Mathematical Modeling of Behavior of Automatic Teller Machine with Respect to Reliability Analysis

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***Abstract* — *This paper presents an overview of results related to the operational behaviour of Automatic Teller system with respect to reliability analysis. ATM system has three major subsystems namely: the ATM stations, the central computer and the computers in bank. These are designated as 'A', 'B' and 'C' respectively. The central computer is connected with all ATM station and bank computers. Initially, as the system works, human cashier enters account and transaction data. ATMs communicate with the appropriate banks. An ATM accepts a cash card, interacts with the user communities with the central system to carry out the transaction, dispenses cash, and prints receipts. The only permanent data are stored in the bank computers.*

Keywords — *Reliability theory, stochastic processes, Laplace transforms and Cost profit function*.

I. INTRODUCTION

Reliability technology is an important phenomenon of this existing Industrial area. This technology has widely used to increase the efficiency of machines of the system. To overcome day-to-day problems, now a day the system analysis and engineers are interested in the analysis of reliability models to implement them for practical utility. Many researchers of this field have studied the system subject to without switching configuration. Firstly, Kumar, A. and Agarwal, M. [5] had discussed the system with four components in parallel with the assumption that at least two must fail to get the system failed, Gopalan and Naidu [2, 3, 4] have considered with inspection rate. However, very few researchers [6, 7] have analysed the real models. The system discussed here deals with the stochastic analysis of an automated teller machine subjected to standby configuration having perfect switch.

The ATM system has three major subsystems namely: the ATM stations, the central computer and the computers in bank. These are designated as '*A*', '*B*' and '*C*' respectively. The central computer is connected with all ATM station and bank computers.

Initially, as the system works, human cashier enters account and transaction data. ATMs communicate with the appropriate banks. An ATM accepts a cash card, interacts with the user communities with the central system to carry out the transaction, dispenses cash, and prints receipts. The only permanent data are stored in the bank computers.

***SUBSYSTEM A* (ATM STATIONS):** This subsystem consists of N non-identical units connected in series and failure of any one of these subject to complete breakdown of the system.

***SUBSYSTEM B* (CENTRAL COMPUTER):** This subsystem consists of two identical units' b_1 and b_2. b_1 initially operates while b_2 is kept stand-by, followed on line through a perfect switching device.

***SUBSYSTEM C* (COMPUTERS IN BANK):** This subsystem composed of M non identical units. These units are connected such that failure of any on causes the system goes in degraded state and further failure of any units results the complete breakdown of system.

All the failures are distributed exponentially while repair rates are generally distributed. The inspection of stand-by is carried out random epochs. Using supplementary variable technique and Laplace transforms, several reliability parameters are obtained to highlight the importance of the model. Some particular cases, steady state behaviour and one numerical example are given at the end to connect the model with physical situations.

II. ASSUMPTIONS

(i). The system has three types of states: normal, degraded, and failed.

(ii). Initially, the system is in good state of full efficiency.

(iii). The repair facility is adopted first-come-first served.

(iv). The system consists of three subsystems namely, *A*(ATM stations), *B* (central computer) and *C* (computers in bank).

(v). All the failures follow exponential time distributed while all the repairs follow general time distribution.

(vi). Switching device is assumed to be perfect.

(vii). All transition rates are statistically independent.

(viii). Repairs are perfect in nature i.e., the repair facility never does any damage to the units.

(ix). The transition rates vary from component to component as all the components are non- identical.

(x). The system goes to complete breakdown if any unit of subsystem *A* fails, both units of subsystem *B* fail and more than one unit of subsystem *C* fail.

(xi). When a unit fails, repair for the failed unit and the installation of the standby unit for operation starts.

III. MATHEMATICAL SYMBOL

The following notations are used throughout in this paper

Symbol	Description
$D/D_t/D_x/D_y$	$\frac{d}{dt}/\frac{\partial}{\partial t}/\frac{\partial}{\partial x}/\frac{\partial}{\partial y}$
$\bar{F}(s)$	Laplace transform of $F(t)$
ϕ	Constant switching rate
$P_{0,0,0}(t)$	The probability that at time t, the system is in good state.
$P_{0,s,0}(t)$	The probability that at time t, the system is in operable state due to working of standby B-unit.
$P_{0,F,j}(z,t)\Delta$	The probability that at time t the system is in failed state due to the failure of subsystem B and j^{th}-C unit. The elapsed repair time lies in the interval $(z, z+\Delta)$
$P_{0,s,F}(z,t)\Delta$	The probability that time t, the system is in failed state due to the failure of subsystem C. The elapsed repair time lies in the interval
$P_{0,s,j}(t)$	The probability that at time t, the system degraded state due to the failure of j^{th}-C unit and standby B-unit is working.

ATM Network

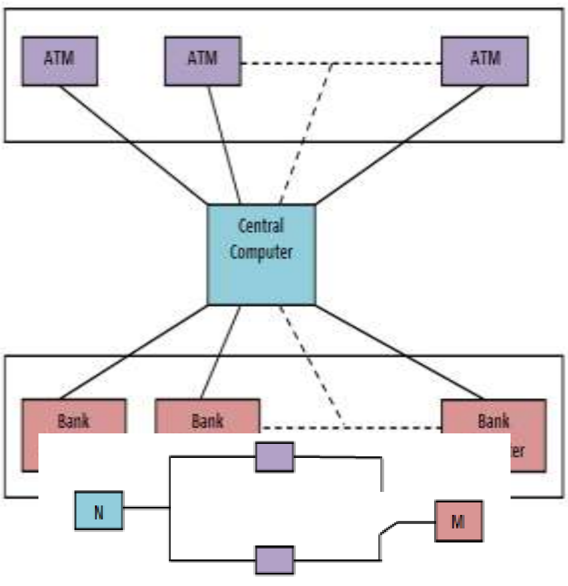


Figure 2.2

Transition state diagram

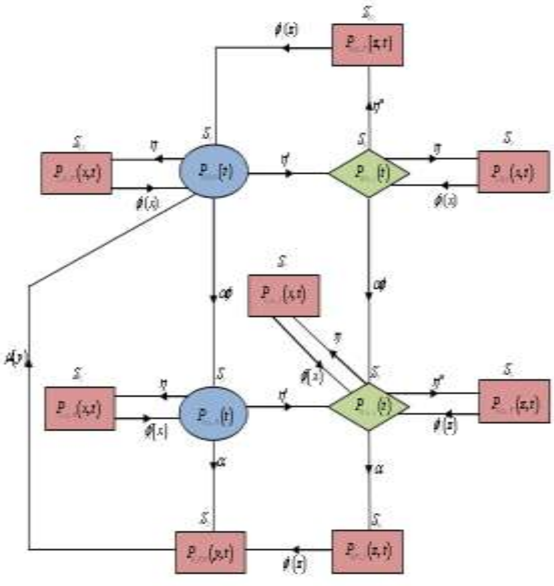
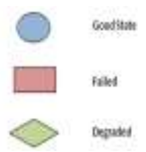


Figure 2.3

Where



IV. FORMULATION OF MATHEMATICAL MODEL

The analysis crucially depends on the method of supplementary variables technique and the supplementary variable x denotes the time that a unit has been elapsed undergoing repair. Viewing the nature of the problem, we obtain the following set of difference-differential equations:

$$\left(D_t+\eta+\eta'+\alpha\phi\right)P_{0,0,0}(t)=\sum_i\int P_{i,0,0}(x,t)\,\phi_i(x)\,dx+\sum_j\int P_{0,0,F}(z,t)\,\phi_j(z)$$
$$+\int P_{0,F,0}(y,t)\,\mu(y)\,dy \quad \text{... (1)}$$

$$\left(D_t+\eta+\eta'+\alpha\right)P_{0,s,0}(t)=\sum_i\int P_{i,s,0}(x,t)\,\phi_i(x)\,dx+\alpha\phi P_{0,0,0}(t) \quad \text{... (2)}$$

$$\left[D_y+D_t+\mu(y)\right]P_{0,F,0}(y,t)=0 \quad \text{... (3)}$$

$$\left[D_x+D_t+\phi_i(x)\right]P_{i,s,0}(x,t)=0 \quad \text{... (4)}$$

$$\left[D_z+D_t+\phi_j(z)\right]P_{0,F,j}(z,t)=0 \quad \text{... (5)}$$

$$\left(D_t+\eta+\eta''+\alpha\right)P_{0,s,j}(t)=\sum_i\int P_{i,s,j}(x,t)\,\phi_i(x)\,dx+$$
$$\sum_j\int P_{0,s,F}(z,t)\,\phi_j(z)\,dz+\alpha\phi P_{0,0,j}(t)+\eta' P_{0,s,0}(t) \quad \text{... (6)}$$

$$\left[D_z+D_t+\phi_j(z)\right]P_{0,s,F}(z,t)=0 \quad \text{... (7)}$$

$$\left[D_x+D_t+\phi_i(x)\right]P_{i,s,j}(x,t)=0 \quad \text{... (8)}$$

$$D_t + \eta + \eta'' + \alpha\phi \;\; P_{0,0,j} \;\; t \;\; = \eta' P_{0,0,0} \;\; t \;\; + \sum_j \int P_{i,0,j} \;\; x,t \;\; \phi_j \;\; x \;\; dx \quad ... (9)$$

$$\left[D_x + D_t + \phi_i \;\; x \;\; \right] P_{i,0,j} \;\; x,t \;\; = 0 \quad ... (10)$$

$$\left[D_z + D_t + \phi_j \;\; z \;\; \right] P_{i,0,0} \;\; z,t \;\; = 0 \quad ... (11)$$

$$\left[D_x + D_t + \phi_i \;\; x \;\; \right] P_{i,0,0} \;\; x,t \;\; = 0 \quad ... (12)$$

A. Boundary Conditions:

$$P_{0,F,0} \;\; 0,t \;\; = \alpha P_{0,s,0} \;\; t \;\; + \int P_{0,F,j} \;\; z,t \;\; \phi_j \;\; z \;\; dz \quad ... (13)$$

$$P_{i,s,0} \;\; 0,t \;\; = \eta P_{0,s,0} \;\; t \quad ... (14)$$

$$P_{0,F,j} \;\; 0,t \;\; = \alpha P_{0,s,j} \;\; t \quad ... (15)$$

$$P_{0,s,F} \;\; 0,t \;\; = \eta'' P_{0,s,j} \;\; t \quad ... (16)$$

$$P_{i,s,j} \;\; 0,t \;\; = \eta P_{0,s,j} \;\; t \quad ... (17)$$

$$P_{i,0,j} \;\; 0,t \;\; = \eta P_{0,0,j} \;\; t \quad ... (18)$$

$$P_{0,0,F} \;\; 0,t \;\; = \eta'' P_{0,0,j} \;\; t \quad ... (19)$$

$$P_{i,0,0} \;\; 0,t \;\; = \eta P_{0,0,0} \;\; t \quad ... (20)$$

B. Initial Conditions:

$$P_{0,0,0} \;\; 0 \;\; = 1 \text{ Otherwise zero} \quad ... (21)$$

V. Solution of Mathematical Model

Taking Laplace transforms of equations (1) through (12) and using Boundary as well as initial conditions one may obtain:

$$s + \eta + \eta' + \alpha\phi \;\; \bar{P}_{0,0,0} \;\; s \;\; = 1 + \sum_i \int \bar{P}_{i,0,0} \;\; x,s \;\; \phi_i \;\; x \;\; dx$$

$$+ \sum_j \int \bar{P}_{0,0,F} \;\; z,s \;\; \phi_j \;\; z \;\; + \int \bar{P}_{0,F,0} \;\; y,s \;\; \mu \;\; y \;\; dy \quad ... (22)$$

$$s + \eta + \eta' + \alpha \;\; \bar{P}_{0,s,0} \;\; s \;\; = \sum_i \int \bar{P}_{i,s,0} \;\; x,s \;\; \phi_i \;\; x \;\; dx + \alpha\phi \bar{P}_{0,0,0} \;\; s \quad ... (23)$$

$$\left[D_y + s + \mu \;\; y \;\; \right] \bar{P}_{0,F,0} \;\; y,s \;\; = 0 \quad ... (24)$$

$$\left[D_s + s + \phi_i \;\; x \;\; \right] \bar{P}_{i,s,0} \;\; x,s \;\; = 0 \quad ... (25)$$

$$\left[D_z + s + \phi_j \;\; z \;\; \right] \bar{P}_{0,F,j} \;\; z,s \;\; = 0 \quad ... (26)$$

$$s + \eta + \eta'' + \alpha \;\; \bar{P}_{0,s,j} \;\; s \;\; = \sum_i \int \bar{P}_{i,s,j} \;\; x,s \;\; \phi_i \;\; x \;\; dx + \sum_j \int \bar{P}_{0,s,F} \;\; z,s \;\; \phi_j \;\; z \;\; dz + \alpha\phi \bar{P}_{0,0,j} \;\; s \;\; + \eta' \bar{P}_{0,s,0} \;\; s \quad ... (27)$$

$$\left[D_z + s + \phi_j \;\; z \;\; \right] \bar{P}_{0,s,F} \;\; z,s \;\; = 0 \quad ... (28)$$

$$\left[D_x + s + \phi_i \;\; x \;\; \right] \bar{P}_{i,s,j} \;\; x,s \;\; = 0 \quad ... (29)$$

$$s + \eta + \eta'' + \alpha\phi \;\; \bar{P}_{0,0,j} \;\; s \;\; = \eta' P_{0,0,0} \;\; s \;\; + \sum_i \int \bar{P}_{i,0,j} \;\; x,s \;\; \phi_i \;\; x \;\; dx \quad ... (30)$$

$$\left[D_x + s + \phi_i \;\; x \;\; \right] \bar{P}_{i,0,j} \;\; x,s \;\; = 0 \quad ... (31)$$

$$\left[D_z + s + \phi_j \;\; z \;\; \right] \bar{P}_{0,0,F} \;\; z,s \;\; = 0 \quad ... (32)$$

$$D_x + s + \phi_i \;\; \bar{P}_{i,0,0} \;\; x,s \;\; = 0 \quad ... (33)$$

$$\bar{P}_{0,F,0} \;\; 0,s \;\; = \alpha \bar{P}_{0,s,0} \;\; s \;\; + \int \bar{P}_{0,F,j} \;\; z,s \; \phi_j \;\; z \;\; dz \quad ... (34)$$

$$P_{i,s,0} \;\; 0,t \;\; = \eta P_{0,s,0} \;\; s \quad ... (35)$$

$$\bar{P}_{0,F,j} \;\; 0,s \;\; = \alpha \bar{P}_{0,s,j} \;\; s \quad ... (36)$$

$$\bar{P}_{0,s,F} \;\; 0,s \;\; = \eta'' \bar{P}_{0,s,j} \;\; s \quad ... (37)$$

$$\bar{P}_{i,s,j} \;\; 0,s \;\; = \eta \bar{P}_{0,s,j} \;\; s \quad ... (38)$$

$$\bar{P}_{i,0,j} \;\; 0,s \;\; = \eta \bar{P}_{0,0,j} \;\; s \quad ... (39)$$

$$\bar{P}_{0,0,F} \;\; 0,s \;\; = \eta'' \bar{P}_{0,0,j} \;\; s \quad ... (40)$$

$$\bar{P}_{i,0,0} \;\; 0,s \;\; = \eta \bar{P}_{0,0,0} \;\; s \quad ... (41)$$

After solving the above equations, we get finally

$$\bar{P}_{0,0,0} \;\; s \;\; = \frac{1}{Z \;\; s} \quad ... (42)$$

$$\bar{P}_{0,0,j} \;\; s \;\; = \frac{A \;\; s}{Z \;\; s} \quad ... (43)$$

$$\bar{P}_{0,0,F} \;\; s \;\; = \eta'' \frac{A \;\; s}{Z \;\; s} D_{\phi_j} \;\; s \quad ... (44)$$

$$\bar{P}_{i,0,j} \;\; s \;\; = \eta \frac{A \;\; s}{Z \;\; s} D_{\phi_j} \;\; s \quad ... (45)$$

$$\bar{P}_{i,0,0} \;\; s \;\; = \frac{\eta}{Z \;\; s} D_{\phi_j} \;\; s \quad ... (46)$$

$$\bar{P}_{0,s,0} \;\; s \;\; = \frac{B \;\; s}{Z \;\; s} \quad ... (47)$$

$$\bar{P}_{0,s,j} \;\; s \;\; = \frac{C \;\; s}{Z \;\; s} \quad ... (48)$$

$$\bar{P}_{0,s,F} \;\; s \;\; = \eta'' \frac{C \;\; s}{Z \;\; s} D_{\phi_j} \;\; s \quad ... (49)$$

$$\bar{P}_{j,s,0} \;\; s \;\; = \eta \frac{B \;\; s}{Z \;\; s} D_{\phi_j} \;\; s \quad ... (50)$$

$$\bar{P}_{0,F,0} \;\; s \;\; = \frac{\alpha}{Z \;\; s} \left[B \;\; s \;\; + C \;\; s \;\; \sum_j \bar{S}_{\phi_j} \;\; s \;\; \right] D_{\mu} \;\; s \quad ... (51)$$

$$\bar{P}_{0,F,j} \;\; s \;\; = \alpha \frac{C \;\; s}{Z \;\; s} D_{\phi_j} \;\; s \quad ... (52)$$

$$\bar{P}_{i,s,j} \;\; s \;\; = \eta \frac{C \;\; s}{Z \;\; s} D_{\phi_j} \;\; s \quad ... (53)$$

Where

$$D_k \;\; s \;\; = \frac{1 - \bar{S}_k \;\; S}{s}, \quad \forall k \quad ... (54)$$

It is interesting to note that sum of relation (42) through (53) $= \frac{1}{s}$

VI. ERGODIC BEHAVIOUR OF SYSTEM

Using Abel's Lemma $\lim_{s\to 0} s\bar{F}(s) = \lim_{t\to\infty} F(t) = F$ (say), provided the limit on the R.H.S. exists, the time independent probabilities are obtained as follows by making use above lemma in the relations (42) through (53)

$$P_{0,0,0} = \frac{1}{Z'(0)} \quad \dots (55)$$

$$P_{0,0,j} = \frac{A(0)}{Z'(0)} \quad \dots (56)$$

$$P_{0,0,F} = \eta'' \frac{A(0)}{Z'(0)} M_{\phi_j} \quad \dots (57)$$

$$P_{i,0,j} = \eta \frac{A(0)}{Z'(0)} M_{\phi_j} \quad \dots (58)$$

$$P_{i,0,0} = \frac{\eta}{Z'(0)} M_{\phi_j} \quad \dots (59)$$

$$P_{0,s,0} = \frac{B(0)}{Z'(0)} \quad \dots (60)$$

$$P_{0,s,j} = \frac{C(0)}{Z'(0)} \quad \dots (61)$$

$$P_{0,s,F} = \eta'' \frac{C(0)}{Z'(0)} M_{\phi_j} \quad \dots (62)$$

$$P_{i,s,0} = \eta \frac{B(0)}{Z'(0)} M_{\phi_i} \quad \dots (63)$$

$$P_{0,F,0} = \frac{\alpha}{Z'(0)} \left[B(0) + C(0)\right] M_{\mu} \quad \dots (64)$$

$$P_{0,F,j} = \alpha \frac{C(0)}{Z'(0)} M_{\phi_j} \quad \dots (65)$$

$$P_{i,s,j} = \eta \frac{C(0)}{Z'(0)} M_{\phi_i} \quad \dots (66)$$

Where, $Z'(0) = \left[D_s Z(s)\right]_{s=0}$, $A(0) = \left[A(s)\right]_{s=0}$, $B(0) = \left[B(s)\right]_{s=0}$, $C(0) = \left[C(s)\right]_{s=0}$ and M_k = Mean time to repair k^{th} unit

VII. EVALUATION OF UP AND DOWN STATE PROBABILITIES

We have,

$$\bar{P}_{up}(s) = \bar{P}_{0,0,0}(s) + \bar{P}_{0,0,j}(s) + \bar{P}_{0,s,0}(s) \quad \dots (67)$$

On both sides taking inverse Laplace transform, we get

$$P_{up}(t) = A\exp.\left[-(\eta+\eta'+\alpha\phi)t\right] + B\exp.\left[-(\eta+\eta''+\alpha\phi)t\right] + C\exp.\left[-(\eta+\eta'+\alpha)t\right] \quad \dots (68)$$

Where $A = 2 + \left(\frac{\eta'}{\eta''-\eta'}\right)\left(\frac{\phi}{1-\phi}\right)$, $B = \frac{\eta'}{\eta'-\eta''}$ and $C = \frac{\phi}{\phi-1}$

and $P_{down}(t) = 1 - P_{up}(t)$ ... (69)

VIII. RELIABILITY OF THE SYSTEM

The Reliability of the system is

$$R(s) = \frac{1}{s+\eta+\eta'+\alpha\phi}$$

On both sides taking inverse Laplace transform, we get

$$R(t) = \exp.\left[-(\eta+\eta'+\alpha\phi)t\right] \quad \dots (70)$$

IX. M.T.T.F OF ATM SYSTEM

The MTTF of ATM system is defined as

$$M.T.T.F. = \lim_{s\to 0} R(s)$$

$$\Rightarrow \quad M.T.T.F. = \frac{1}{\eta+\eta'+\alpha\phi} \quad \dots (71)$$

X. NUMERICAL COMPUTATION

Substituting $\eta = \eta' = 0.01$, $\eta'' = 0.02$, $\alpha = 0.03$, $\phi = 0.04$ and $c_1 = 2$,
$c_2 = 1$and all repair rates are zero, then from equations (68), (70) and (71), we get

A. *Availability of system*

$$P_{up}(t) = 2.042\exp(-0.0212t) - \exp(-0.0312t) - 0.042\exp(-0.05t)$$

B. *Reliability o fATM system*

$$R(t) = \exp(-0.0212t)$$

C. *MTTF of ATM system*

$$M.T.T.F. = \frac{1}{\eta+0.01+0.0012}$$

XI. TABLE I AND FIGURE 2.4
COMPUTATION OF AVAILABILITY OF SYSTEM WITH RESPECT TO TIME

S.No.	t	Pup(t)
1	0	1
2	1	0.989931919
3	2	0.979718863
4	3	0.969375842
5	4	0.958917131
6	5	0.948356308
7	6	0.937706274

8	7	0.926979288
9	8	0.916186991
10	9	0.905340433
11	10	0.894450096
12	11	0.883525924
13	12	0.872577337
14	13	0.86161326
15	14	0.850642141
16	15	0.839671975

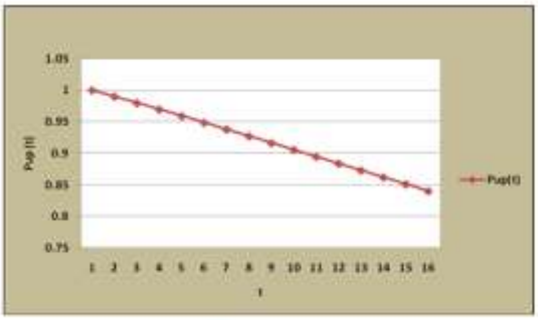


Figure 2.4

TABLE III AND FIGURE 2.5
COMPUTATION OF RELIABILITY FUNCTION WITH RESPECT TO TIME

S.No.	*t*	*R(t)*
1	0	1
2	1	0.97902314
3	2	0.958486309
4	3	0.938380277
5	4	0.918696005
6	5	0.899424648
7	6	0.880557543
8	7	0.862086211
9	8	0.84400235
10	9	0.826297831
11	10	0.808964698
12	11	0.791995159
13	12	0.775381587
14	13	0.759116517
15	14	0.743192636
16	15	0.727602788

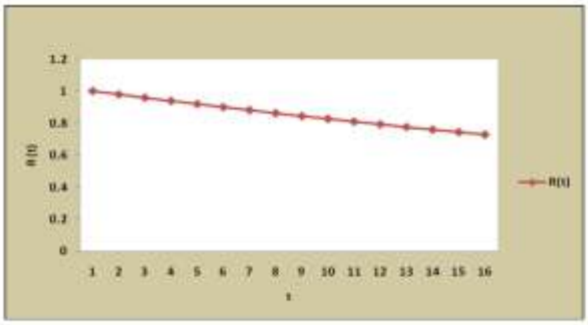


Figure 2.5

TABLE IIIII AND FIGURE 2.6
COMPUTATION OF MTTF WITH η OF SYSTEM WITH RESPECT TO TIME

S.No.	*η*	*MTTF*
1	0.01	47.16981132
2	0.02	32.05128205
3	0.03	24.27184466
4	0.04	19.53125
5	0.05	16.33986928
6	0.06	14.04494382
7	0.07	12.31527094
8	0.08	10.96491228
9	0.09	9.881422925
10	0.10	8.992805755
11	0.11	8.250825083
12	0.12	7.62195122
13	0.13	7.082152975
14	0.14	6.613756614
15	0.15	6.203473945
16	0.16	5.841121495

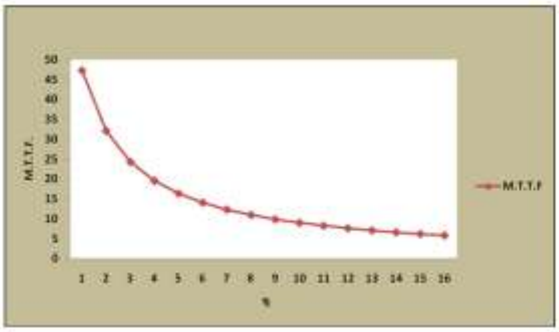


Figure 2.6

XII. CONCLUSIONS

The *Table-I* and *Figure 2.4* provide information how availability of the complex engineering repairable system change with respect to time when failure rate increases, then availability of system is decreases.

The *Table-II* & *Figure 2.5* provide information how reliability of the complex engineering repairable system change with respect to time when failure rate increases, then reliability of system is decreases.

The *Table-III* and *Figure 2.6* provide the information of mean time to failure with the variation of η. It can be seen that as failure increases initially MTTF decreases vigorously, after particular of η it decreases slightly.

Hence the present study clearly proves the importance of operational behaviour system of ATM system in comparison of [8] which seem to be possible in many engineering systems when it is analysed with the help of the network.

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