Soret-Dufour and Radiation Effects on Unsteady MHD Flow of Dusty Fluid over Inclined Porous Plate Embedded in Porous Medium

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Abstract— *An unsteady two dimensional MHD (Magneto Hydro Dynamics) flow of an incompressible viscous and electrically conducting dusty fluid past a continuously moving inclined plate with Soret-Dufour and Radiation effects is analyzed. The governing equations are solved numerically using Crank-Nicolson finite difference method for different values of flow parameters. The velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number are explained through graphs and tables.*

Keywords— *MHD, Thermophoresis effect, Dufour effect, porous medium, Hall effect, radiation effect, Heat and Mass transfer, Crank-Nicolson method.*

I. Introduction

During the last many decades, large number of mathematician has been attracted toward study of unsteady MHD flow of dusty fluid with effects of Thermophoresis, Dufour, radiation because of a large number of avenues for research work. The investigation of behavior of such flow have application in chemical engineering, MHD generators, in designing of underground water energy storage system, soil sciences, nuclear power reactors, aeronautics and so on. MHD convection flow past a semi-infinite vertical plate with thermal radiation was discussed by Venkataramana [1]. Sparrow and Cess [2] have discussed the effect of magnetic field on free convection heat transfer. Effect of magnetic field on heat and mass transfer by natural convection from vertical surface in porous medium with Soret and Dufour effects was discussed by Postelnicu [3].

Rajesh et al. [4] studied effects of radiation and mass transfer on MHD flow with exponentially accelerated vertical plate. Interaction of mixed convection with thermal radiation in laminar flow had been analyzed by Bestman et al. [5]. Olanrewaju [8] discussed Soret-Dufour and radiation effects on transient free convection flow past a moving plate. N. Pandya and A. K. Shukla [9] analyzed Soret-Dufour and Radiation Effects on Unsteady MHD Flow past an Impulsively Started Inclined Porous Plate with Variable Temperature and Mass Diffusion. N. Pandya and A. K .Shukla [10] Effects of Thermophoresis, Dufour, Hall and Radiation on an Unsteady MHD flow past an Inclined Plate with Viscous Dissipation. N. Pandya and A. K. Shukla [11] has been discussed Soret-Dufour and Radiation effect on unsteady MHD flow over an inclined porous plate embedded in porous medium with viscous dissipation. Sudhakarraiah et al. [12] examined Soret effect on unsteady MHD flow past a vertical plate with heat source. Dubey et. al. [6] have studied on Effect of the dusty viscous fluid on unsteady free convective flow along a porous hot vertical plate with thermal diffusion and mass transfer solved by perturbation techniques. Recently, Dubey et. al. [7] have analyzed Effect of Dusty Viscous Fluid on Unsteady Laminar Free Convective Flow through Porous Medium along a Moving Porous Hot Vertical Plate with Thermal Diffusion.

The aim of this research work is to analyze combined effects of Soret or Thermophoresis, Dufour, and radiation on unsteady MHD flow of dusty fluid past an inclined porous plate embedded in porous medium with variable temperature and mass diffusion. Partial differential equations of dimensionless governing equations of flow have been solved by Crank-Nicolson implicit finite difference method. Effects of different physical parameters of flow field on velocity, temperature, concentration, coefficients of skin-friction, Nusselt number and Sherwood number have been discussed through graphs and tables.

II. Mathematical Analysis

An unsteady MHD flow of a viscous incompressible dusty fluid past an infinite inclined plate with variable temperature and mass diffusion has been analyzed. The plate is inclined at angle λ to vertical and embedded in porous medium. x'-axis has been considered along plate, y'-axis normal to it. A uniform magnetic field $B0$ is taken along y'-axis and plate is electrically non-conducting. Magnetic Reynolds number and transversely applied magnetic file are very small so induced magnetic field is negligible in comparison to applied magnetic field, Cowling [14].

Due to infinite length in x' direction, flow variables are function of t' and y' only. Consider usual Boussinesq approximation, governing equations of flow field are:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0 \qquad (1)$$

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=\nu\frac{\partial^2 u'}{\partial y'^2}+g\beta(T'-T'_\infty)\cos(\lambda)$$
$$+g\beta'(C'-C'_\infty)\cos(\lambda)+\frac{KN_0(V-u')}{\rho} \quad (2)$$
$$-\frac{\sigma B_0^2 u'}{\rho}-\frac{\nu u'}{K'}$$

$$m_1\frac{\partial V}{\partial t'}=K(u'-V) \quad (3)$$

$$\rho C_p\left(\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}\right)=k\frac{\partial^2 T'}{\partial y'^2}-\frac{\partial q_r}{\partial y'}+\frac{\rho D_m K_T}{c_s}\frac{\partial^2 C'}{\partial y'^2} \quad (4)$$

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}+\frac{D_m K_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} \quad (5)$$

where γ^* is coefficient of volume expansion for mass transfer, γ is volumetric coefficient of thermal expansion, v' is velocity along y'-axis, K' is permeability of porous medium, σ is electrical conductivity, D_m is molecular diffusivity, g is acceleration due to gravity, K_T is thermal diffusion ratio, V is the velocity of dust particles, K is the stokes resistance coefficient, N_0 is the number density of the dust particles which is taken to be constant, m_1 is the mass of dust particles , ρ is fluid density, k is thermal conductivity of fluid, C' and T' are dimensional concentration and temperature, C'_∞ and T'_∞ are concentration and temperature of free stream, q_r is radiative heat along y'-axis, ν is kinematic viscosity and T_m is mean fluid temperature.

Boundary and initial conditional for this model are given as:

$$\begin{aligned}
&t'\le 0 \quad u'=0 \quad T'=T'_\infty \quad C'=C'_\infty \quad \forall y' \\
&t'\ge 0 \quad u'=u_0 \quad v'=-v_0 \quad T'=T'_\infty+(T'_\infty-T'_\infty)e^{-At'} \\
&C'=C'_\infty+(C'_w-C'_\infty)e^{-At'} \quad at\ y'=0 \\
&u'=0 \quad T'\to T'_\infty \quad C'\to C'_\infty \quad y'\to\infty
\end{aligned} \quad (6)$$

Where T'_w and C'_w are concentration and temperature respectively of plate and $A=\frac{v_0^2}{\nu}$.

The radiative heat flux term by using the Roseland approximation is given by

$$q_r=-\frac{4\sigma}{3k_m}\frac{\partial T'^4}{\partial y'} \quad (7)$$

where σ and k_m are Stefan Boltzmann constant and mean absorption coefficient respectively. It is assumed that the temperature difference within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about T'_∞ and neglecting the higher order terms, thus

$$T'^4 \cong 4T'^3_\infty T'-3T'^4_\infty \quad (8)$$

so, by equations 7 and 8, equation 4 is reduced

$$\rho C_p\left(\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}\right)=k\frac{\partial^2 T'}{\partial y'^2}+\frac{16\sigma T'^3_\infty}{3k_m}\frac{\partial^2 T'}{\partial y'^2}+\frac{\rho D_m K_T}{c_s}\frac{\partial^2 C'}{\partial y'^2} \quad (9)$$

In order to obtain dimensionless partial differential equations, we introduce following quantities:

$$\begin{aligned}
&u=\frac{u'}{u_0}, t=\frac{t'v_0^2}{\nu}, y=\frac{y'v_0}{\nu}, \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty}, \\
&C=\frac{C'-C'_\infty}{C'_w-C'_\infty}, Gm=\frac{\nu g\beta'(C'_w-C'_\infty)}{u_0v_0^2} \\
&, Gr=\frac{\nu g\beta(T'_w-T'_\infty)}{u_0v_0^2}, K=\frac{v_0^2K'}{\nu^2}, \Pr=\frac{\mu c_p}{k}, \\
&M=\frac{\sigma B_0^2\nu}{\rho v_0^2}, Du=\frac{D_mK_T(C'_w-C'_\infty)}{c_sc_p\nu(T'_w-T'_\infty)}, Sc=\frac{\nu}{D_m}, \\
&Sr=\frac{D_mK_T(T'_w-T'_\infty)}{T_m\nu(C'_w-C'_\infty)}, R=\frac{4\sigma T'^3_\infty}{k_mk}, v=\frac{V}{u_0}, \\
&B_1=\frac{\nu KN_0}{\rho u_0^2}, B=\frac{m_1u_0^2}{VK}
\end{aligned} \quad (10)$$

Using quantities of equation 10, we get non-dimensional form of equations 2, 3, 9 and 5 respectively:

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+Gr\cos(\lambda)\theta+Gm\cos(\lambda)C$$
$$+B_1(v-u)-(M+\frac{1}{K})u \quad (11)$$

$$B\frac{\partial V}{\partial t}=u-V \quad (12)$$

$$\frac{\partial\theta}{\partial t}-\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\left(1+\frac{4R}{3}\right)\frac{\partial^2\theta}{\partial y^2}+Du\frac{\partial^2 C}{\partial y^2} \quad (13)$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}+Sr\frac{\partial^2\theta}{\partial y^2} \quad (14)$$

dimensionless boundary and initial conditions are:

$$t \le 0 \quad u=0 \quad \theta=0 \quad C=0 \quad \forall y$$
$$t \ge 0 \quad u=1 \quad \theta=e^{-t} \quad C=e^{-t} \quad at \ y=0$$
$$u=0 \quad \theta \to 0 \quad C \to 0 \quad y \to \infty \qquad (15)$$

Further, there is primary interest for research workers to calculate physical quantities skin-friction coefficients *t* along wall *x*-axis, Nusselt number *Nu* and Sherwood number *Sh*. Non-dimensional form of these physical quantities are:

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \qquad (16)$$
$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

III. Method of Solution

Partial differential equations 11 to 14 are solved with initial and boundary conditions 15. To find exact solution of these partial differential equations are impossible. So, these are solved numerically using Crank-Nicolson implicit finite difference method. First, equations 11, 12,13 and 14 are expressed as:

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t} - \frac{u_{i+1,j}-u_{i,j}}{\Delta y} = \frac{1}{2}\left(\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}+u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}}{2(\Delta y)^2}\right)$$
$$+Gr\cos(\lambda)\left(\frac{\theta_{i,j+1}+\theta_{i,j}}{2}\right)+Gm\cos(\lambda)\left(\frac{C_{i,j+1}+C_{i,j}}{2}\right) \qquad (17)$$
$$+B_1\left(\left(\frac{v_{i,j+1}+v_{i,j}}{2}\right)-\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)\right)$$
$$-\left(M+\frac{1}{K}\right)\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)$$

$$B\left(\frac{v_{i,j+1}-v_{i,j}}{\Delta t}\right)=\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)-\left(\frac{v_{i,j+1}+v_{i,j}}{2}\right) \qquad (18)$$

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t} - \frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta y} = \frac{1}{2\Pr}\left(1+\frac{4R}{3}\right)\left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}+\theta_{i-1,j+1}-2\theta_{i,j+1}+\theta_{i+1,j+1}}{2(\Delta y)^2}\right) \qquad (19)$$
$$+\frac{Du}{2}\left(\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}+C_{i-1,j+1}-2C_{i,j+1}+C_{i+1,j+1}}{2(\Delta y)^2}\right)$$

$$\frac{C_{i,j+1}-C_{i,j}}{\Delta t} - \frac{C_{i+1,j}-C_{i,j}}{\Delta y} = \frac{1}{2Sc}\left(\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}+C_{i-1,j+1}-2C_{i,j+1}+C_{i+1,j+1}}{2(\Delta y)^2}\right) \qquad (20)$$
$$+\frac{Sr}{2}\left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}+\theta_{i-1,j+1}-2\theta_{i,j+1}+\theta_{i+1,j+1}}{2(\Delta y)^2}\right)$$

initial and boundary conditions are also expressed as:

$$u_{i,0}=0,\ \theta_{i,0}=0,\ C_{i,0}=0 \quad \forall i$$
$$u_{0,j}=1,\ \theta_{0,j}=e^{-j\Delta t},\ C_{0,j}=e^{-j\Delta t}$$
$$u_{L,j}=0,\ \theta_{L,j}\to 0,\ C_{L,j}\to 0 \qquad (21)$$

where index i refers to y, j refers to time t, $\Delta t = t_{j+1}-t_j$ and $\Delta y = y_{i+1}-y_i$. Known values of u,θ and C at t, we solved above equations for values $t+\Delta t$ as follows: We obtain these values to substitute $i=1,2,....,L-1$, where L pertains to ∞ then equations 16 to 19 give tridiagonal system of equations with initial and boundary conditions in equation 20 are solved using Thomos algorithm as discussed in Carnahan et al. [15], we have found values of θ and C for all values of y at $t+\Delta t$. Equation 16 and 17 are solved by same to substitute these values of θ and C, we get solution for u till desired time t. calculation were execute for $\Delta y = 0.1, \Delta t = 0.001$ and repeated till $y=4$.

IV. Result and Discussion

In order to analysis, we would like to see numerical results for velocity profile u, temperature profile θ and concentration profile C by giving numerical values of thermal Grashof number Gr, Radiation parameter R, Thermophoresis or Soret number Sr, Dufour number Du, solutal Grashof number Gm, Schmidt number, Sc magnetic parameter M, dusty fluid parameter B_1, dusty particle parameter B, permeability of porous medium K, prandtl number $\Pr$, and inclination angle λ with help of graphs. As well as coefficients of skin-friction *t* along wall x-axis, Nusselt number Nu and Sherwood number Sh are discussed through tables.

Figures 1 and 3 depict that on increasing Sr, velocity profile u and concentration profile C increase rapidly when Sr increases. velocity profiles u in figure 4 decreases and concentration profile in figure 6 increases when Schmidt number Sc increases. Figure 8 show that temperature profile θ first decreases after then increases when Dufour number Du increases while concentration profile C first increases, at some distance from plate it decreases and at last it again increases in figure 9.

Velocity profile u decreases as inclination angle λ increases in figure 10. Figures 2, 11 and 12 explain that velocity profile u, temperature profile θ and concentration profile C increase as time t increases. on increasing dusty particle parameter B, in figure 5 velocity profile u decreases, in figure 7 velocity profile u decreases rapidly when dusty fluids parameter B_1 increases.

From table 1, it is clear that on increasing Dusty fluids parameter, Dusty particle parameter, inclination angle and Schmidt number, skin-friction coefficient τ decreases. Further skin-friction coefficient τ increases as Dufour number, Soret number and time increase. In table 2, we have seen that Nusselt number Nu decreases as Schmidt number increases and Sherwood number Sh increases as Dufour number, Soret number and time increase. On other hand, Nusselt number increases as Dufour number, Soret number and time increase and Sherwood number decreases as Schmidt number increases.

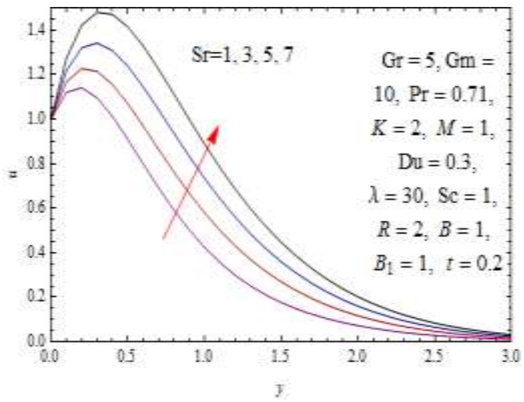

Fig. 1 Velocity Profile for Different Values of Sr

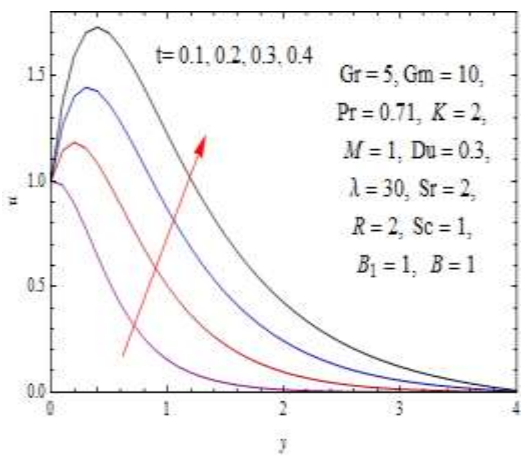

Fig. 2 Velocity Profile for Different Values of t

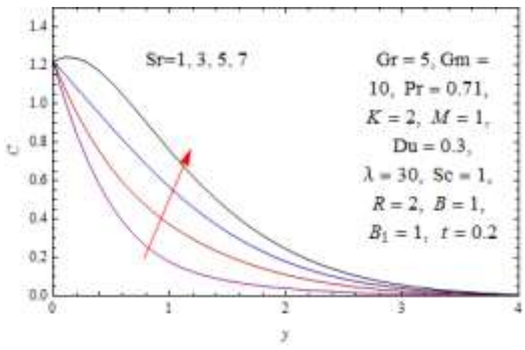

Fig. 3 Concentration Profile for Different Values of Sr

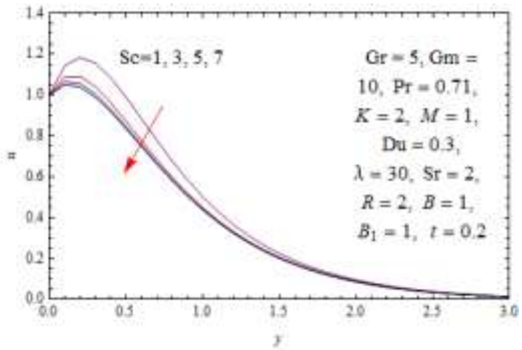

Fig. 4 Velocity Profile for Different Values of Sc

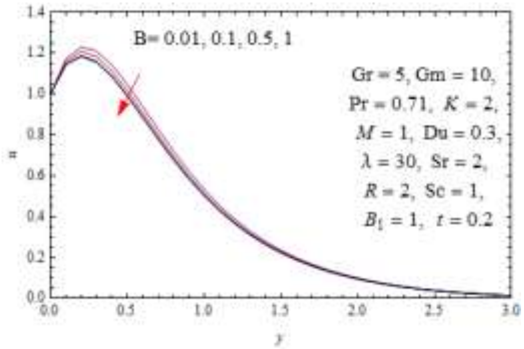

Fig. 5 Velocity Profile for Different Values of B

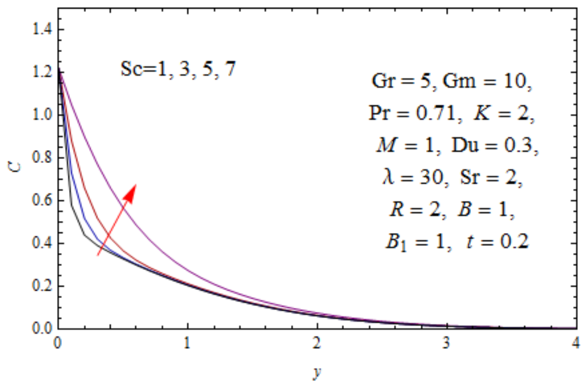

Fig. 6 Concentration Profile for Different Values of Sc

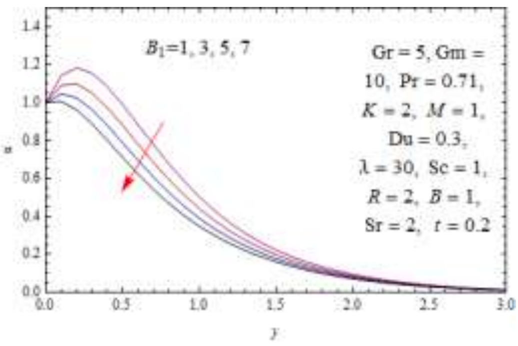

Fig. 7 Velocity Profile for Different Values of B_1

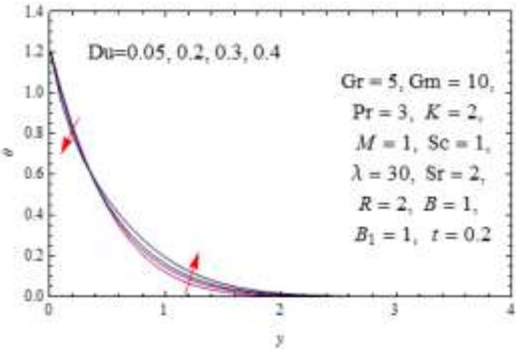

Fig. 8 Temperature Profile for Different Values of Du

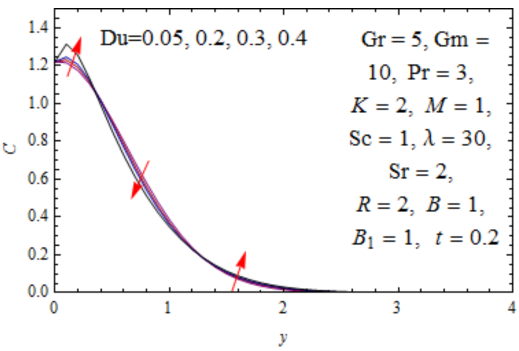

Fig. 9 Concentration Profile for Different Values of Du

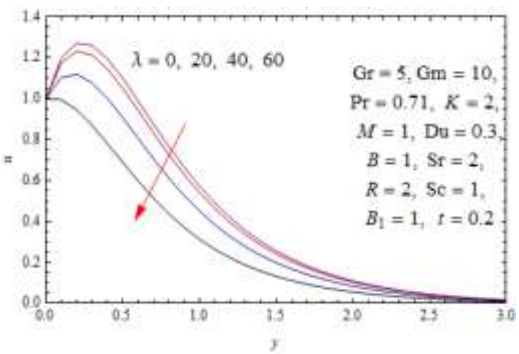

Fig. 10 Velocity Profile for Different Values of λ

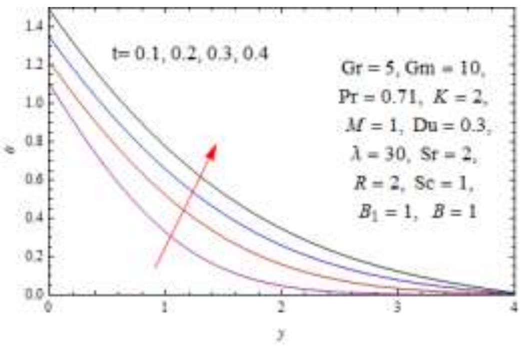

Fig. 3 Temperature Profile for Different Values of t

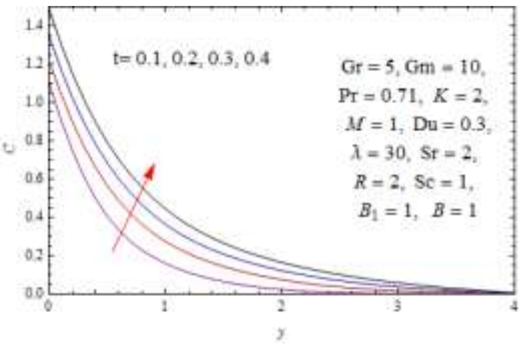

Fig. 12 Concentration Profile for Different Values of t

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Table-1: Skin friction coefficient τ for different values of parameter

B	B1	Du	λ	Sc	Sr	t	t
0	1	0.3	30	1	2	0.2	1.71249
0.1	1	0.3	30	1	2	0.2	1.59062
0.5	1	0.3	30	1	2	0.2	1.46506
1	1	0.3	30	1	2	0.2	1.43461
1	3	0.3	30	1	2	0.2	0.918609
1	5	0.3	30	1	2	0.2	0.466621
1	1	0.1	30	1	2	0.2	1.69639
1	1	0.2	30	1	2	0.2	1.70803
1	1	0.4	30	1	2	0.2	1.75473
1	1	0.3	0	1	2	0.2	1.98485
1	1	0.3	20	1	2	0.2	1.73897
1	1	0.3	40	1	2	0.2	1.03097
1	1	0.3	30	3	2	0.2	0.90124
1	1	0.3	30	5	2	0.2	0.663712
1	1	0.3	30	7	2	0.2	0.512079
1	1	0.3	30	1	1	0.2	1.21065
1	1	0.3	30	1	3	0.2	1.67561
1	1	0.3	30	1	5	0.2	2.18054
1	1	0.3	0	1	2	0.1	-0.188516
1	1	0.3	0	1	2	0.3	2.65398
1	1	0.3	0	1	2	0.4	3.74994

Table-2: Nusselt number Nu and Sherwood number Sh for different values of parameter

B	B1	Du	λ	Sc	Sr	t	Nu	Sh
1	1	0.05	30	1	2	0.2	2.05055	0.0512616
1	1	0.2	30	1	2	0.2	2.12441	-0.093489
1	1	0.4	30	1	2	0.2	2.57864	-0.935715
1	1	0.3	30	3	2	0.2	0.74608	3.40116
1	1	0.3	30	5	2	0.2	0.660977	4.91395
1	1	0.3	30	7	2	0.2	0.575411	6.42578
1	1	0.3	30	1	1	0.2	0.824218	2.00704
1	1	0.3	30	1	3	0.2	0.854969	1.39934
1	1	0.3	30	1	5	0.2	0.892275	0.672936
1	1	0.3	30	1	2	0.1	0.96496	1.90341
1	1	0.3	30	1	2	0.3	0.827407	1.72851
1	1	0.3	30	1	2	0.4	0.855602	1.81265