# Properties of nano b-regular spaces

Dhanis Arul Mary[#1] A, Arockiarani I*[2]

*Department of Mathematics,*

*Nirmala College for women, Coimbatore, Tamilnadu, India*

***Abstract:***
*The purpose of this paper is to introduce a new class of regular spaces namely nano b-regular and nano gb-regular spaces. Some basic properties of these separation axioms are studied by utilizing nano b-open and nano gb-open sets.*

**Mathematics Subject Classification:** 54D10, 54D15, 54C08, 54C10

**Key words:** nano b-regular space, nano gb-regular space, almost nano gb-closed map, nano pre-b-closed map.

## 1. INTRODUCTION

By using semi open sets due to Levine[14] Jin Han Park[11] studied the properties of s-normal spaces and some functions. As a generalization of closed sets, in 1970, Levine [13] initiated the study of so called g-closed sets. Using g-closed sets, Munshi [16] introduced g-regular and g-normal spaces in topological spaces. And in 1999, Takashi Noiri[19] investigated the characterizations of $\alpha$ -regular spaces and the preservation theorems in a topological spaces. Since then many topologists have utilized these concepts to the various notions of subsets, weak separation axioms, weak regularity, weak normality and weaker and stronger forms of covering axioms in the literature. The concept of g-regular spaces in topological spaces was proposed and studied by V. Popa and T. Noiri[20]. Further, Vigilino[22] established the notion of semi -normal spaces and C-compact spaces and obtained its basic properties.

In this paper, we define the two new classes of spaces called nano b-regular spaces and nano gb-regular spaces in nano topological spaces. We bring out several characterizations of these spaces along with already existing weaker forms of regularity and obtain some of the preservation theorems.

## 2. PRELIMINARIES

**Definition 2.1[23]:** Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.

Let $X \subseteq U$

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is

$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by $x \in U$.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is

$$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$$

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is

$$B_R(X) = U_R(X) – L_R(X).$$

**Definition 2.2[15]:** Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let $X \subseteq U$. Let $\tau_R(X)$ = {U, $\phi$, $L_R(X)$, $U_R(X)$, $B_R(X)$}. Then $\tau_R(X)$ is a topology on U, called as the nano topology with respect to X. Elements of the nano topology are known as the nano-open sets in U and $(U, \tau_R(X))$ is called the nano topological space. $[\tau_R(X)]^c$ is called as the dual nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called as nano closed sets.

**Definition 2.3[5]**: Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be nano b-open if $A \subseteq$ Ncl(NintA) $\cup$ Nint(NclA).

**Definition 2.4[6]:** Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be nano continuous if $f^{-1}(B)$ is nano open in U for every nano-open set B in V.

**Definition 2.7:** A space X is said to be p-normal [21] (resp.s-normal [11]) if for any pair of disjoint closed sets A and B, there exist disjoint preopen (resp. semi open) sets U and V such that A $\subset$ U and B $\subset$ V.

**Definition 2.8[7]:** A nano topological space $(U,\tau_R(X))$ is said to nano b-normal if for any pair of disjoint nano closed sets A and B, there exist disjoint nano b-open sets M and N such that A $\subset$ M and B $\subset$ N.

## 3. NANO b-REGULAR SPACES

**Definition 3.1:** A nano topological space U is said to be nano b-regular if for each nano closed set F and a point u $\notin$ F, there exist disjoint nano b-open sets M and N such that u $\in$ M and F $\subset$ N.

**Theorem 3.2:** Every nano regular space is nano b-regular

**Remark 3.3:** The reverse implication of the above Theorem need not be true can be seen from the following example.

**Example 3.4:** Let U = {a, b, c, d} with $U/R$ = {{a}, {c}, {b, d}} and X = {a, b}. Then the nano topology $\tau_R(X)$ = {U, $\phi$, {a}, {b, d}, {a, b, d}} is nano b-regular but not nano regular space.

**Definition 3.5:** A subset A of a nano topological space $(U,\tau_R(X))$ is called

(a) nano generalized b-closed [5](briefly, nano gb-closed), if Nbcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U.
(b) nano gb-open if the complement of A is nano gb-closed in $(U,\tau_R(X))$.

and a subset A of a nano topological space $(U,\tau_R(X))$ is nano gb-open if and only if G $\subseteq$ Nbint(A) whenever G $\subset$ A and G is nano closed in $(U,\tau_R(X))$.

**Definition 3.6:** A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is said to be

(a) almost nano b-open if f(M) is nano b-open in V for every nano regular open set M of U
(b) almost nano gb-closed if f(F) is nano gb-closed in V for every nano regular closed set F of U and
(c) nano pre b-closed if f(F) is nano b-closed for every nano b-closed set F of U.

**Theorem 3.7:** In a nano topological space, the following conditions are equivalent:

(a) U is nano b-regular.
(b) For every point u $\in$ U and every nano open set N containing u there exists a nano b-open set M such that u $\in$ M $\subset$ NbCl(M) $\subset$ N.
(c) For every nano closed set A, the intersection of all the nano b-closed b-neighbourhoods of A is A.
(d) For every set A and a nano open set B such that A $\cap$ B $\neq\phi$, there exists a nano b-open set O such that A $\cap$ O $\neq\phi$ and NbCl(O) $\subset$ B.
(e) For every non-empty set A and nano closed set B such that A $\cap$ B = $\phi$, there exists disjoint nano b-open sets P and Q such that A $\cap$ P $\neq\phi$ and B $\subset$ Q.

**Proof:** (a) $\Rightarrow$ (b): Let N be a nano open set containing u. Then U-N is nano closed and u $\notin$ U-N. Since U is nano b-regular there exist nano b-open sets L and M such that U - N $\subset$ L, u $\in$ M and L $\cap$ M= $\phi$. Now NbCl(U - L) = U-L, for M $\subset$ U-L and U-L is nano b-closed. Hence, NbCl(M) $\subset$ N.
(b) $\Rightarrow$(c): Let A be nano closed and u $\notin$ A. Then, U-A is nano open and contains u. By (b) there is a nano b-open set M such that u $\in$ M $\subset$ NbCl(M) $\subset$ U- A. And so, U-M $\supset$ U- NbCl(M) $\supset$ A. Consequently, U-M is nano b-closed nano b-neighbourhood of A to which u does not belong. Hence (c) holds.
(c) $\Rightarrow$(d): Let A $\cap$ B $\neq\phi$ and B is nano open. Let u $\in$ A $\cap$ B. Since u does not belong to the nano closed set U-B, there exists a nano b-closed nano b-neighbourhood of U-B, say N such that u $\notin$ N. Let U-B $\subset$ M $\subset$ N, where M is nano b-open. Then O = U-N is nano b-open set which contains u and so A $\cap$ O $\neq\phi$. Also, U-M being nano b-closed, NbCl(O) = NbCl(U-N) $\subset$ U-M $\subset$ B.
(d) $\Rightarrow$(e): If A $\cap$ B = $\phi$, where A is non-empty and B is nano closed then A $\cap$ (U-B) $\neq\phi$ and U-B is nano open. Therefore by (d) there exists a nano b-open set P such that A $\cap$ P $\neq\phi$, P $\subset$ NbCl(P) $\subset$ U-B. Put Q = U- NbCl(P). Then B $\subset$ Q. P and Q are nano b-open sets, such that Q = U- NbCl(P) $\subset$ U – P.
(e) $\Rightarrow$(a): Obvious.

**Theorem 3.8:** If f:U $\to$ V is a nano continuous, nano b-open and nano gb-closed surjection and if U is a nano regular space then V is nano b-regular.
**Proof:** Let v $\in$ V and N be a nano open set containing v of $(V,\tau_{R'}(Y))$. Let u be a point of $(U,\tau_R(X))$ such that v = f(u). Since $(U,\tau_R(X))$ is nano regular and f is nano continuous, there is a nano open

set M such that u $\in$ M $\subset$ Ncl(M) $\subset$ $f^{-1}(N)$. Hence, v $\in$ f(M) $\subset$ f(Ncl(M)) $\subset$ N. Since f is nano gb-closed map, f(Ncl(M)) is a nano gb-closed set contained in the nano open set N. Hence we have, $\tau_{R'}$ -Nbcl(f(Ncl(M))) $\subset$ N. Therefore, v $\in$ f(M) $\subset$ $\tau_{R'}$ -Nbcl(f(M)) $\subset$ $\tau_{R'}$ -Nbcl(f(Ncl(M))) $\subset$ N. This implies v $\in$ f(M) $\subset$ $\tau_{R'}$ -Nbcl(f(M))) $\subset$ N and f(M) is nano b-open. Hence $(V, \tau_{R'}(Y))$ is nano b-regular by Theorem 3.7.

**Corollary 3.9:** If f:U $\to$ V is nano continuous, nanoMb-open, nano pre gb-closed bijection and U is a nano b-regular space then V is nano b-regular.

**Proof:** Let v $\in$ V and N be a nano open set containing v of $(V, \tau_{R'}(Y))$. Let u be a point of $(U, \tau_R(X))$ such that v = f(u). By assumptions and the Theorem 3.12, there exists a nano b-open set M such that u $\in$ M $\subset$ $\tau_R$-Nbcl(M) $\subset$ $f^{-1}(N)$. Hence, v $\in$ f(M) $\subset$ f( $\tau_R$ -Nbcl(M)) $\subset$ N and

f( $\tau_R$ -Nbcl(M)) is a nano gb-closed set contained in the nano open set N. Hence we have, $\tau_{R'}$ -Nbcl(f( $\tau_R$ -Nbcl(M))) $\subset$ N. Therefore, v $\in$ f(M) $\subset$ $\tau_{R'}$ -Nbcl(f(M)) $\subset$ $\tau_{R'}$ -Nbcl(f(Ncl(M))) $\subset$ N. This implies v $\in$ f(M) $\subset$ $\tau_{R'}$ -Nbcl(f(M))) $\subset$ N and f(M) is nano b-open. Hence by Theorem 3.7, $(V, \tau_{R'}(Y))$ is nano b-regular.

## 4. NANO gb-REGULAR SPACES

**Definition 4.1:** A nano topological space $(U, \tau_R(X))$ is said to be nano gb-regular if for each nano gb-closed set F and a point u $\notin$ F, there exists disjoint nano b-open sets M and N such that u $\in$ M and F $\subset$ N.

**Theorem 4.2:** A nano topological space $(U, \tau_R(X))$, th following conditions are equivalent:

(a) $(U, \tau_R(X))$ is nano gb-regular.
(b) Every nano gb-open set M is a union of nano b-regular sets.
(c) Every nano gb-closed set A is an intersection of nano b-regular sets.

**Proof:** (a) $\Rightarrow$(b): Let M be a nano gb-open set and let u $\in$ M. If A= U-M, then A is nano gb-closed. By assumption there exist disjoint nano b-open subsets $W_1$ and $W_2$ of U such that u $\in$ $W_1$ and A $\subset$ $W_2$. If N = NbCl($W_1$), then N is nano b-closed and N $\cap$ A $\subset$ N $\cap$ $W_2$ = $\phi$. It follows that u $\in$ N $\subset$ M. Thus M is union of nano b-regular sets.
(b) $\Rightarrow$(c): This is obvious.
(c) $\Rightarrow$(a): Let A be nano gb-closed and let u $\notin$ A. By the hypothesis there exist a nano b-regular set N such that A $\subset$ N and u $\notin$ N. If M = U \ N, then M is nano b-open set containing u and M $\cap$ N = $\phi$. Thus, $(U, \tau_R(X))$ is nano gb-regular.

**Definition 4.3:** A subset N of U is said to be a nano gb-neighbourhood of a point u in U, if there exists a nano gb-open set M such that u $\in$ M $\subset$ N.

**Theorem 4.4:** If B $\subseteq$ A $\subseteq$ U, B is nano gb-closed relative to A and that A is nano open and nano gb-closed in $(U, \tau_R(X))$, then B is nano gb-closed in $(U, \tau_R(X))$.

**Proof:** Let B $\subseteq$ A and A is both nano gb-closed and nano open set, then Nbcl(A) $\subseteq$ A and thus Nbcl(B) $\subseteq$ Nbcl(A) $\subseteq$ A. Now from the fact that A $\cap$ Nbcl(B) = $Nbcl_A$(B) we have, Nbcl(B) = $Nbcl_A$(B) $\subseteq$ A. If B is nano gb-closed relative to A and G is nano open subset of U such that B $\subseteq$ G, then B = B $\cap$ A $\subseteq$ G $\cap$ A, where G $\cap$ A is nano open in A. As B is nano gb-closed relative to A, Nbcl(B) = $Nbcl_A$(B) $\subseteq$ G $\cap$ A $\subseteq$ G. Therefore, B is nano gb-closed in U.

**Theorem 4.5:** If B $\subseteq$ A $\subseteq$ U, B is nano gb-closed in $(U, \tau_R(X))$ and that A is nano open and nano gb-closed in $(U, \tau_R(X))$, then B is nano gb-closed relative to A.

**Proof:** Conversely, if B is nano gb-closed in U and G is an nano open subset of A such that B $\subseteq$ G, then G = N $\cap$ A for some nano open subset Nof U. As B $\subseteq$ N and B is nano gb-closed in U, Nbcl(B) $\subseteq$ N. Thus, $Nbcl_A$(B) = Nbcl(B) $\cap$ A $\subseteq$ N $\cap$ A = G. Therefore, B is nano gb-closed relative to A.

**Theorem 4.6:** If $(U, \tau_R(X))$ is a nano gb-regular space and V is a nano open and nano gb-closed subset of $(U, \tau_R(X))$, then the space V is nano gb-regular.

**Proof:** Let F be any nano gb-closed set of V and v $\in$ Fc. By the Theorem 4.4 F is nano gb-closed in $(U, \tau_R(X))$. Since $(U, \tau_R(X))$ is nano gb-regular, there exist disjoint nano b-open sets M and N of $(U, \tau_R(X))$ such that v $\in$ M, and F $\subseteq$ N. Since V is nano open and hence nano b-open we get M $\cap$ V and N $\cap$ V are disjoint nano b-open sets of the subspace V such that v $\in$ M $\cap$ V and F $\subseteq$ N $\cap$ V. Hence the subspace V is nano gb-regular.

**Theorem 4.7:** Let $(U, \tau_R(X))$ be a nano topological space. Then the following statements are equivalent:

(a) $(U, \tau_R(X))$ is nano gb-regular.

(b) For each point u $\in$ U and for each nano gb-open neighbourhood W of u, there exists a nano b-open set M of U such that NbCl(M) $\subseteq$ W.

(c) For each point u $\in$ U and for each nano gb-closed set F not containing u, there exists a nano b-open set N of U such that NbCl(N) $\cap$ F = $\phi$.

**Proof:**

(a) $\Rightarrow$ (b): Let W be any nano gb-open neighbourhood of u. Then there exists a nano gb-open set G such that u $\in$ G $\subseteq$ W. Since $G^c$ is nano gb-closed and u $\notin$ $G^c$, by assumption, there exist nano b-open sets M and N such that $G^c$ $\subseteq$ N, u $\in$ M and M $\cap$ N = $\phi$ and so M $\subseteq$ $N^c$. Now NbCl(M) $\subseteq$ NbCl($N^c$) = $N^c$ and $G^c$ $\subseteq$ N implies $N^c$ $\subseteq$ G $\subseteq$ W. Thus, NbCl(M) $\subseteq$ W.

(b) $\Rightarrow$(a): Let F be any nano gb-closed set and u $\notin$ F. Then u $\in$ $F^c$ and $F^c$ is nano gb-open and so $F^c$ is a nano gb-neighbourhood of u. By assumption, there exists a nano b-open set N of u such that u $\in$ N and NbCl(N) $\subseteq$ $F^c$, which implies F $\subseteq$ $(\text{NbCl}(N))^c$. Then $(\text{NbCl}(N))^c$ is nano b-open set containing F and N $\cap$ $(\text{NbCl}(N))^c$ = $\phi$. Therefore U is nano gb-regular.

(b) $\Rightarrow$(c): Let u $\in$ U and F be a nano gb-closed set such that u $\notin$ F. Then $F^c$ is a nano gb-neighbourhood of u and by the assumption, there exists a nano b-open set N of U such that NbCl(N) $\cap$ F = $\phi$.

(c) $\Rightarrow$ (b): Let u $\in$ U and W be a nano gb-neighbourhood of u. Then there exists a nano gb-open set G such that u $\in$ G $\subseteq$ W. Since $G^c$ is a nano gb-closed set and u $\notin$ $G^c$, by assumption there exists a nano b-open set M of U such that NbCl(M) $\cap$ $G^c$ = $\phi$. Therefore NbCl(M) $\subseteq$ G $\subseteq$ W.

**Theorem 4.8:** If $(U, \tau_R(X))$ is nano gb-regular space and $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is bijective, nano gb-irresolute and nano Mb-open, then $(V, \tau_{R^1}(Y))$ is nano gb-regular.

**Proof:** Let v $\in$ V and F be any nano gb-closed subset of $(V, \tau_{R^1}(Y))$ with v $\notin$ F. Since f is nano gb-irresolute, $f^{-1}(F)$ is nano gb-closed set in $(U, \tau_R(X))$. Since f is bijective, let f(u) = v, then u $\neq f^{-1}(v)$. By assumption, there exist nano b-open sets M and N such that u $\in$ M and $f^{-1}(F)$ $\subseteq$ N. Since f is nano Mb-open and bijective we have, v $\in$ f(M) and F $\subseteq$ f(N) and f(M) $\cap$ f(N)= f(M $\cap$ N) = $\phi$. Hence $(V, \tau_{R^1}(Y))$ is nano gb-regular space.

**Definition 4.9:** A function $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called nano gc-irresolute if $f^{-1}(F)$ is nano g-closed in U for every nano g-closed set F in V.

**Theorem 4.10:** If $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano gc-irresolute, nano Mb-closed and A is a nano gb-closed subset of $(U, \tau_R(X))$, then f(A) is nano gb-closed.

**Theorem 4.11:** If $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano gc-irresolute, nano Mb-closed and injective and $(V, \tau_{R^1}(Y))$ is nano gb-regular, then $(U, \tau_R(X))$ is nano gb-regular.

**Proof:** Let F be any nano gb-closed set of $(U, \tau_R(X))$ and u $\notin$ F. Since f is nano gc-irresolute, nano Mb-closed then by Theorem 4.10**,** f(F) is nano gb-closed in V and f(u) $\notin$ f(F). Since $(V, \tau_{R^1}(Y))$ is nano gb-regular, and so there exist disjoint nano b-open sets M and N in $(V, \tau_{R^1}(Y))$ such that f(u) $\in$ M and f(F) $\subseteq$ N. By assumption, $f^{-1}(M)$ and $f^{-1}(N)$ $\in$ NbO(U), such that u $\in$ $f^{-1}(M)$ and F $\subseteq$ $f^{-1}(N)$ and $f^{-1}(M)$ $\cap$ $f^{-1}(N)$ = $\phi$. Therefore, $(U, \tau_R(X))$ is nano gb-regular.

**REFERENCES:**

[1] Ahmad Al-Omari and Mohd. Salmi Md. Noorani, On Generalized b-closed sets, Bull. Malays. Math. Sci. Soc. (2)32(1)(2009), 19-30.

[2] D. Andrijevic - Semi pre open sets, Mat. Vesnik, 38(1986), 24-32.

[3] D. Andrijevic , 'On b-open sets', Mat. Vesnik, 48(1996), 59-64.

[4] S.P. Arya and T. Nour, Characterizations of s-normal spaces, Indian J. Pure Appl. Math., 21 (8) (1990), 717-719.

[5] Dhanis Arul Mary A and Arockiarani I "On b-open sets and b-continuous functions in nano topological spaces" IJIRS, ISSN 2319-9725, Vol 13, Iss.11(2014), 98-116.

[6] Dhanis Arul Mary A and Arockiarani I, On nano gb-closed sets in nano topological spaces, IJMA, 6 (2)(2015) , 54-58.

[7] Dhanis Arul Mary A and Arockiarani I, On quasi nano b-normal spaces, Bulletin of Mathematics and Statistics Research, ISSN: 2348-0580, Vol.3, Iss.3(2015), 124-136.

[8] Dhanis Arul Mary A and Arockiarani I "Properties of nano gb-closed maps" IOSR, e-ISSN: 2278-5728, p-ISSN: 2319-765X. Vol.11, Iss.2 (2015), 21-24.

[9] M. Ganster and I. L. Reilly, Locally closed sets and LC-Continuous functions, Int. J. Math. Sci., 3 (1989), 417- 424.

[10] M. Ganster, S. Jafari and G. B. Navalagi, On semi -g-regular and semi-g-normal spaces, Demonstratio Math., 35 (2) (2002), 415 – 421.

[11] Jin Han Park, On s-normal spaces and some functions, Indian J. Pure appl. Math. 30(6)(1999), 575-580.

[12] J. K. Kohli and A. K. Das, New Normality Axioms and decompositions of normality, Glasnik Mat., 37(57) (2002), 163-173.

[13] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat, Polermo, 19 (2) (1970), 89 – 96.

[14] N. Levine, Semi open sets and semi continuity in topological spaces, Amer.Math. Monthly 70 (1963), 36-41.

[15] M. Lellis Thivagar and Carmel Richard, On nano forms of weakly open sets, International Journal of Mathematics and statistics Invention, (2013), 31-37.

[16] Munchi B. M.: Separation axioms, Acta Ciencia Indica.,12 (1986), 140-144.

[17] T. Noiri, semi-normal spaces and some functions, acta Math. Hungar., 65 (3) (1994), 305-311.

[18] T. Noiri, Almost continuity and some separation axioms, Glasnik Math., 9 (29) (1974), 131 – 135.

[19] T. Noiri,A note on $\alpha$ -regular spaces, Indian J.pure appl. Math. 30(9) (1999) 889-891.

[20] T. Noiri and V.Popa, On g-regular spaces and some functions, Mem.Fac.Sci. Kochi Univ. Ser. A. Math. 20(1999), 67-74.

[21] Paul and Bhattacharya, On p-normal spaces, Soochow J. Math., 21 (3) (1995), 273-289.

[22] G. Vigilino, Semi normal and C-compact spaces, Duke J. Math., 38 (1971), 57-61.

[23] Z. Pawlak (1982) "Rough Sets", International Journal of Information and Computer Sciences, 11(1982), 341-356.