

# Finite Element Analysis of Convective Heat Transfer of Micropolar and Viscous Fluids in a Vertical Channel with Variable Width

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**Abstract** - In this paper we present the novel of two immiscible fluids flow and the effect of heat transfer along a oppositely moving vertical plates. The vertical channel is subjected to the transverse magnetic field and one plate is moving with constant velocity  $w_0$  towards the center. The coupled governing equations of the flow and heat transfer with appropriate boundary conditions are solved using Finite Element method. The profiles of velocity, Micro rotation and temperature are studied for various parameters Grashof Number ( $Gr$ ), magnetic field ( $H$ ), Reynolds Number( $R$ ), Eckert Number ( $Ec$ ), Material Parameter ( $K$ ) and represented graphically.

**Keywords** — Micropolar fluid, Viscous Fluid, Vertical Channel, Moving plates, Viscous Dissipation, MHD, FEM, Magnetic field.

## I. INTRODUCTION

The subject of two-fluid flow and heat transfer has been extensively studied due to its importance in chemical and nuclear industries. The design of two-fluid heat transport system for space application requires knowledge of heat and mass transfer processes and fluid mechanics under reduced gravity conditions. Identification of the two-fluid flow region and determination of the pressure drop, void fraction, quality reaction and two-fluid heat transfer coefficient are of great importance for the design of two-fluid systems. Lohrasbi and Sahai [3] studied two-phase MHD flow and heat transfer in a parallel plate channel with the fluid in one phase being electrically conducting. Malashetty and Leela [7] have analyzed the Hartmann flow characteristics of two fluids in horizontal channel. The study of two-phase flow and heat transfer in an inclined channel has been studied by Malashetty and Umavathi [5] and Malashetty et al [6]. Micropolar fluids are non-Newtonian fluids with microstructures such as polymeric additives, colloidal suspensions, liquid crystals, etc.

### A. Nomenclature

$U_1$  :velocity in the region 1

$U_2$  :velocity in the region 2

$U_0$  :Average Velocity

$T_1$  :Temperature of the plate at  $y = -h_1$

$T_2$  :Temperature of the plate at  $y = h_2$

$T_0$  : Average temperature.

$k_1$  :Thermal conductivity in Region 1

$k_2$  :Thermal conductivity in Region 2

$n$  :Micro rotation parameter

$K$  :Vertex viscosity

$H_0$  :Magnetic Field Intensity

$N$  :Micro rotation number

$h$  :channel width ratio,

$m$  :viscosity ratio

$b$  :Thermal expansion coefficient ratio

$B_0$  :Magnetic induction

Greek letters:

$\rho_1$  :density of fluid in Region 1

$\rho_2$  :density of fluid in Region 2

$\beta_1$  :Coefficient of Thermal expansion in Region 1

$\beta_2$  :Coefficient of Thermal expansion in Region 2

$\mu_1$  :Viscosity of fluid in Region 1

$\mu_2$  :Viscosity of fluid in Region 2

$\sigma$  : electrical conductivity

$\gamma$  :Spin gradient

$\gamma_1$  :Dynamic viscosity of Micropolar fluid

$\mu_e$  :Magnetic Field Permeability

$\alpha$  :Thermal conductivity ratio

$\rho$  :density ratio

Eringen [1] developed the theory of Micropolar fluids, in which the microscopic effects arising from the local structure and the micro motions of the fluids elements are taken into account. The study of viscous dissipation is applicable to polymer technology involving the stretching of plastic sheets. The Problems of Micropolar fluid flow between two vertical plates (channel) are of great technical interest. A lot of attention has been given by many researchers. Suresh babu et.al [2] studied the heat transfer of micropolar and viscous fluids in a vertical channel.

Sastry and Rao [11] have studied the effect of suction in the laminar flow of a Micropolar fluid in a channel. Bhargava and Rani [8] have examined the convective heat transfer in Micropolar fluid flow between parallel plates. Stamonkovic et.al [9] studied the heat transfer of two immiscible fluids between horizontally moving plates.

Keeping in view of the wide area of practical importance of multi fluid flows as mentioned, the objective of this study to investigate the effect of heat transfer in a vertical channel with variable width.

**II. MATHEMATICAL FORMULATION**

The two infinite parallel plates are placed at  $Y = -h_1$  and  $Y = h_2$  along  $Y$ - direction initially as shown in Figure 1.

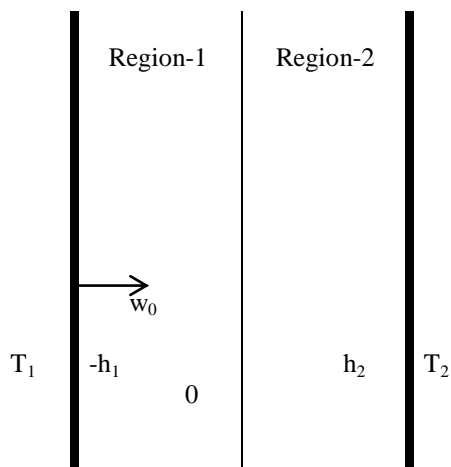


Fig 1 Physical Configuration

The plate at  $Y = -h_1$  is moving with uniform velocity  $w_0$  towards  $Y = 0$  and the second plate is fixed at  $Y = h_2$ , both plates are isothermal with different temperatures  $T_1$  and  $T_2$  respectively. The distance from  $(-h_1$  to  $0)$  represents region 1 and distance from  $(0$  to  $h_2)$  represents region 2 where the first region is filled with Micropolar fluid and the second is with viscous fluid. The fluid flow in the channel is due to buoyancy forces. The transport properties of both fluids are assumed to be constant.

We consider the fluids to be incompressible and immiscible and the flow is steady laminar and fully developed.

The governing equations are

$$\frac{\partial U_1}{\partial Y} = 0 \quad \frac{\partial U_2}{\partial Y} = 0 \quad \text{[Continuity]} \tag{1}$$

$$\rho_1 = \rho_0 [1 - \beta_1 (T_1 - T_0)] \tag{2}$$

$$\rho_2 = \rho_0 [1 - \beta_2 (T_2 - T_0)] \quad \text{[State]} \tag{3}$$

$$\mu_1 + K \frac{d^2 U_1}{dY^2} + K \frac{dn}{dY} + \rho_1 g \beta_1 (T_1 - T_0) = \frac{\sigma B_0^2 U_1}{\rho_1} \tag{4}$$

$$\mu_2 \frac{d^2 U_2}{dY^2} + \rho_2 g \beta_2 (T_2 - T_0) = \frac{\sigma B_0^2 U_2}{\rho_2} \tag{5}$$

[Momentum]

$$\gamma \frac{d^2 n}{dY^2} - K \left[ 2n + \frac{dU_1}{dY} \right] = 0 \tag{6}$$

[Angular Momentum]

$$\frac{d^2 T_1}{dY^2} + \frac{\mu_1}{k_1} \left( \frac{dU_1}{dY} \right)^2 = 0 \tag{7}$$

$$\frac{d^2 T_2}{dY^2} + \frac{\mu_2}{k_2} \left( \frac{dU_2}{dY} \right)^2 = 0 \quad \text{[Energy]} \tag{8}$$

The above system of equations are solved by using the following boundary and interface conditions proposed by T.Arimen et al [10]

$$U_1 = w_0 \text{ at } Y = -h_1, \quad U_2 = 0 \text{ at } Y = h_2$$

$$U_1 = 0 = U_2$$

$$\mu_1 + K \frac{dU_1}{dY} + Kn = \mu_2 \frac{dU_2}{dY}, \quad \frac{dn}{dY} = 0 \text{ at } Y = 0$$

$$n = 0 \text{ at } Y = -h_2$$

For the corresponding temperature boundary conditions it is assumed that the temperature and heat flows are continuous at the interface.

$$T = T_1 \text{ at } Y = -h_1, \quad T = T_2 \text{ at } Y = h_2$$

$$T_1 = 0 = T_2$$

$$k_1 \frac{dT_1}{dY} = k_2 \frac{dT_2}{dY} \text{ at } Y = 0$$

We assume that

$$\gamma = \left( \mu_1 + \frac{K}{2} \right) j$$

$$\text{and } T_2 > T_1$$

By introducing the following non dimensional variables,

$$y_1 = \frac{Y_1}{h_1}, \quad y_2 = \frac{Y_2}{h_2}, \quad u_1 = \frac{U_1}{U_0}, \quad u_2 = \frac{U_2}{U_0},$$

$$\theta_1 = \frac{T_1 - T_0}{\Delta T}, \quad \theta_2 = \frac{T_2 - T_0}{\Delta T}, \quad N = \frac{h_1}{U_0} n, \quad K' = \frac{\mu_1}{K}$$

The governing equations becomes

$$\frac{d^2 u_1}{dy^2} + \frac{K'}{1+K'} \frac{dN}{dy} + \frac{1}{1+K'} \left[ \frac{Gr}{R} \theta_1 \right] - \frac{1}{1+K'} M^2 u_1 = 0 \tag{9}$$

$$\frac{d^2 N}{dy^2} - \frac{2K'}{2+K'} \left( 2N + \frac{du_1}{dy} \right) = 0 \quad (10)$$

$$\frac{d^2 u_2}{dy^2} + \left[ \frac{Gr}{R} \theta_2 \right] \frac{bm}{h\rho} - \frac{M^2 m}{h} u_2 = 0 \quad (11)$$

$$\frac{d^2 \theta_1}{dy^2} + Pr Ec \left( \frac{du_1}{dy} \right)^2 = 0 \quad (12)$$

$$\frac{d^2 \theta_2}{dy^2} + Pr Ec \frac{\alpha}{m} \left( \frac{du_2}{dy} \right)^2 = 0 \quad (13)$$

where  $Gr = \frac{g\beta_1\Delta T h_1^3}{\nu_1^2}$ ,  $R = \frac{U_0 h_1}{\nu_1}$ ,

$$h = \frac{h_1}{h_2}, m = \frac{\mu_1}{\mu_2}, \alpha = \frac{k_1}{k_2}, \rho = \frac{\rho_1}{\rho_2}, b = \frac{\beta_1}{\beta_2},$$

$$M = \frac{\sigma\mu_e^2 H_0^2 h_1^2}{\mu_1}, Pr = \frac{\mu_1 Cp}{k_1}, Ec = \frac{U_0^2}{Cp \Delta T}$$

Subject to the boundary conditions:

$$u_1(0) = u_2(0) \quad , \quad u_2 = 0 \quad \text{at } y = 1 \quad ,$$

$$u_1 = 1 \quad \text{at } y = -1,$$

$$\frac{du_1}{dy} + \frac{K'}{1+K'} N = \frac{1}{mh} \frac{1}{1+K'} \frac{du_2}{dy} \quad \text{at } y = 0$$

$$\frac{dN}{dy} = 0 \quad \text{at } y = 0 \quad \quad N = 0 \quad \text{at } y_1 = -1$$

$$\theta_1 = 1 \quad \text{at } y_1 = -1 \quad , \quad \theta_2 = 0 \quad \text{at } y_2 = 1 \quad ,$$

$$\theta_1(0) = \theta_2(0)$$

$$\frac{d\theta_1}{dy} = \frac{1}{h\alpha} \frac{d\theta_2}{dy} \quad \text{at } y = 0 \quad (14)$$

### III. SOLUTION OF THE PROBLEM

The coupled governing equations are solved numerically using the regular Galerkin Finite Element method as given by J.N. Reddy [3]. For computational purpose each region is divided into 100 linear elements. Each element is 3 noded.

The shape functions at each node of a typical  $i^{th}$  element are the Langrange's interpolation polynomial given by

$$S_i^1 = \frac{\left( y - \left( \frac{2i-101}{100} \right) \right) \left( y - \left( \frac{2i-100}{100} \right) \right)}{\left( \frac{2i-102}{100} - \left( \frac{2i-101}{100} \right) \right) \left( \frac{2i-102}{100} - \left( \frac{2i-100}{100} \right) \right)}$$

$$S_i^2 = \frac{\left( y - \left( \frac{2i-102}{100} \right) \right) \left( y - \left( \frac{2i-100}{100} \right) \right)}{\left( \frac{2i-101}{100} - \left( \frac{2i-102}{100} \right) \right) \left( \frac{2i-101}{100} - \left( \frac{2i-100}{100} \right) \right)}$$

$$S_i^3 = \frac{\left( y - \frac{2i-101}{100} \right) \left( y - \left( \frac{2i-102}{100} \right) \right)}{\left( \frac{2i-100}{100} - \left( \frac{2i-102}{100} \right) \right) \left( \frac{2i-100}{100} - \left( \frac{2i-101}{100} \right) \right)}$$

The stiffness matrix equations corresponding to the governing equations (9) to (13) for  $i^{th}$  element are evaluated by using the following equations:

Region-1

$$\left[ \int_{\Omega} \frac{dS_i^j}{dY} \frac{dS_i^p}{dY} dY + \frac{M^2}{1+K'} \int_{\Omega} S_i^p S_i^j dY \right] U_{ii}^j =$$

$$- \left[ \frac{K'}{1+K'} \int_{\Omega} \frac{dS_i^j}{dY} N_i^j S_i^j dY - \frac{Gr}{1+K'} \frac{1}{R} \int_{\Omega} \theta_{ii}^j S_i^j S_i^p dY \right]$$

$$+ \left[ S_i^j \frac{dU_{ii}^j}{dY} + \frac{K'}{1+K'} N_i^j S_i^j \right]$$

$$\left( S_i^p \frac{d N_i^j S_i^j}{dY} - \frac{2K'}{2+K'} S_i^p U_{ii}^j S_i^j \right) - \int_{\Omega} \frac{dS_i^j}{dY} \left( \frac{d N_i^j S_i^j}{dY} \right) dY$$

$$- \frac{2K'}{2+K'} \left[ 2 \int_{\Omega} N_i^j S_i^j S_i^p dY + \int_{\Omega} \frac{dS_i^j}{dY} U_{ii}^j S_i^j dY \right] = 0$$

$$S_i^p \frac{d \theta_{ii}^j S_i^j}{dY} - \int_{\Omega} \left( \frac{dS_i^j}{dY} \frac{d \theta_{ii}^j S_i^j}{dY} \right) dY$$

$$+ Pr Ec \int_{\Omega} \left( \frac{dU_{ii}^j S_i^j}{dY} \right)^2 S_i^p dY = 0$$

Region - 2

$$S_i^p \frac{d U_{2i}^j S_i^j}{dY} - \int_{\Omega} \left( \frac{dS_i^j}{dY} \frac{d U_{2i}^j S_i^j}{dY} \right) dY$$

$$+ \frac{Gr}{R} \frac{bm}{h\rho} \int_{\Omega} \theta_{2i}^j S_i^j S_i^p dY - \frac{M^2 m}{h} \int_{\Omega} S_i^p U_{2i}^j S_i^j dY = 0$$

$$S_i^p \frac{d \theta_{2i}^j S_i^j}{dY} - \int_{\Omega} \left( \frac{dS_i^j}{dY} \frac{d \theta_{2i}^j S_i^j}{dY} \right) dY$$

$$+ Pr Ec \frac{\alpha}{m} \int_{\Omega} S_i^j \left( \frac{dU_{2i}^j S_i^j}{dY} \right)^2 dY = 0$$

Where  $\Omega$  is the typical element region  $\left(\frac{2i-102}{100}, \frac{2i-100}{100}\right)$

These coupled governing equations are solved iteratively subject to the boundary conditions given in (14) until the desired accuracy of  $10^{-5}$  is attained.

The Nusselt Number and Shear Stress can be calculated at both walls by using the expressions

$$Nu_1 = \left[ \frac{\partial \theta}{\partial Y} \right]_{Y=-1}, \quad Nu_2 = \left[ \frac{\partial \theta}{\partial Y} \right]_{Y=1}$$

$$St_1 = \left[ \frac{\partial U_1}{\partial Y} \right]_{Y=-1}, \quad St_2 = \left[ \frac{\partial U_2}{\partial Y} \right]_{Y=1}$$

**IV. RESULTS AND DISCUSSION**

The graphical results are displayed in fig 2 to fig 16. The velocity profiles are displayed in fig 2 to 6. The movement of the left plate of the channel enhances the momentum boundary in the second region. The buoyancy force enhances the velocity from fig 2. The movement of left plate dominates the inertial force from fig 3. The Lorentz force retards the velocity across the channel almost uniformly from fig 4. The viscous dissipation enhances the velocity from fig 5. The material property shows slight significance in velocity variation in the first region only from fig 6. The micro rotation profiles for the region 1 are in fig 7 to fig 11. The reverse of momentum has observed for all variations due to the moment of the plate. The angular momentum is significant for variations of Gr, H, Ec and K almost uniformly. The enhancement of buoyancy, viscous dissipation retards the angular momentum. From fig 8 the inertial force effect is significant near the centre. The Lorentz force retards the angular momentum.

The temperature profiles are from fig 12 to 16. The temperature is more pronounced with the variations of buoyancy, magnetic field and Eckert number from fig 12, fig 13, and fig 14 respectively. The viscous force dominates in temperature distribution across the channel. The Lorentz force retards the temperature. Enthalpy difference dominates the temperature distribution across the channel. The Reynolds number and spin are not having much effect on temperature.

**A. Nusselt Number and shear stress**

The Nusselt Number and the Shear Stress values are given in Table I. The buoyancy reduces the heat transfer rate in micropolar fluid boundary and enhances at the viscous boundary. The reverse effect is observed for shear stress. The Lorentz force enhances the heat transfer rate at the micropolar fluid boundary but reduces at the viscous boundary. The viscous dissipation is also dominant in heat transfer rate and shear stress. Material property reduces the stress on the boundary.

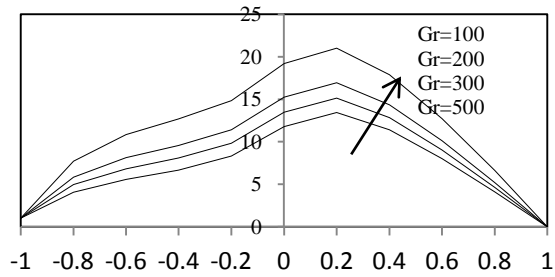


fig 2. Variations of Velocity with Gr

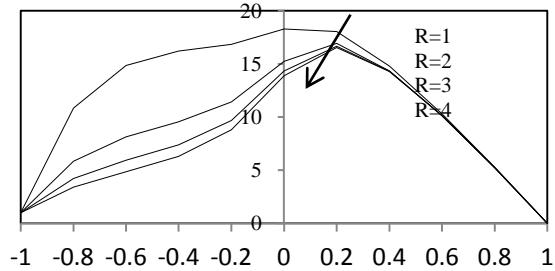


fig 3. Variations of Velocity with R

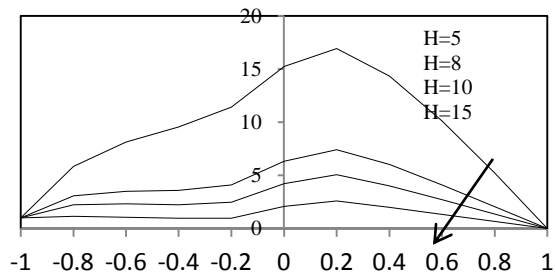


fig 4. Variations of Velocity with H

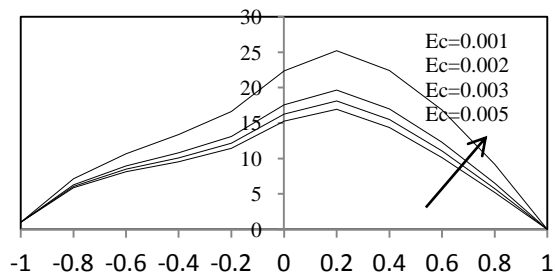


fig 5. Variations of Velocity with Ec

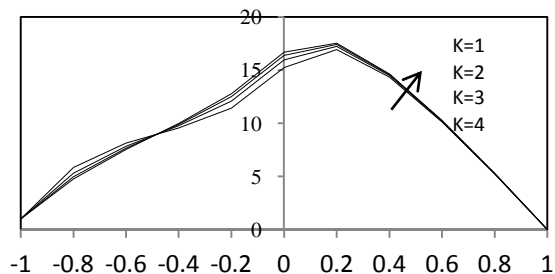


fig 6. Variations of Velocity with K

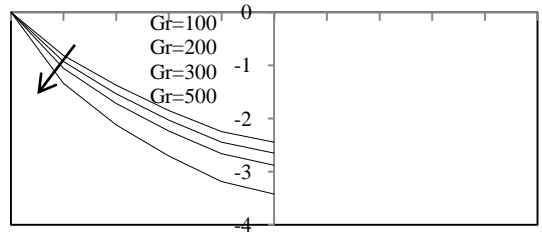


fig 7. Variations of Micro rotation with Gr

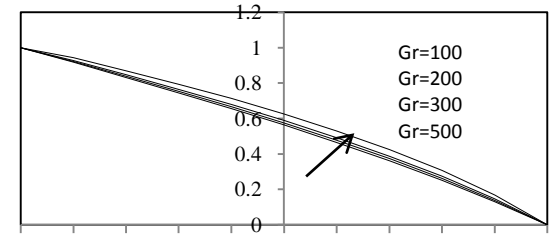


fig 12. Variations of Temperature with Gr

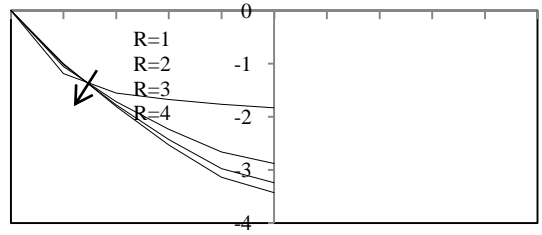


fig 8. Variations of Micro rotation with R

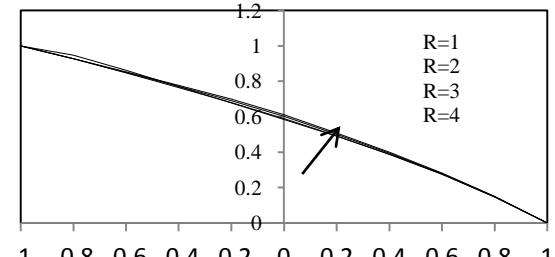


fig 13. Variations of Temperature with R

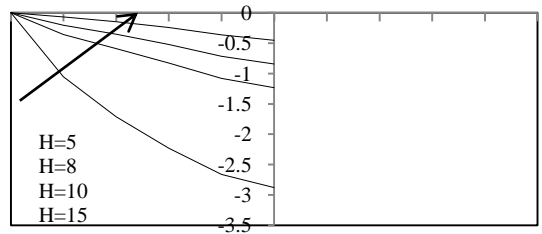


fig 9. Variations of Micro rotation with H

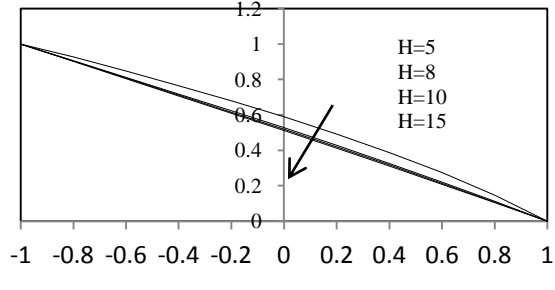


fig 14. Variations of Temperature with H

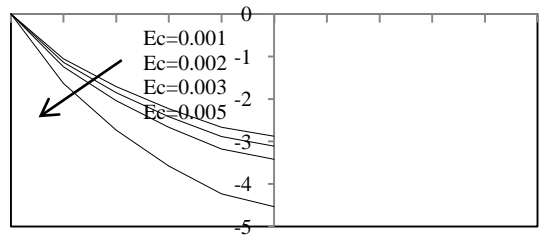


fig 10. Variations of Micro rotation with Ec

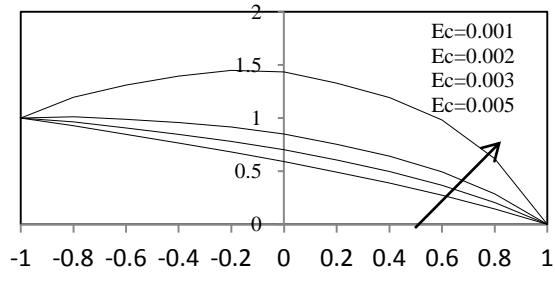


fig 15. Variations of Temperature with Ec

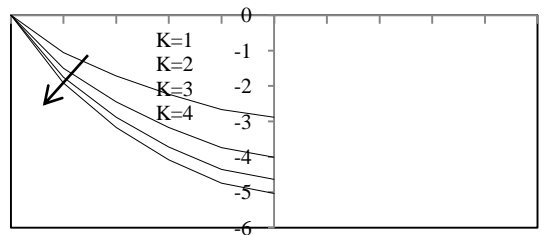


fig 11. Variations of Micro rotation with K

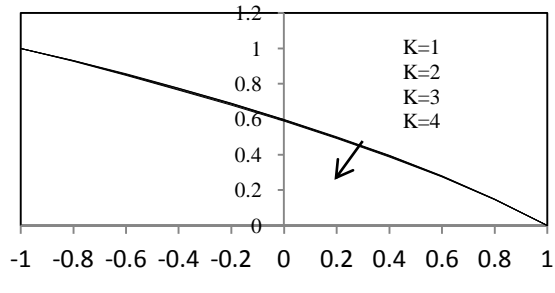


fig 16. Variations of Temperature with K

Nusselt number and Shear stress								
Gr	R	H	Ec	K	St-I	St-II	Nu-I	Nu-II
100	2	5	0.001	1	36.2402	-28.7413	-0.395748	-0.663513
200	2	5	0.001	1	42.3718	-31.408	-0.361873	-0.702001
300	2	5	0.001	1	48.5982	-34.352	-0.319367	-0.748942
300	1	5	0.001	1	84.3885	-34.5898	-0.0843623	-0.755735
300	3	5	0.001	1	36.8886	-34.4316	-0.347081	-0.761117
300	4	5	0.001	1	31.0417	-34.5025	-0.351712	-0.770067
300	2	8	0.001	1	32.1439	-16.1166	-0.458378	-0.556303
300	2	10	0.001	1	26.2391	-11.321	-0.476314	-0.534776
300	2	15	0.002	1	17.2485	-5.43871	-0.476903	-0.5424
300	2	5	0.003	1	50.6399	-40.785	0.185761	-1.52488
300	2	5	0.001	2	42.738	-34.4623	-0.321857	-0.75571
300	2	5	0.001	3	39.4375	-34.5435	-0.32035	-0.761079

Table. I

**V. REFERENCES**

- [1] A.C.Eringen, "Simple microfluids", International Journal Of Engineering And Science" Vol 2(2),pp.205-217, 1964.
- [2] B Suresh Babu , G Srinivas, B.R.K. Reddy, Finite element analysis of free convection flow with MHD micropolar and viscous fluids in a vertical channel with dissipative effects, Journal of Naval Architecture and Marine Engineering , Vol 8, No 1,2011
- [3] J.Lohsasbi, V.Sahai, "Magneto hydro dynamic heat transfer in two phase flow between parallel plates", Journal of Applied Sciences and research ,Vol 45, pp.53-66.1988.
- [4] J.N.Reddy, "An Introduction to the Finite Element Method" Third edition, McGraw-Hill, New York,2005.
- [5] M.S.Malashetty, J.C.Umavathi, J.Prathap Kumar, "Two fluid Magneto Convection Flow in an inclined Channel", International Journal of Transport Phenomena, Vol3,pp.73-84,1997.
- [6] M.S.Malashetty, J.C.Umavathi, J.PrathapKumar, "Convective Magneto hydro dynamic two fluid flow and heat transfer in an inclined Channel", Heat Mass transfer, Vol 37, pp.259-264,2000.
- [7] M.S.Malashetty, V.Leela, "Magneto hydro dynamic heat transfer in two- phase flow", International Journal of Engineering science, Vol 30, pp.371-377.
- [8] R.Bhargava and M.Rani, "Numerical solution of heat transfer of Micropolar fluid flow in a channel with porous walls", International Journal Of Engineering And Science, Vol 23, pp.409-513. 1985.
- [9] Stamenkovic m zivojin , Dragisa D.Nikodijevic, Bratislav D.Bлагоjevic, "MHD Flow and Heat transfer of two immiscible fluids between moving plates, Transactions of Canadian Society for Mechanical Engineering,vol 3-4,2010.
- [10] T.Ariman. M.A.Turk. N.D.Sylvester, "Micro continuum fluid Mechanics, A review", International Journal Of Engineering Science, Vol 11,pp.905-930, 1973.
- [11] V.U.K.Sastry and V.R.M.Rao, "Numerical solution of Micropolar fluid in a channel with porous wall" International Journal of Engineering and Science, Vol20, pp.631-642, 1982.