Mathematics for Human Nervous System

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Abstract: *Modern digital computers outperform humans in tasks based on precise and fast arithmetic operations. However, people are much better and faster than computers in solving complex perceptual problems, such as recognizing images, often in the presence of disturbances. Also, humans can perform complex movements with precision and grace, even in the presence of disturbances, and can generalize from past experience. In a computer, usually there exists a single processor implementing a sequence of arithmetic and logical operations, nowadays at speeds approaching billion operations per second. However this type of devices has ability neither to adapt their structure nor to learn in the way that human being does. An Artificial Neural Model (ANM) is an information processing paradigm that is inspired by the way biological nervous systems, such as the brain, process information. The key element of this paradigm is the novel mathematical structure of the information processing system. In this paper we focus on the mathematical modeling aspects of the basic unit of the human nervous system and Artificial Neural Model.*

Keywords: *Human Nervous System (CNS), Artificial Neural Model (ANM), Basic Mathematics.*

I THE HUMAN NERVIOUS SYSTEM

The human nervous system is divided into two major divisions: The peripheral nervous system (PNS) and the central nervous system (CNS). The PNS consists of a. sensory neurons running from stimulus receptors that inform the CNS of the stimuli. b. Motor neurons running from the CNS to the muscles and glands - called effectors - that take action. The peripheral nervous system is subdivided into the a. sensory-somatic nervous system and the autonomic nervous system (ANS). The peripheral nervous system carry information to and from the central nervous system. The central nervous system is composed of the brain and spinal cord.

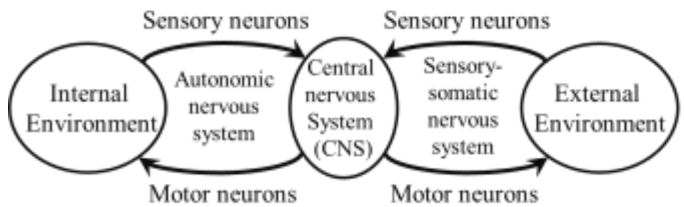


Figure 1 The Human Nervous System

The central nervous system is so named because of its anatomical location along the central axis of the body and because it is central in function. If we use a computer analogy to understand that it is central in function, the CNS would be the central processing unit and the other two parts of the nervous system would supply inputs and transmit outputs. Figure 3.2 shows the central nervous system.

a. Major Subdivisions of the Central Nervous System. The major subdivisions of the central nervous system are the brain and spinal cord.

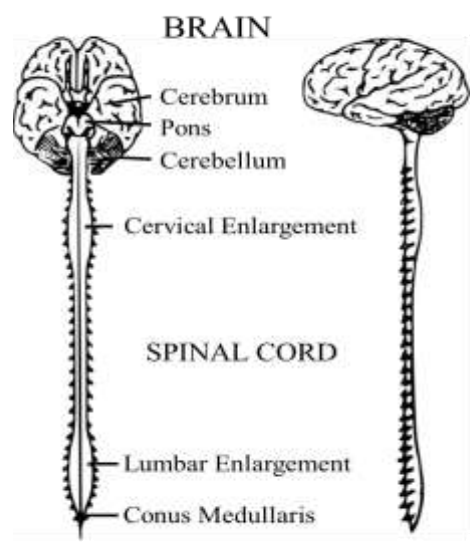


Figure 2 The central nervous system (CNS).

b. Coverings of the Central Nervous System. Bone and fibrous tissues cover the parts of the central nervous system. These coverings help to protect the delicate tissue of the CNS.

c. Cerebrospinal Fluid. The cerebrospinal fluid (CSF) is a liquid that is thought to serve as a cushion and circulatory vehicle within the central nervous system.

II THE HUMAN BRAIN

The human brain is one of the most complex objects in the universe. Many attempts have been made to investigate and model the functionalities of the brain. We still do not know how exactly brain learns. However, the human brain has three major subdivisions: brainstem, cerebellum, and the cerebrum. The central nervous system is first formed as a simple tube like structure in the embryo. The concentration of nervous tissues at one end of the human embryo to produce the brain and head is referred to as cephalization. When the embryo is about four weeks old, it is possible to identify the early forms of the brainstem, cerebellum, and the cerebrum, as well as the spinal cord. As development continues, the brain is located within the cranium in the cranial cavity. Figure 3.3 for illustrations of the adult brain.

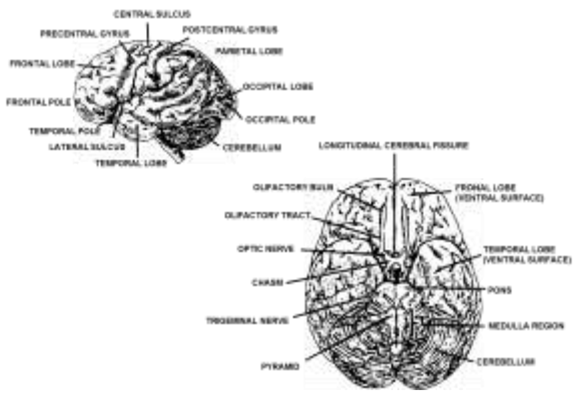


Figure 3 Human brain: A side view, B bottom view.

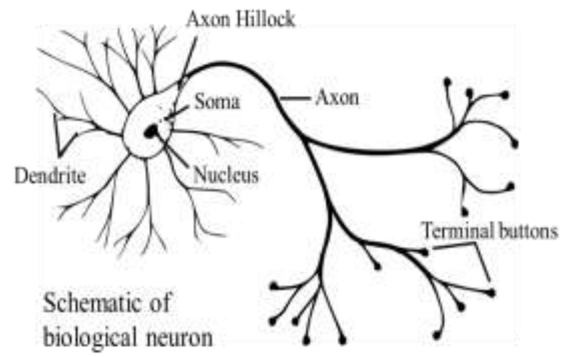
III Neuron and Neuron "Connections"

b. Dendrite. The dendrite is a neuron process which is a tree-like structure that receives the signals from other neurons and carries the signals toward the cell body. Each neuron may have one or more dendrites. Dendrites receive information and transmit it to the cell body.

c. Axon. The axon is a single long fiber that carries the signal from the cell body out to other neurons. Each neuron has only one axon. An axon, having a length varying from a fraction of a millimeter to a meter in human body, prolongs from the cell body at the point called *axon hillock*. At the other end, the axon is separated into several branches, at the very end of which the axon enlarges and forms terminal *buttons*.

d. Synapses. The terminal buttons of an axon are placed in special structures called the synapses which are the junctions transmitting signals from one

A neuron is a nerve cell body and all of its branches.



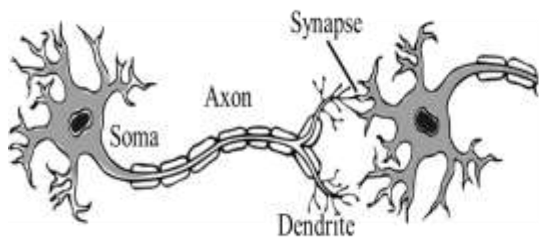


Figure 4 A neuron

Figure 5 Soma, Dendrite, Axon and Synapse of a Neuron.

There are four types of neuron branches: The cell body, dendrites, axons and synapsis.

a. The Cell Body. A neuron has a roughly spherical cell body called soma. The cell body is the heart of the cell, containing the nucleus and maintaining protein synthesis. The cell body is fully responsible for growth and maintenance of the neuron.

neuron to another . A neuron typically drive 10^3 to 10^4 synaptic junctions. Although it is not very common, synapses may also take place between two axons or two dendrites of different cells or between an axon and a cell body.

The human central nervous system(CNS) is comprised of about $1.3x10^{10}$ neurons and that about $1x10^{10}$ of them takes place in the brain i.e. the brain has 10^{10}= 100 billion neurons.The thickness of a bank note is approx. 0.1 mm, i.e., the stack of 100 billion bank notes has the length of 100 km. Each neuron has 10^4 connections to other neurons in the brain or spinal cord. i.e, the network is sparsely connected. At any time, some of these neurons are firing and the power dissipation due this electrical activity is estimated to be in the order of 10 watts. Monitoring the activity in the brain has shown that, even when asleep, $5x10^7$ nerve impulses per second are being relayed back and

forth between the brain and other parts of the body. This rate is increased significantly when awake. Electrical signals travel along these connections, and each neuron processes its inputs and generates a set of output signals which are then sent to neurons that it is connected to.
A neuron may "connect" either with another neuron or with a muscle fiber. A phrase used to describe such "connections" is "continuity without contact." Neurons do not actually touch. There is just enough space to prevent the electrical transmission from crossing from the first neuron to the next. This space is called the synaptic cleft. Information is transferred across the synaptic cleft by chemicals called neurotransmitters. Neurotransmitters are manufactured and stored on only one side of the cleft. Because of this, information flows in only one direction across the cleft.
We conclude the biological neuron model with the following six points:
1. A neuron has a **single output** conducted through its axon and **multiple inputs** conducted through dendrites.

Though the nerve cell frequently possesses more than one dendrite,
the axon is single… The dendrite is the receptive process of the neuron;
the axon is the discharging process,….

2. The single neuronal output, produced by a pulse known as the action potential, can be **fanned out** to connect to many other neurons. Signal flow is **unilateral** at junctions.

An axon gives rise to many expanded terminal branches (presynaptic terminal boutons)…
A single neuron may be involved in many thousands of synaptic connections,
but in every case the impulse transmission can occur only in one direction….

3. The dendrites (and a neuron may have as many as 150,000 of them) are the major source of inputs although it is noted that inputs can also be made through the soma (body) and axon and the neuron as a whole can be responsive to fields in its environment. Hence, the model has **multiple inputs**, and there are many of them.

4. The **neuronal response depends upon a summation of inputs**.
The two subliminal volleys are sent in over the same nerve, each volley
produces an effect which is manifested by an EPSP
[excitatory postsynaptic potential]. TheEPSPs will the sum, and if the
critical level of depolarization is reached, an impulse will be set off.

5. The **neuronal response is all or none** in that the characteristics of the pulse (referred to as the **action potential**) do not depend on detailed characteristics of the input.

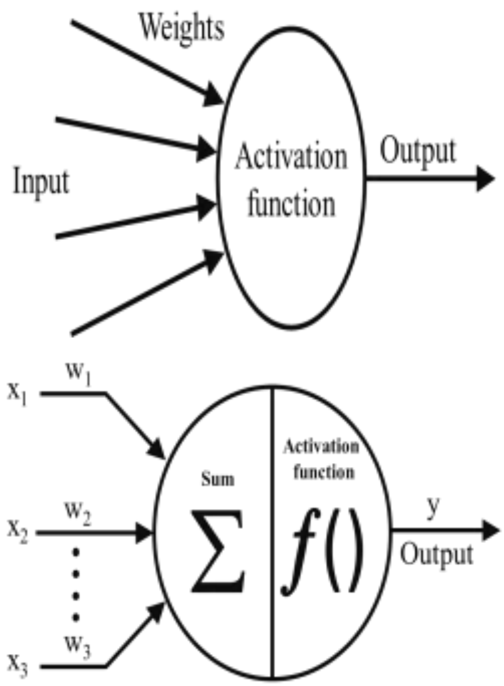
Thus, the propagated disturbance established in a single nerve fiber
cannot be varied by grading the intensity or duration of the stimulus,
i.e., the nerve fiber under a given set of conditions gives a maximal
response or no response at all.

Hence, our model should exhibit theresholding and a limited range set of possible outputs. The single pulse case is that there are only two possible outputs with “1” representing the action potential and “0” or “-1” representing the quiescent state.

6. The mechanisms for changing the response of a neural network include the use of chemicals like calcium and neurotransmitters to change the characteristics of a neuron (open and close selected ion channels) as well as changing the synaptic strengths. Although only the second possibility is exploited in artificial neural networks but the first mechanism is the most important.

4. Artificial Neuron Model and Basic Mathematics

Artificial neuron model proposed by McCulloch and Pitts, [McCulloch and Pitts 1943] attempt to reproduce. An artificial neuron is an information processing unit that is fundamental to the operation of a neural network. Each neuron represents a map, typically with multiple inputs and a single output. Specifically, the output of the neuron is a function of a sum of the inputs. The function at the output of the neuron is called the *activation function*.



(a)

Figure 1 (a), (b) Mathematical representation of an artificial neuron model

Single-Input Artificial Neuron Model

A single-input artificial neuron is shown in Figure 2. The scalar input x is multiplied by the scalar *weight* w to form wx, one of the terms that is sent to the summer. The other input, 1, is multiplied by a *bias* b and then passed to the summer. The summer output n, often referred to as the *net input,* goes into a *activation function f,* which produces the scalar neuron output y.

If we relate this simple model back to the biological neuron that the input x is a single dendrite, the weight *w* corresponds to the strength of a synapse, the cell body is represented by the summation and the activation function, and the neuron output *y* represents the signal on the axon;

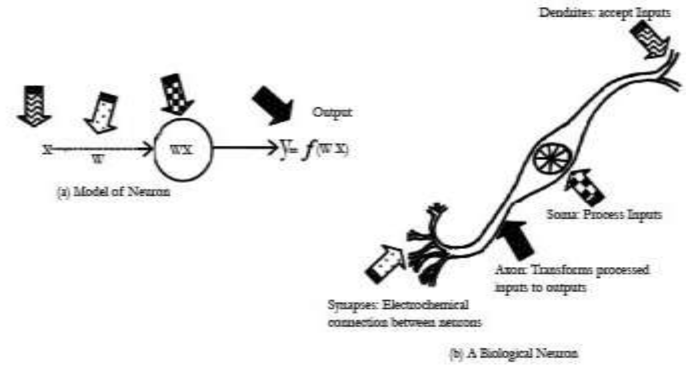


Figure 2 Activity similarities between (a) a computing neuron and (b) a biological neuron

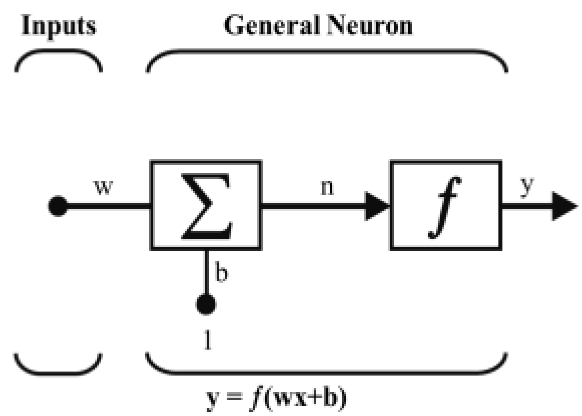


Figure 3 Single-Input Neuron

Then the neuron output is calculated as

$$y = f(wx+b).$$

If, for instance, w = 3, x =2 and b = -1.5, then

$$y = f(3(2)-1.5) = f(4.5)$$

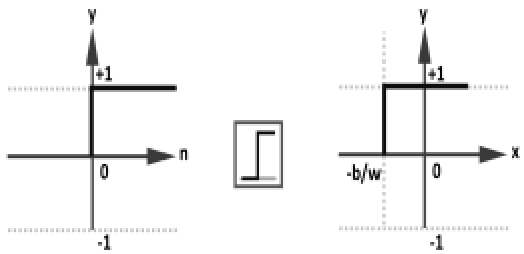
The actual output depends on the particular activation function that is chosen.

The bias is much like a weight, except that it has a constant input of 1. One can choose neurons with or without biases. The bias gives the network an extra variable, and so we might expect that networks with biases would be more powerful than those without, and that is true. Note, for instance, that y neuron without a bias will always have a net input **n** of zero when the network inputs **x** are zero. This may not be desirable and can be avoided by the use of a bias. In fact, w and b are both *adjustable* scalar parameters of the neuron. Typically the activation function is chosen by the designer and then the parameters w and b will be adjusted by some learning rule so that the neuron input/output relationship meets some specific goal.

As described in the following section, we have different activation functions for different purposes.

5 Activation Functions

A particular activation function is chosen to satisfy some specification of the problem that the neuron is attempting to solve. A variety of activation functions have been included in this section. Three of the most commonly used functions are discussed below. The *hard limit activation function*, shown on the left side of Figure 4, sets the output of the neuron to 0 if the function argument is less than 0, or 1 if its argument is greater than or equal to 0. We will use this function to create neurons that classify inputs into two distinct categories.



y = hardlim(n) *y = hardlim(wx+b)*

Hard Limit activation Function

Single-Input *hardlim* Neuron

Figure 4 Hard Limit Activation Function

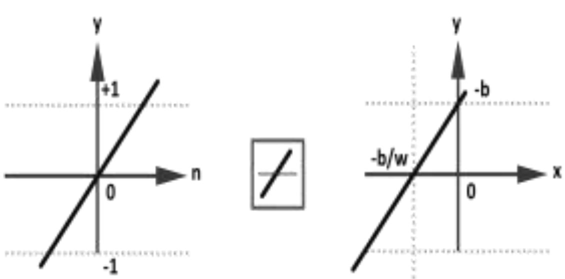
The graph on the right side of Figure 4 illustrates the input/output characteristic of a single-input neuron that uses a hard limit activation function. Here we can see the effect of the weight and the bias. Note that an icon for the hard limit activation function is shown between the two figures. Such icons will replace the general *f* in network diagrams to show the particular activation function that is being used.

The output of a *linear activation function* is equal to its input:

$$y = n$$

as illustrated in Figure 5.

Neurons with this activation function are used in the ADALINE networks.



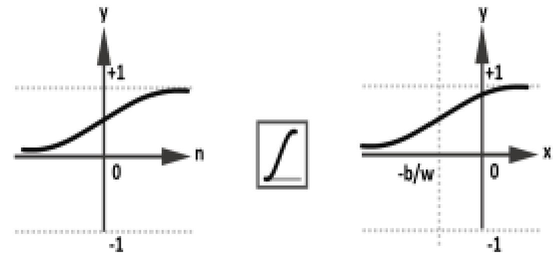
y = purelin(n) *y = purelin(wx+b)*

Linear Activation Function

Single-Input *purelin* Neuron

Figure 5 Linear Activation Function

The *log-sigmoid activation function* is shown in Figure 6.



y =logsig(n) *y = logsig(wx+b)*

Log-Sigmoid Activation Function

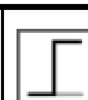
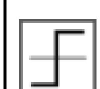
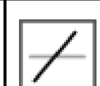
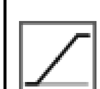
Single-Input *logsig* Neuron

Figure 6 . Log-Sigmoid Activation Function

This activation function takes the input (which may have any value between plus and minus infinity) and squashes the output into the range 0 to 1, according to the expression:

$$y = \frac{1}{1+e^{-n}}$$

The log-sigmoid activation function is commonly used in multilayer networks that are trained using the backpropagation algorithm, in part because this function is differentiable. Most of the activation functions used in the artificial neural networks are summarized in Table 1.

Name	**Input/Output Relation**	**Icon**	**Matlab Function**
Hard Limit	$\mathbf{y = 0 \quad n < 0}$ $\mathbf{y = 1 \quad n \geq 0}$		**hardlim**
Symmetrical Hard Limit	$\mathbf{y = -1 \quad n < 0}$ $\mathbf{y = +1 \quad n \geq 0}$		**hardlims**
Linear	$\mathbf{y = n}$		**purelin**
Saturating Linear	$\mathbf{y = 0 \quad n < 0}$ $\mathbf{y = n \quad 0 \leq n \leq 1}$ $\mathbf{y = 1 \quad n > 1}$		**satlin**

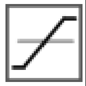
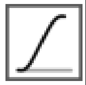
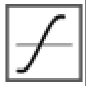
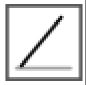
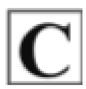
Symmetric Saturating Linear	$\mathbf{y = -1 \quad n < -1}$ $\mathbf{y = n \quad -1 \leq n \leq}$ $\mathbf{y = 1 \quad n > 1}$		**satlins**
Log-Sigmoid	$\mathbf{y = \frac{1}{1 + e^{-n}}}$		**logsig**
Hyperbolic Tangent Sigmoid	$\mathbf{y = \frac{e^{n} - e^{-n}}{e^{n} + e^{-n}}}$		**tansig**
Positive Linear	$\mathbf{y = 0 \quad n < 0}$ $\mathbf{y = n \quad n \geq 0}$		**posin**
Competitive	$\mathbf{y = 1}$ **neuron wi** $\mathbf{y = 0}$ **all other r**		**compet**

Table 1 Activation Functions

Multiple-Input Artificial Neuron Model

Typically, a neuron has more than one input. A neuron with inputs $x_1, x_2, \ldots\ldots, x_N$ is shown in Figure 7. The individual inputs are each weighted by corresponding elements $w_{11}, w_{12}, \ldots\ldots, w_{1N}$ of the *weight matrix* **W**.

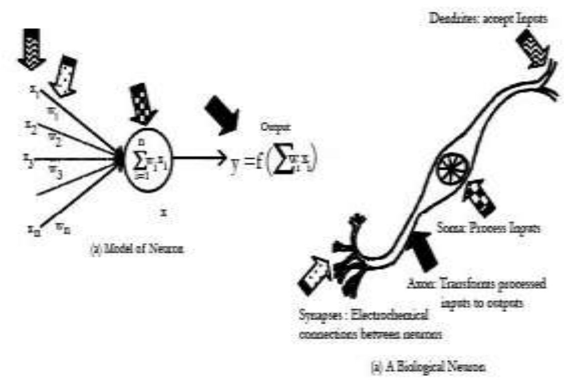
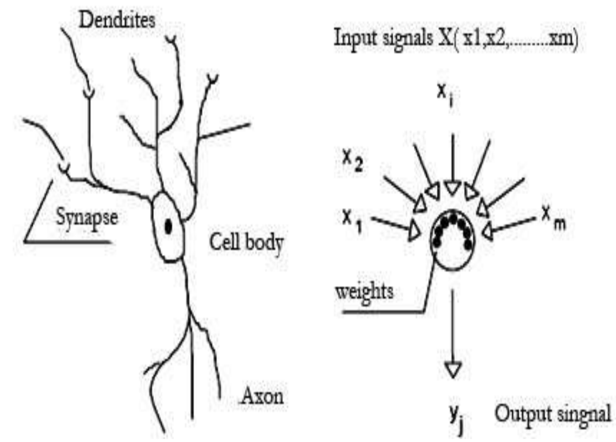


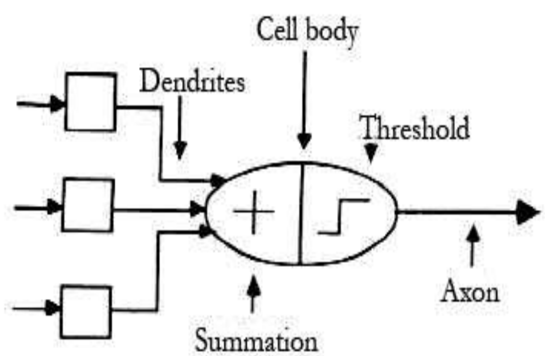
Figure 7 Comparison between (a) a artificial neuron and (b) a biological neuron

endrites $\quad$ $\mathbf{x_i}$ $\quad$ **inputs synaptic efficacy** $\mathbf{w_i}$ **weights**



excitation level $\quad$ **b** $\quad$ **noise(bias,threshold) process input** $\quad$ ***f*** $\quad$ **activation function axon signal** $\quad$ **y** $\quad$ **network output**

(a)



(b)

Figure 8 (a) Comparison between the biological and artificial neuron

(b) Structure of artificial neuron model from biological model

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Biography

Muhammad Hanif received his B.Sc and M.Sc degree in Mathematics from Chittagong University, Chittagong, Bangladesh in 1996 and 1998 respectively and M.Phil and PhD degree in Applied Mathematics from Research Centre for Mathematical and Physical Sciences, Chittagong University, Bangladesh in 2007 and 2012 respectively under the renowned cosmologist Professor J N Islam (1939 – 2013). He did Post Doctoral Research in Applied Mathematics from University Grants Commission Bangladesh in 2014. From 2000 to 2006 he worked as a Lecturer and Assistant Professor in Mathematics at International Islamic University Chittagong and Chittagong University of Engineering and Technology. In 2006, he joined Noakhali Science and Technology University, Noakhali, Bangladesh where he is currently an Associate Professor in the department of Applied Mathematics. His current research interests include number theory, nonlinear programming, biomedical mathematics, neural networks and industrial mathematics.