. From Table 1-2, it can be observed that the solutions obtained by the ADM match well with the exact solutions of the transistor circuit problem irrespective of time-varying cases, but the STHW method yield a little error.

From Fig. 2-3, it can be observed that the ADM yields less error when compared to the STHW method. Hence, one can get the solution for any length of time for this observer design of singular systems (transistor circuits) problem using ADM easily.

Acknowledgment

The authors gratefully acknowledge the Dr. A. Murugesan, Assistant Professor, Department of Mathematics, Government Arts College (Autonomous), Salem - 636 007, for encouragement and support. The authors also heartfelt thank to Dr. S. Mehar Banu, Assistant Professor, Department of Mathematics, Government Arts College for Women (Autonomous), Salem - 636 008, Tamil Nadu, India, for her kind help and suggestions.

References

[1] G. Adomian, "Solving Frontier Problems of Physics: Decomposition method", Kluwer, Boston, MA, 1994.

[2] J. C. Butcher, "*The Numerical Methods for Ordinary Differential Equations*", 2003, John Wiley & Sons, U.K.

[3] S. L. Campbell, *Singular systems of differential equations*, Pitman, London, (1980).

[4] S. L. Campbell, *Singular systems of differential equations II*, Pitman, London. (1982).

[5] S. L. Campbell, "Bilinear nonlinear descriptor control systems", *CRSC Tech. Rept. 102386-01*, Department of Mathematics, N. C. State University, Raleigh, (1987) NC 27695.

[6] S. L. Campbell and J. Rodriguez, "Non-linear singular systems and contraction mappings", *Proceedings of the American Control Conference*, (1984), 1513-1519.

[7] S. L. Campbell and L. R. Petzold, "Canonical forms and solvable singular systems of differential equations", *SIAM J. Algebraic and Discrete Methods*, 4, (1983), 517-521.

[8] W. L. Chen and Y. P. Shih, "Analysis and optimum control of time-varying linear systems via Walsh functions", *Int. J. Control*, 27, 917-932, 1978.

[9] C. H. Hsiao and W. J. Wang, "State analysis of time-varying singular bilinear systems via Haar Wavelets", *Math. and Computers in simulation*, 52, 11-20, 2000.

[10] S. Sekar and A. Kavitha, "Numerical Investigation of the Time Invariant Optimal Control of Singular Systems Using Adomian Decomposition Method", *Applied Mathematical Sciences*, vol. 8, no. 121, pp. 6011-6018, 2014.

[11] S. Sekar and A. Kavitha, "Analysis of the linear time-invariant Electronic Circuit using Adomian Decomposition Method", *Global Journal of Pure and Applied Mathematics*, vol. 11, no. 1, pp. 10-13, 2015.

[12] S. Sekar and K. Jaganathan, "Numerical investigation of time-varying transistor circuit using single-term Haar wavelet series", *International Journal of Computer, Mathematical Sciences and Applications* , Vol. 4, Nos. 1-2, 2010, pp. 99-104.

[13] S. Sekar and M. Nalini, "Numerical Analysis of Different Second Order Systems Using Adomian Decomposition Method", *Applied Mathematical Sciences*, vol. 8, no. 77, pp. 3825-3832, 2014.

[14] S. Sekar and M. Nalini, "Numerical Investigation of Higher Order Nonlinear Problem in the Calculus of Variations Using Adomian Decomposition Method", *IOSR Journal of Mathematics*, vol. 11, no. 1 Ver. II, (Jan-Feb. 2015), pp. 74-77.

[15] S. Sekar and M. Nalini, "A Study on linear time-invariant Transistor Circuit using Adomian Decomposition Method", *Global Journal of Pure and Applied Mathematics*, vol. 11, no. 1, pp. 1-3, 2015.

[16] S. Sekar and K. Jaganathan, "Numerical investigation of time-varying transistor circuit using single-term Haar wavelet series", *International Journal of Computer, Mathematical Sciences and Applications* , Vol. 4, Nos. 1-2, 2010, pp. 99-104.

[17] C. J. Wang, "State feedback impulse elimination of linear time-varying singular systems", Automatica, 32(1), 133-136, 1996.