A Brief Study of LD-VAM to Find Initial Feasible Solution of Transportation Problem

Raghbir Dyal

Department of Mathematics, Govt. College, Sri Muktsar Sahib, INDIA

Abstract

This paper gives note on LD-VAM (logical development of Vogel approximation method), which is given by Das et al, in 2014. This paper shows that LD-VAM method does not always gives better initial solution than VAM. A counter example is given to prove the claim.

Keywords: Transportation Problem, Initial Basic Feasible Solution, Optimal Solution, LD-VAM.

1. Introduction

A certain class of linear programming problem know as transportation problems arises very frequently in practical applications. The classical transportation problem received its name because it arises naturally in the contacts of determining optimum shipping pattern. For example: A product may be transported from factories to retail stores. The factories are the sources and the store are the destinations. The amount of products that is available is known and the demands are also known. The problem is that different legs of the network joining the sources to the destination have different costs associated with them. The aim is to find the minimum cost routing of products from the supply point to the destination. The general transportation problem can be formulated as: A product is available at each of m origin and it is required that the given quantities of the product be shipped to each of n destinations. The minimum cost of shipping a unit of the product from any origin to any destination is known. The shipping schedule which minimizes the total cost of shipment is to be determined. The problem can be formulated as:

$$\text{Min } Z=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}$$

subject to

$$\sum_{j=1}^{n}x_{ij}=a_i, i=1,2,...m$$

$$\sum_{i=1}^{m}x_{ij}=b_j, j=1,2,...n$$

$$x_{ij}\geq 0, \text{ for all } i,j$$

For each supply point $i, (i = 1, 2, ...m)$ and demand point $j, (j = 1, 2, ...n)$
c_{ij}=unit transportation cost from i^{th} source to j^{th} destination
x_{ij}=amount of homogeneous product transported from i^{th} source to j^{th} destination
a_i=amount of supply at i^{th} source.
b_j=amount of demand at j^{th} destination.
where a_i and b_j are given non-negative numbers and assumed that total supply is equal to total demand, i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, then transportation problem is called balanced otherwise it is called unbalanced. The aim is to minimize the objective function satisfying the above mentioned constraints. In the classical transportation problem of linear programming, the traditional objective is one of minimizing the total cost.

Because of the special structure of the transportation model, the problem can also be represented as Table 1.

Table 1: Tabular representation of model (α)

Destination $\rightarrow$ source $\downarrow$	D_1	D_2	$\dots$	D_n	supply(a_i)
S_1	c_{11}	c_{12}	$\dots$	c_{1n}	a_1
S_2	c_{21}	c_{22}	$\dots$	c_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
S_m	c_{m1}	c_{m2}	$\dots$	c_{mn}	a_m
Demand (b_j)	b_1	b_2	$\dots$	b_n	

2. Numerical Examples

Numerical example: Mr. kamal has four factories manufacturing machines and Mr. keshav requires these machines at six different destinations.The transportation cost, supply and demand are shown in the Table.
Input data and initial basic feasible solution obtained by applying VAM and LD-VAM method is given in table 2

Table 2: Input data and optimal solution

Ex.	Input Data	Obtained Allocations by LD- VAM	Obtained Cost
1	$[c_{ij}]_{4\times6}$=[1 2 1 4 5 2; 3 3 2 1 4 3; 4 2 5 9 6 2; 3 1 7 3 4 6;]; $[a_i]_{4\times1}$=[30 50 75 20]; $[b_j]_{1\times6}$=[20 40 30 10 50 25]	x_{11} = 20, x_{13} = 10, x_{23} = 20, x_{24} = 10, x_{25} = 20, x_{32} = 20, x_{35} = 30, x_{36} = 25, x_{42} = 20,	470

Table 3: Input data and optimal solution

Ex.	Input Data	Obtained Allocations by VAM	Obtained Cost
1	$[c_{ij}]_{4\times6}$=[1 2 1 4 5 2; 3 3 2 1 4 3; 4 2 5 9 6 2; 3 1 7 3 4 6;]; $[a_i]_{4\times1}$=[30 50 75 20]; $[b_j]_{1\times6}$=[20 40 30 10 50 25]	$x_{11} = 20$, $x_{13} = 10$, $x_{23} = 20$, $x_{24} = 10$, $x_{25} = 20$, $x_{32} = 20$, $x_{35} = 30$, $x_{36} = 25$, $x_{42} = 20$,	470

Thus we have concluded that initial basic feasible solution using LD-VAM is not always better than VAM.

References

[1] Arora, S., Puri, M.C.(1997) On lexicographic optimal solutions in transportation problem. *Optimization* 39, 383-403.

[2] Arora, S., Puri, M.C.(2001) On a standard time transportation problem. *ASOR Bulletin* 20, 2-14.

[3] Bansal, S., Puri, M.C.(1980) A min max problem. ZOR 24, 191-200.

[4] Bhatia, H.L., Swarup, K. Puri, M.C. (1977) A procedure for time minimizing transportation problem. *Indian Journal of Pure and Applied Mathematics* 8, 920-929.

[5] Dantzig, G. B. (1951). Linear Programming and Extensions. *Princeton,NJ:Princeton University Press.*

[6] Das et al (2014) Logical development of Vogel approximation method (LD-VAM); An approach to find basic feasible solution of transportation problems. *An International Journal of Scientific and Technology Reasearch* 3(2), 42-48

[7] Gupta, A. Khanna S. Puri, M. C. (1993). A Paradox in Linear Transportation Problems. *Optimization.* 27, 375-387

[8] Hitchcock FL (1941) The Distribution of a product from several sources to numerous localities. *J. Math. Phy.* 20: 224- 230.