Triple Connected Domination Number and Strong Triple Connected Domination Number of a Connected Graph

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Abstract: *The concept of connectedness plays crucial role in any meshing. A variety of connectedness has been studied in the literature by considering the existence of a path between any two vertices. A communication network in which a communicating node can send a message to two stations at one stretch will be more effective and economic. Such an optimization leads to the concept of triple connected graphs. In [1] G. Mahadevan et. al. introduced the concept of triple connected domination number of a graph. In this paper, we discuss the result about triple connected domination number, strong triple connected domination number of graph G and their relationship with other graph theoretical parameters.*

Keywords: *Adjacency matrix, degree of vertex, Path.*

I. Introduction

In *[5]* J. Paulraj Joseph, M.K. Angel Jebitha, P. Chithra Devi and G.Sudhana introduced the concept of Triple connected graphs by considering the existence of a path containing any three vertices of a graph G. We shall Let $G(V,E)$ be a simple graph with vertex set V and edge set E. Two vertices are said to be adjacent (or neighbours) if they are connected by an edge; the corresponding relation between vertices is called adjacency relation. The number of adjacent vertices of a vertex v_i, denoted by d_i is its degree. We denote the maximum and minimum degree of vertex by Δ and δ. A graph G is said to be connected if there is a path between every pair of vertices. A maximal connected subgraph of a graph G is said to be a component of G. We denote a path on n vertices by P_n and a cycle on n vertices by C_n. A vertex cut in a graph is a subset X of V such that $G\backslash X$ is disconnected. Vertex cuts of size one, two and three are called cut vertices. The *connectivity* $\kappa(G)$ of a connected graph G is the minimum number of vertices that need to be removed to disconnect the graph. The chromatic number $\chi(G)$ is the smallest number of colors needed to color all the vertices of a graph G in which adjacent vertices receive different colors. The adjacency matrix A represents the adjacency relation between vertices. The entries a_{ij} of the adjacency matrix is 1 if vertices v_i and v_j are adjacent, and 0 otherwise.

In *[1]* G. Mahadevan et. al., considered triple connected domination number of a graph. Many authors have introduced different types of domination arguments by imposing conditions on the domination set. They have studied the characteristics of triple connected graphs and established many results on them. A graph G is said to be triple connected if any three vertices lie on a path in G. All paths, cycles, complete graphs and wheels are some example of triple connected graphs. A subset T of vertices of a connected graph G is said to be triple connected dominating set, T is a dominating set and the induced subgraph $< T >$ is triple connected. The triple connected domination number of G is the minimum number taken over all triple connected dominating set and is denoted by γ_{tc}.

II. Results

Our main aim here is to study connected domination parameters and its relationship with other graph theoretical parameters. Throughout this paper, we consider connected graph without self-loop and multiple edges.

Theorem 2.1: If the induced subgraph of each connected dominating set of G has more than two pendant vertices, then G does not contain a strong triple connected dominating set.

Theorem 2.2: A tree T is triple connected if and only if $T \cong P_n;\ n \geq 3$.

Observation 2.1: The complement of the triple connected dominating set may or may not be a triple connected dominating set.

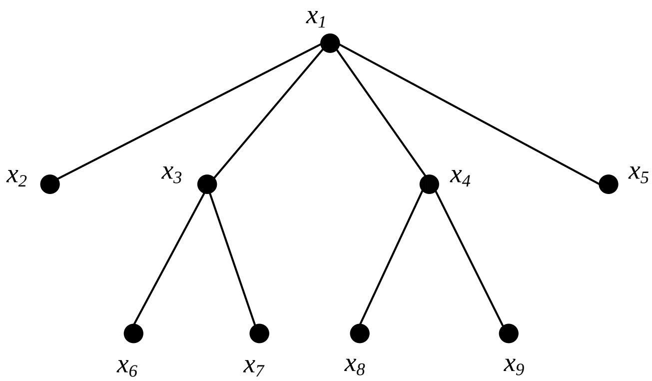

Figure 2.1

For a Graph G with 9 vertices in figure 2.1, $T= \{x_1, x_3, x_4\}$ forms a triple connected dominating set. However, the complement $V\backslash T$ = $\{x_2, x_5, x_6, x_7, x_8, x_9\}$ is not a triple connected dominating set.

Theorem 2.3: If G is connected graph of order 4 and $\gamma_{tc} =3$ then G has common adjacency matrix with P_4 and C_4.

Proof: Let G be a connected graph with 4 vertices and $\gamma_{tc} =3$. Let $T= \{x, y, z\}$ be the triple connected dominated set with 3 vertices and $V\backslash T = \{u\}$. Since T is a triple connected dominated set with γ_{tc} vertices of G this implies $< T >$ is either P_3 or C_3.

If $< T > = C_3 = xyzx$, since G is connected graph this implies u is adjacent to x (or y or z). Now by adding edges to P_4 or C_3 without affecting γ_{tc} we have, G has common adjacency matrix with C_4.

If $< T > = P_3 = xyz$, since G is connected graph this implies u is adjacent to x (or y or z). We have, G has common adjacency matrix with P_4. □

Strong triple connected domination number

A subset T of vertices of a connected graph G is said to be strong triple connected dominating set, if T is a strong dominating set and the induced subgraph $< T >$ is triple connected. The strong triple connected domination number of G is the minimum number taken over all strong triple connected dominating set and we denote it by $\mu(G)$.
Example: For a graph G with 5 vertices in figure 2.2 $\mu(G)=3$

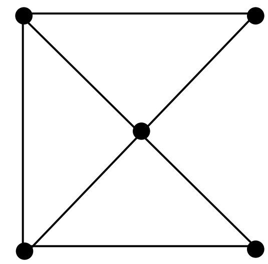

Figure 2.2

Theorem 2.4: If G is a connected graph of order 5 then $\mu(G) = 3$ if and only if G has common adjacency matrix with P_5, C_5, F_2.

Proof: Firstly suppose G has common adjacency matrix with P_5, C_5, F_2 then clearly $\mu(G) = 3$.
Conversely let G be a connected graph of order 5 and $\mu(G) = 3$. Let $T=\{x, y, z\}$ be a strong triple connected dominating set with μ vertices then clearly $< T > = P_3$ or C_3. Let $V - T = V(G) - V(T) =\{u, v\}$ then $< V\text{-}T >$ is either K_2 or $\overline{K_2}$.

If $< T > = C_3 =xyzx$ and $< V\text{-}T > = K_2 = uv$, since G is connected, there exists a vertex say x (or y, z) in C_3 is adjacent to u(or v) in K_2. Then $T= \{x, y, z\}$ forms a strong triple connected dominating set with μ vertices of G so that $\mu = 3$. If $d(x) =4, d(y)= d(z) =2$, then G and F_2 have common adjacency matrix. In all cases, no new graph exists.

If $< T > = C_3 =xyzx$ and $< V\text{-}T > =\overline{K_2}$, since G is connected, there exists a vertex say x (or y, z) in C_3 is adjacent to u and v in $\overline{K_2}$. Then $T= \{x, y, z\}$ forms a strong triple connected dominating set with μ vertices of G so that $\mu = 3$. In all other cases, no new graph exists. Since G is connected there exists a vertex say x (or y, z) in C_3 which is adjacent to u in $\overline{K_2}$and y (or z) in C_3 is adjacent to v in $\overline{K_2}$, then $T= \{x, y, z\}$ forms a strong triple connected dominating set with μ vertices of G so that $\mu = 3$. In this case, no new graph exists.

If $< T > = P_3 = xyz$ and $< V\text{-}T > = K_2 =uv$, since G is connected, there exists a vertex say x in P_3 which is adjacent to u or v in K_2. Then $T = \{x, y ,u\}$ forms a strong triple connected dominating set with μ vertices of G, so that $\mu = 3$. If u is adjacent to x and $d(x)= d(y)= 2, d(z)=1$ then G and P_5 have common adjacency matrix. Since G is connected there exists a vertex say y in P_3 to adjacent to u(or v) in K_2. Then $T= \{y, u, v\}$ forms a strong triple connected dominating set with μ vertices of G so that $\mu= 3$. If $d(x)= d(z)=1, d(y)= 3$, then G and P_4 have common adjacency matrix. Now by increasing the degrees of vertices, by the above argument, we have G and C_5 have common adjacency matrix.

If $< T > = P_3 = xyz$ and $< V\text{-}T > = \overline{K_2}$, since G is connected there exists a vertex say x (or z) in P_3 is adjacent to u and v in $\overline{K_2}$. Then $T= \{x, y, z\}$ forms a strong triple connected dominating set with μ vertices of G so that $\mu= 3$. Since G is connected, there exists a vertex say x in P_3 which is adjacent to u in $\overline{K_2}$and y in P_3 is adjacent to v in $\overline{K_2}$. Then $T= \{x, y, z\}$ forms a strong triple connected dominating set with μ vertices of G so that $\mu = 3$. Since G is connected, there exists a vertex say x in P_3 which is adjacent to u in $\overline{K_2}$ and z in P_3 is adjacent to v in $\overline{K_2}$. Then $T= \{x, y, z\}$ forms a strong triple connected dominating set with μ vertices of G so that $\mu = 3$. In all the above cases, no new graph exists. □

Theorem 2.5: If G is a connected graph of order $n>2$ and exactly one vertex has maximum degree $n\text{-}2$ then G is isomorphic to P_5 and C_5.

Proof: Let G be a connected graph of order $n > 2$ and exactly one vertex has maximum degree $n\text{-}2$. Let x be the vertex of maximum degree $n\text{-}2$. Let $v_1, v_2, \ldots, v_{n-2}$ be the vertices which are adjacent to vertex x and v_{n-1} be the vertex which is not adjacent to x. since G is connected, v_{n-1} is adjacent to a vertex say y. then $T=\{x, y, v_{n-1}\}$ forms a minimum strong triple connected dominating set of G. Hence, G is isomorphic to P_5 and C_5. □

Theorem 2.6: Let G be a connected graph of order $n>3$. If the bound is sharp and $\mu(G) + \kappa(G) \leq 2n - 2$ then $\mu(G) + \chi(G) \leq 2n - 1$.

Proof: If G is connected graph of order $n > 3$ then $\chi(G) \leq n$. Let us assume $\mu(G) + \kappa(G) \leq 2n - 2$ and the bounds are sharp then G is isomorphic to K_4 this will imply $\mu(G) \leq$ n-1. Hence $\mu(G) + \chi(G) \leq 2n - 1$. □

III. Conclusion

In this survey, we attempt to bring together most of the results and papers that dealt with it. We discussed the results of triple connected domination number and strong triple connected domination number with other graph parameters.

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