Neighbourhood Cordial and Neighbourhood Product Cordial Labeling of Graphs

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Abstract *- In this paper, we introduce the various types of neighbourhood cordial labeling of graphs and present the neighbourhood cordial labeling of* kP_2, kP_n, $K_{1,n}\cup P_n$, $K_{1,n}\cup C_n$, $C_n\cup P_n\cup K_{1,n,n}$, $C_n\cup K_{1,n}\cup K_{1,n,n}$ *and* $P_n\cup K_{1,n}\cup K_{1,n,n}$. *Finally, we investigate the total neighbourhood cordial labeling of* $P_n\cup K_{1,n,n}$, $C_n\cup K_{1,n,n}$, $K_{1,n}\cup K_{1,n,n}$ *and total neighbourhood product cordial labeling of* $K_{1,n}$.

Keywords - Neighbourhood cordial graph, Total neighbourhood cordial graph, Neighbourhood product cordial graph, Total neighbourhood cordial graph.,

I. Introduction

By a graph G, we mean a finite, connected, undirected graph without loops and multiple edges, suppose graph G is disconnected means each component of G must contain at least one edge, for terms not defined here, we refer to Harary [3]. For standard terminology and notations related to graph labeling, we refer to Gallian [2]. In [1], Cahit introduce the concept of cordial labeling of graph. The concept of product cordial labeling of a graph is introduced by Sundaram et.al., [5]. In [6], Sundaram et al. also introduce the concept of total product cordial labeling of graph. Motivated by the study of various types of cordial labeling and neighbourhood concept in Graph theory, we introduce neighbourhood cordial labeling, total neighbourhood cordial labeling, neighbourhood product cordial labeling and total neighbourhood product cordial labeling of G. In [4], Muthaiyan et. al., also prove the graphs $C_n\cup C_m\cup C_r$, $P_n\cup P_m\cup P_r$, $C_n\cup P_m\cup K_{1,r,r}$, $C_n\cup K_{1,m}\cup K_{1,r,r}$ and $P_n\cup K_{1,m}\cup K_{1,r,r}$ are neighbourhood cordial graphs under some conditions, the graphs $P_n\cup P_m$ and $C_n\cup C_m$ are total neighbourhood cordial graphs under some conditions and present the neighbourhood product cordial and total neighbourhood product cordial labeling of path and cycle related disconnected graphs. The brief summaries of definitions which are necessary for the present investigation are provided below.

Definition: 1.1

The set of all vertices adjacent to a vertex v is called the neighbourhood of v and is denoted by N(v).

Definition: 1.2

A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.

Definition: 1.3

A mapping $f:V(G)\rightarrow\{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. If for an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = | f(u) - f(v) |$. Then $v_f(i)$ = number of vertices of having label i under f and $e_f(i)$ = number of edges of having label i under f*.

A binary vertex labeling f of a graph G is called a cordial labeling if $| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1) | \le 1$. A graph G is cordial if it admits cordial labeling.

Definition: 1.4

Let G be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a product cordial labeling of G if $| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1) | \le 1$, where $v_f(i)$ and $e_f(i)$ denote the number of vertices and edges respectively labeled with i (i = 0,1). A graph with a product cordial labeling is called a product cordial graph.

Definition: 1.5

Let G be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a total product cordial labeling of G if $|f(0) - f(1)| \le 1$, where f(i) denotes sum of the number of vertices and the number of edges labeled with i (i = 0,1). A graph with a total product cordial labeling is called a total product cordial graph.

Definition: 1.6

A binary vertex labeling f of a graph G is called a neighbourhood cordial labeling if for every vertex $v \in V(G)$, then all the vertices adjacent to the vertex v have the same label, $| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1) | \le 1$. A graph G is neighbourhood cordial if it admits neighbourhood cordial labeling.

Definition: 1.7

A binary vertex labeling f of a graph G is called a total neighbourhood cordial labeling, if for every vertex $v \in V(G)$, then all the vertices adjacent to the vertex v have the same label and $|f(0)-f(1)| \le 1$, where f(i) denotes sum of the number of vertices and the number of edges labeled with i (i = 0,1). A graph G is total neighbourhood cordial if it admits total neighbourhood cordial labeling.

Definition: 1.8

Let G be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a neighbourhood product cordial labeling of G, if for every vertex $v \in V(G)$, then all the vertices adjacent to the vertex v have the same label, $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(i)$ and $e_f(i)$ denote the number of vertices and edges respectively labeled with i (i = 0,1). A graph with a neighbourhood product cordial labeling is called a neighbourhood product cordial graph.

Definition: 1.9

Let G be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a total neighbourhood product cordial labeling of G, if for every vertex $v \in V(G)$, then all the vertices adjacent to the vertex v have the same label and $| f(0)–f(1)| \leq 1$, where f(i) denotes sum of the number of vertices and the number of edges labeled with i (i = 0,1). A graph with a total neighbourhood product cordial labeling is called a total neighbourhood product cordial graph.

Definition: 1.10

In a graph G, if every vertex of G is labeled by 0, then it is called 0 - type labeling of graph G.

In a disconnected graph G, if every vertex in any one of the component of G is labeled by 0, then it is called 0 - type labeling of that component of G.

Definition: 1.11

In a graph G, if every vertex of G is labeled by 1, then it is called 1 - type labeling of graph G.

In a disconnected graph G, if every vertex in any one of the component of G is labeled by 1, then it is called 1 - type labeling of that component of G.

Definition: 1.12

In a graph G, if the end vertices u_i and v_i of each edge e_i (= u_iv_i) are labeled with distinct label, then it is called 01 - type labeling of graph G.

In a disconnected G, if the end vertices u_i and v_i of each edge e_i (= u_iv_i) are labeled with distinct label in any one of the component of G, then it is called 01 - type labeling of that component graph of G.

Definition: 1.13

A complete bipartite graph $K_{1,n}$ is called a star and it has n+1 vertices and n edges. $K_{1,n,n}$ is the graph obtained by the subdivision of the edges of the star $K_{1,n}$.

Observations: 1.14

(i). If G is a neighbourhood cordial graph, then 0 - type labeling or 1- type labeling is suitable labeling for any (bipartite / non-bipartite) component of G.

(ii). If G is a neighbourhood cordial graph, then 01 - type labeling is not suitable labeling for any non-bipartite component of G.

(iii). If G is a neighbourhood cordial graph, then 01 - type labeling is only suitable labeling for any bipartite component of G.

(iv). If G is a neighbourhood cordial graph, then 0 - type labeling or 1 - type labeling on any one of the component of G induces each edge label is 0.

(v). If G is a neighbourhood cordial graph G, then 01 - type labeling on any one of the component of G induces each edge label is 1.

(vi). If G is a neighbourhood product cordial graph, then 0 - type labeling on any one of the component of G induces each edge label is 0.

(vii). If G is a neighbourhood product cordial graph, then 1 - type labeling on any one of the component of G induces each edge label is 1.

(viii). If G is a neighbourhood product cordial graph, then 01 - type labeling on any one of the component of G induces each edge label is 0.

II. MAIN THEOREMS

Theorem: 2.1

Let G be a (connected or disconnected) neighbourhood cordial graph with n vertices and m edges, then G contains at least one component which is bipartite.

Proof.

Let G be a neighbourhood cordial graph with n vertices and m edges.

Case (i) : G is a connected neighbourhood cordial graph with n vertices and m edges.

Suppose G is non-bipartite graph.

Then by observation 1.14 (i), 0 - type labeling or 1- type labeling is suitable labeling for G, we have $|v_f(0)–v_f(1)|= n > 1$, which is contradiction to our assumption.

Hence G must contain at least one bipartite component.

Case (ii) : G is a disconnected neighbourhood cordial graph with n vertices and m edges.

Since G is a disconnected graph, then $m \geq 2$.

Suppose G has no bipartite components.

Then by observation 1.14 (i), 0 - type labeling or 1- type labeling is suitable labeling for G.

Also, by observation 1.14 (iv), we have $e_f(0) = m$.

Therefore $| e_f(0) - e_f(1) | = m \geq 2$, which is contradiction to our assumption.

Hence G must contain at least one bipartite component.

Therefore, a (connected or disconnected) graph G is neighbourhood cordial with n vertices and m edges, and then G contains at least one component which is bipartite.

Theorem: 2.2

A connected graph G is neighbourhood cordial graph iff G is K_2.

Proof.

Let G = K_2. Let = uv be an edge of G.

Define f : V(G) $\rightarrow$ {0,1} as follows

$f(u) = 0$ and $f(v) = 1$.

Then $f^*(uv) = 1$, $|v_f(0)–v_f(1)| \leq 1$ and $|e_f(0) – e_f(1)| \leq 1$.

Hence, f is neighbourhood cordial labeling of G and G is neighbourhood cordial graph.

Conversely

Assume that the connected graph G is neighbourhood cordial graph.

From the above theorem, we have G is bipartite graph.

Claim : G = K_2.

Suppose G $\neq$ K_2 and G has at least two edges.

Any suitable labeling of G which satisfy the vertex condition, we have $| e_f(0) – e_f(1) | > 2$, which is contradiction to our assumption. Therefore, G = K_2.

Hence, a connected graph G is neighbourhood cordial graph iff G is K_2.

Theorem : 2.3

Let G be a connected graph with n vertices, m edges and $| n–m | \leq 1$, then G is total neighbourhood cordial graph.

Proof.

Let G be a connected graph with n vertices, m edges and $|n – m | \leq 1$.

Let $u_1, u_2, ..., u_n$ and $e_1, e_2, ..., e_m$ be the vertices and edges of G respectively.

Define vertex labeling f : V(G) $\rightarrow$ {0,1} as follows

$f(u_i) = 1$ for $1 \leq i \leq n$.

Then, we have $v_f(1) = n$ and $e_f(0) = m$.

Therefore, $f(0) = m$ and $f(1) = n$ and $|n – m| \leq 1$ implies that $| f(1) – f(0) | \leq 1$.

Therefore, G is total neighbourhood cordial graph.

Example : 2.1

The graph $K_{1,5}$ and its total neighbourhood cordial labeling is given in Figure 2.1.

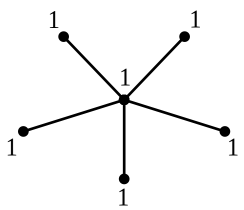

Figure 2.1

Theorem : 2.4

Let n_1, n_2 be the number of vertices and m_1, m_2 be the number of edges of connected graphs G_1 and G_2 respectively, $| n_1–n_2 | \leq 1$ and $|m_1 – m_2| \leq 1$, then $G_1 \cup G_2$ is neighbourhood product cordial graph.

Proof.

Let G_1 and G_2 be two connected graphs with n_1,n_2 vertices and m_1, m_2 edges respectively.

Let $u_1, u_2, ..., u_{n_1}$ and $v_1, v_2, ..., v_{n_2}$ be the vertices of G_1 and G_2 respectively.

Define vertex labeling f : V(G) $\rightarrow$ {0,1} as follows

$f(u_i) = 0$ for $1 \leq i \leq n_1$.

$f(v_i) = 1$ for $1 \leq i \leq n_2$.

Then, we have $v_f(0) = n_1$, $v_f(1) = n_2$, $e_f(0) = m_1$ and $e_f(1) = m_2$.

$| n_1–n_2 | \leq 1$ and $|m_1–m_2| \leq 1$implies that $| v_f(0) – v_f(1) | \leq 1$ and $| e_f(0) – e_f(1) | \leq 1$.

Therefore, $G_1 \cup G_2$ is neighbourhood product cordial graph.

Example : 2.2

The graph $C_5 \cup K_{1,5}$ and its total neighbourhood product cordial labeling is given in Figure 2.2.

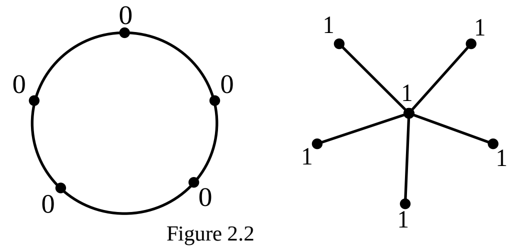

Figure 2.2

Theorem : 2.5

The disconnected graph kP_2 is neighbourhood cordial graph except $k \equiv 2 (\text{mod } 4)$.

Proof.

Let G be the disconnected graph kP_2.

Let $u_{11}, u_{12}, u_{21}, u_{22}, \ldots, u_{k1}, u_{k2}$ be the vertices of kP_2.

Then $|V(G)| = 2k$ and $|E(G)| = k$.

Define vertex labeling f : V(G) $\rightarrow$ {0,1}, we consider following two cases.

Case (i) : $k \equiv 0,1,3 (\text{mod } 4)$.

$f(u_{i1}) = 0$ for $i \equiv 1,2 (\text{mod } 4)$,

$f(u_{i1}) = 1$ for $i \equiv 0,3 (\text{mod } 4)$,

$f(u_{i2}) = 1$ for i is odd,

$f(u_{i2}) = 0$ for i is even.

In view of the above labeling pattern, $v_f(0) = v_f(1) = k$ and $e_f(1) = e_f(0)+1 = \frac{k+1}{2}$, if $k \equiv 1,3 (\text{mod } 4)$ and $v_f(0) = v_f(1) = k$ and $e_f(0) = e_f(1) = \frac{k}{2}$, if $k \equiv 0 (\text{mod } 4)$.

Therefore, $| v_f(0) – v_f(1) | \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence G is neighbourhood cordial graph.

Case (ii) : $k \equiv 2 (\text{mod } 4)$.

In order to satisfy the vertex condition for neighbourhood cordial graph, it is essential to assign label 0 to k vertices and label 1 to k vertices. Then, this labeling will give rise $\frac{k+2}{2}$ edges with label 1 and $\frac{k-2}{2}$ edges with label 0.

Therefore $|e_g(0) - e_g(1)| = 2$. Thus the edge condition for neighbourhood cordial graph is violated.

Hence, G is not neighbourhood cordial graph, when $k \equiv 2(\text{mod } 4)$.

Therefore, the graph kP_2 is neighbourhood cordial graph except $k \equiv 2(\text{mod } 4)$.

Example : 2.3

The disconnected graph $5P_2$ and its neighbourhood cordial labeling is given in Figure 2.3.

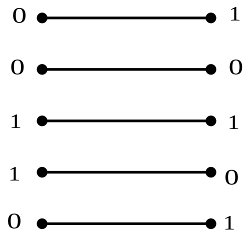

Figure 2.3

Theorem : 2.6

The disconnected graph kP_n is neighbourhood cordial graph when $k \equiv 0(\text{mod } 4)$ and $n \geq 3$.

Proof.

Let G be the disconnected graph kP_n.

Let $u_{11}, u_{12}, \ldots, u_{1n}, u_{21}, u_{22}, \ldots, u_{2n}, u_{k1}, u_{k2}, \ldots, u_{kn}$ be the vertices of kP_n. Then $|V(G)| = 2nk$ and $|E(G)| = (n-1)k$.

Define vertex labeling $f : V(G) \to \{0,1\}$, we consider following two cases.

Case (i) : $k \equiv 0(\text{mod } 4)$.

$f(u_{ij}) = 0$ for $i \equiv 1(\text{mod } 4)$ and j is odd,

$f(u_{ij}) = 1$ for $i \equiv 1(\text{mod } 4)$ and j is even,

$f(u_{ij}) = 0$ for $i \equiv 2(\text{mod } 4)$ and $1 \leq j \leq n$,

$f(u_{ij}) = 1$ for $i \equiv 3(\text{mod } 4)$ and $1 \leq j \leq n$,

$f(u_{ij}) = 1$ for $i \equiv 0(\text{mod } 4)$ and j is odd,

$f(u_{ij}) = 0$ for $i \equiv 0(\text{mod } 4)$ and j is even.

In view of the above labeling pattern we have,

$$e_f(0) = e_f(1) = \frac{(n-1)k}{2}.$$

Therefore, $|e_f(0) - e_f(1)| \leq 1$.

Hence G is neighbourhood cordial graph.

Case (ii) : $k \equiv 1,2,3(\text{mod } 4)$.

Any suitable labeling which satisfy the vertex condition for neighbourhood cordial graph gives $|e_g(0) - e_g(1)| \geq n-1$.

Thus the edge condition for neighbourhood cordial graph is violated.

Hence, G is not neighbourhood cordial graph, when $k \equiv 1,2,3(\text{mod } 4)$.

Therefore, the graph kP_n is neighbourhood cordial graph when $k \equiv 0(\text{mod } 4)$ and $n \geq 3$.

Example : 2.4

The disconnected graph $4P_4$ and its neighbourhood cordial labeling is given in Figure 2.4.

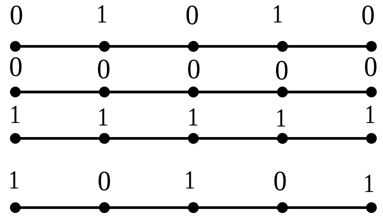

Figure 2.4

Theorem : 2.7

The disconnected graph $K_{1,n} \cup P_n$ is neighbourhood cordial graph, where $n \geq 2$.

Proof.

Let G be the disconnected graph $K_{1,n} \cup P_n$.

Let $v, v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ be the vertices of $K_{1,n}$ and P_n respectively.

Then $|V(G)| = 2n+1$ and $|E(G)| = 2n-1$.

Define vertex labeling $f : V(G) \to \{0,1\}$ as follows.

$f(u_i) = 0$ for $1 \leq i \leq n$,

$f(v) = 0$,

$f(v_i) = 1$ for $1 \leq i \leq n$,

In view of the above labeling pattern we have,

$v_f(0) = v_f(1) + 1 = n+1$ and $e_f(1) = e_f(0) + 1 = n$.

Therefore, $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence G is neighbourhood cordial graph.

Example : 2.5

The graph $K_{1,8} \cup P_8$ and its neighbourhood cordial labeling is given in Figure 2.5.

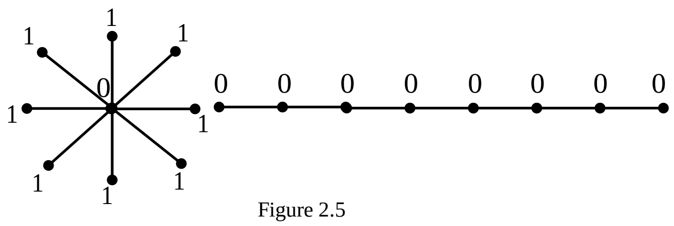

Figure 2.5

Theorem : 2.8

The disconnected graph $K_{1,n} \cup C_n$ is neighbourhood cordial graph, where $n \geq 3$.

Proof.

Let G be the disconnected graph $K_{1,n} \cup C_n$.

Let $v, v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ be the vertices of $K_{1,n}$ and C_n respectively.

Then $|V(G)| = 2n+1$ and $|E(G)| = 2n$.

Define vertex labeling $f : V(G) \to \{0,1\}$ as follows.

$f(u_i) = 0$ for $1 \leq i \leq n$,

$f(v) = 0$,

$f(v_i) = 1$ for $1 \leq i \leq n$,

In view of the above labeling pattern we have,

$v_f(0) = v_f(1) + 1 = n+1$ and $e_f(1) = e_f(0) = n$.

Therefore, $| v_f(0) – v_f(1) | \leq 1$ and $| e_f(0) – e_f(1) | \leq 1$.

Hence G is neighbourhood cordial graph.

Example : 2.6

The graph $K_{1,6} \cup C_6$ and its neighbourhood cordial labeling is given in Figure 2.6.

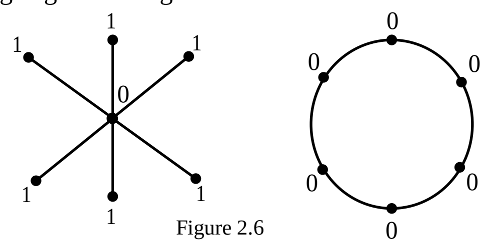

Figure 2.6

Theorem: 2.9

The disconnected graph $C_n \cup P_n \cup K_{1,n,n}$ is neighbourhood cordial graph, where $n \geq 3$.

Proof.

Let G be the disconnected graph $C_n \cup P_n \cup K_{1,n,n}$.

Let $w_1, w_2, ..., w_n, u_1, u_2, ..., u_n$ and $v, v_1, v_2, ..., v_n, v_{n+1}, v_{n+2}, ..., v_{2n}$ be the vertices of C_n, P_n and $K_{1,n,n}$ respectively.

Then $|V(G)| = 4n+1$ and $|E(G)| = 4n – 1$.

Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

$f(w_i) = 1$ for $1 \leq i \leq n$,

$f(u_i) = 0$ for $1 \leq i \leq n$,

$f(v) = 0$,

$f(v_i) = 1$ for $1 \leq i \leq n$,

$f(v_i) = 0$ for $n+1 \leq i \leq 2n$,

In view of the above labeling pattern we have,

$v_f(0) = v_f(1) + 1 = 2n+1$ and $e_f(1) = e_f(0) + 1 = 2n$.

Therefore, $| v_f(0) – v_f(1) | \leq 1$ and $| e_f(0) – e_f(1) | \leq 1$.

Hence G is neighbourhood cordial graph.

Example : 2.7

The graph $C_5 \cup P_5 \cup K_{1,5,5}$ and its neighbourhood cordial labeling is given in Figure 2.7.

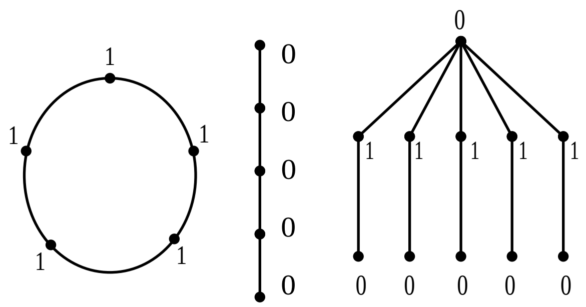

Figure 2.7

Theorem : 2.10

The disconnected graph $C_n \cup K_{1,n} \cup K_{1,n,n}$ is neighbourhood cordial graph, where $n \geq 3$.

Proof.

Let G be the disconnected graph $C_n \cup K_{1,n} \cup K_{1,n,n}$.

Let $w_1, w_2, ..., w_n, u, u_1, u_2, ..., u_n$ and $v, v_1, v_2, ..., v_n, v_{n+1}, v_{n+2}, ..., v_{2n}$ be the vertices of C_n, $K_{1,n}$ and $K_{1,n,n}$ respectively.

Then $|V(G)| = 4n+2$ and $|E(G)| = 4n$.

Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

$f(w_i) = 0$ for $1 \leq i \leq n$,

$f(u) = 1$,

$f(u_i) = 1$ for $1 \leq i \leq n$,

$f(v) = 0$,

$f(v_i) = 1$ for $1 \leq i \leq n$,

$f(v_i) = 0$ for $n+1 \leq i \leq 2n$,

In view of the above labeling pattern we have,

$v_f(0) = v_f(1) = 2n+1$ and $e_f(1) = e_f(0) = 2n$.

Therefore, $| v_f(0) – v_f(1) | \leq 1$ and $| e_f(0) – e_f(1) | \leq 1$.

Hence G is neighbourhood cordial graph.

Example: 2.8

The graph $C_5 \cup K_{1,5} \cup K_{1,5,5}$ and its neighbourhood cordial labeling is given in Figure 2.8.

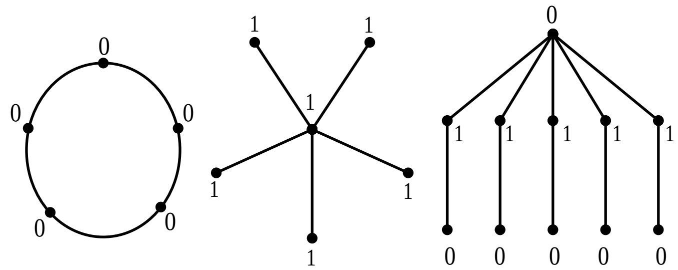

Figure 2.8

Theorem : 2.11

The disconnected graph $P_n \cup K_{1,n} \cup K_{1,n,n}$ is neighbourhood cordial graph, where $n \geq 2$.

Proof.

Let G be the disconnected graph $P_n \cup K_{1,n} \cup K_{1,n,n}$.

Let $w_1, w_2, ..., w_n, u, u_1, u_2, ..., u_n$ and $v, v_1, v_2, ..., v_n, v_{n+1}, v_{n+2}, ..., v_{2n}$ be the vertices of P_n, $K_{1,n}$ and $K_{1,n,n}$ respectively.

Then $|V(G)| = 4n+2$ and $|E(G)| = 4n – 1$.

Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

$f(w_i) = 0$ for $1 \leq i \leq n$,

$f(u) = 1$,

$f(u_i) = 1$ for $1 \leq i \leq n$,

$f(v) = 0$,

$f(v_i) = 1$ for $1 \leq i \leq n$,

$f(v_i) = 0$ for $n+1 \leq i \leq 2n$,

In view of the above labeling pattern we have,

$v_f(0) = v_f(1) = 2n+1$ and $e_f(1) = e_f(0) + 1 = 2n$.

Therefore, $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence G is neighbourhood cordial graph.

Example : 2.9

The graph $P_5 \cup K_{1,5} \cup K_{1,5,5}$ and its neighbourhood cordial labeling is given in Figure 2.9.

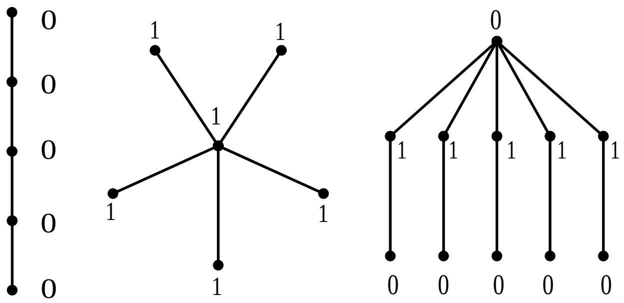

Figure 2.9

Theorem : 2.12

The disconnected graph $P_n \cup K_{1,n,n}$ is total neighbourhood cordial graph, where $n \geq 2$.

Proof.

Let G be the disconnected graph $P_n \cup K_{1,n,n}$.

Let $u_1, u_2, ..., u_n$ and $v, v_1, v_2, ..., v_n, v_{n+1}, v_{n+2}, ..., v_{2n}$ be the vertices of P_n and $K_{1,n,n}$ respectively.

Then $|V(G)| = 3n+1$ and $|E(G)| = 3n - 1$.

Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

$f(u_i) = 0$ for $1 \leq i \leq n$,

$f(v) = 0$,

$f(v_i) = 1$ for $1 \leq i \leq n$,

$f(v_i) = 0$ for $n+1 \leq i \leq 2n$,

In view of the above labeling pattern we have,

$v_f(0) = 2n+1$, $v_f(1) = n$, $e_f(1) = 2n$ and $e_f(0) = n-1$.

Here, $f(0) = 3n$ and $f(1) = 3n$. Therefore, $|f(0) - f(1)| \leq 1$.

Hence G is total neighbourhood cordial graph.

Example : 2.10

The graph $P_5 \cup K_{1,5,5}$ and its total neighbourhood cordial labeling is given in Figure 2.10.

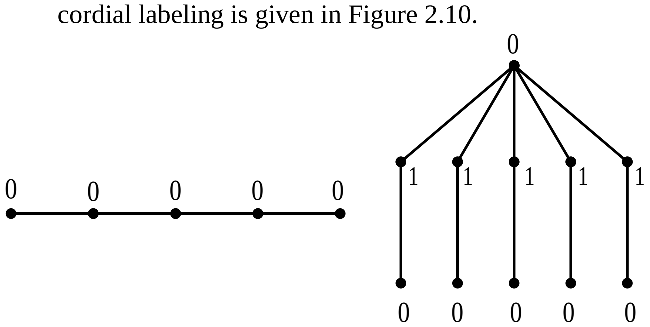

Figure 2.10

Theorem : 2.13

The disconnected graph $C_n \cup K_{1,n,n}$ is total neighbourhood cordial graph, where $n \geq 3$.

Proof.

Let G be the disconnected graph $C_n \cup K_{1,n,n}$.

Let $w_1, w_2, ..., w_n$, and $v, v_1, v_2, ..., v_n, v_{n+1}, v_{n+2}, ..., v_{2n}$ be the vertices of C_n, and $K_{1,n,n}$ respectively.

Then $|V(G)| = 3n+1$ and $|E(G)| = 3n$.

Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

$f(w_i) = 0$ for $1 \leq i \leq n$,

$f(v) = 0$,

$f(v_i) = 1$ for $1 \leq i \leq n$,

$f(v_i) = 0$ for $n+1 \leq i \leq 2n$,

In view of the above labeling pattern we have,

$v_f(0) = 2n+1$, $v_f(1) = n$, $e_f(1) = 2n$ and $e_f(0) = n$.

Here, $f(0) = 3n+1$ and $f(1) = 3n$. Therefore, $|f(0) - f(1)| \leq 1$.

Hence G is total neighbourhood cordial graph.

Example: 2.11

The graph $C_5 \cup K_{1,5,5}$ and its total neighbourhood cordial labeling is given in Figure 2.11.

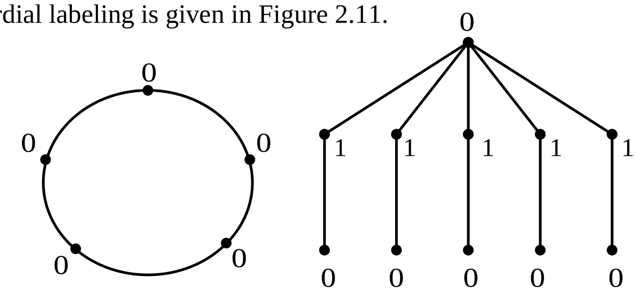

Figure 2.11

Theorem : 2.14

The disconnected graph $K_{1,n} \cup K_{1,n,n}$ is total neighbourhood cordial graph, where $n \geq 2$.

Proof.

Let G be the disconnected graph $K_{1,n} \cup K_{1,n,n}$.

Let $u, u_1, u_2, ..., u_n$ and $v, v_1, v_2, ..., v_n, v_{n+1}, v_{n+2}, ..., v_{2n}$ be the vertices of $K_{1,n}$ and $K_{1,n,n}$ respectively.

Then $|V(G)| = 3n+2$ and $|E(G)| = 3n$.

Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

$f(u) = 0$,

$f(u_i) = 0$ for $1 \leq i \leq n$,

$f(v) = 1$,

$f(v_i) = 0$ for $1 \leq i \leq n$,

$f(v_i) = 1$ for $n+1 \leq i \leq 2n$,

In view of the above labeling pattern we have,

$v_f(0) = 2n+1$, $v_f(1) = n+1$, $e_f(0) = n$ and $e_f(1) = 2n$.

Then $f(0) = 3n+1$ and $f(1) = 3n+1$.

Therefore, $|f(0) - f(1)| \leq 1$.

Hence G is total neighbourhood cordial graph.

Example : 2.12

The graph $K_{1,5} \cup K_{1,5,5}$ and its total neighbourhood cordial labeling is given in Figure 2.12.

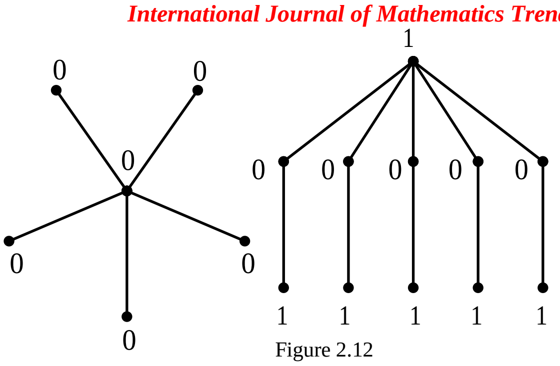

Figure 2.12

Theorem : 2.15

The graph $K_{1,n}$ is total neighbourhood product cordial graph, where $n \geq 2$.

Proof.

Let G be the disconnected graph $K_{1,n}$.

Let v, v_1, v_2, ..., v_n be the vertices of $K_{1,n}$ respectively.

Then $|V(G)| = n+1$ and $|E(G)| = n$.

Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

$f(u) = 0,$

$f(u_i) = 1 \qquad \text{for } 1 \leq i \leq n,$

In view of the above labeling pattern we have,

$v_f(0) = 1$, $v_f(1) = n$, $e_f(0) = n$ and $e_f(1) = 0$.

Then $f(0) = n+1$ and $f(1) = n$.

Therefore, $| f(0) – f(1) | \leq 1$.

Hence G is total neighbourhood product cordial graph.

Example : 2.13

The graph $K_{1,5}$ and its total neighbourhood product cordial labeling is given in Figure 2.13.

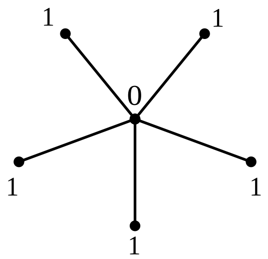

Figure 2.13

III. Conclusions

In this paper, the various types of neighbourhood cordial graphs are introduced and the neighbourhood cordial labeling of kP_2, kP_n, $K_{1,n}\cup P_n$, $K_{1,n}\cup C_n$, $C_n\cup P_n\cup K_{1,n,n}$, $C_n\cup K_{1,n}\cup K_{1,n,n}$, $P_n\cup K_{1,n}\cup K_{1,n,n}$ and total neighbourhood cordial labeling of $P_n\cup K_{1,n,n}$, $C_n\cup K_{1,n,n}$, $K_{1,n}\cup K_{1,n,n}$ are proved. Finally, the total neighbourhood product cordial labeling of $K_{1,n}$ is presented.

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