Effect of Heat Generation/Absorption on Heat and Mass Transfer in A Micropolar Fluid Over A Stretching Sheet with Newtonian Heating and Chemical Reaction

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ABSTRACT

The steady, two dimensional flow of an incompressible, electrically conducting and chemically reacting micropolar fluid over a stretching sheet with Newtonian heating in the presence of MHD, Heat source/sink, mass transfer and chemical reaction. Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. The similarity ordinary differential equations were then solved by MATLAB routine ***bvp4c****. Numerical results are obtained for the skin-friction coefficient, the couple wall stress, the local Nusselt number and Sherwood number as well as the velocity, microrotation, temperature and concentration profiles for different values of the governing parameters, namely, material parameter, magnetic parameter, heat source/sink parameter and chemical reaction parameter.*

Keywords: *Heat and Mass Transfer, micropolar fluid, MHD, Stretching surface, heat source or sink, chemical reaction.*

1. Introduction

Understanding the flow of non-Newtonian fluids is a problem of great interest of researchers and practical importance. There are several natural and industrial applications of such fluids, for instance volcanic lava, molten polymers, drilling mud, oils, certain paints, poly crystal melts, fluid suspensions, cosmetic and food products and many others. The flow dynamics of non-Newtonian fluids can be described by non-linear relationships between the shear stress and shear rate. Further these fluids have sheared dependent viscosity. In literature there exist many mathematical models with different constitutive equations involving different set of empirical parameters. The micropolar fluid model is adequate for exocitic lubricants, animal blood, liquid crystals with rigid molecules, certain biological fluids and colloidal or suspensions solutions. The micromotion of fluid elements, spin inertia and the effects of the couple stresses are very important in micropolar fluids [1,2]. The fluid motion of the micropolar fluid is characterized by the concentration laws of mass, momentum and constitutive relationships describing the effect of couple stress, spin-inertia and micromotion. Hence the flow equation of micropolar fluid involves a micro -rotation vector in addition to classical velocity vector. In micropolar fluids, rigid particles in a small volume element can rotate about the centred of the volume element. The micropolar fluids in fact can predict behaviour at microscale and rotation is independently explained by a microrotation vector. More interesting aspects of the theory and application of micropolar fluids can be found in the books of Eringen [3] and Lukazewicz [4] and in some studies of Peddieson and McNitt [5] Willson [6], Siddheshwar and Manjunath [7].

In Newtonian heating, the rate of heat transfer from the bouncing surface with a finite heating capacity is proportional to the local temperature surface which is usually termed as conjugate convective flow (see Merkin [8], Lesnic et al. [9], Chaudhary and Jain [10]). Salleh et al. [11] numerically investigated the boundary layer flow of viscous fluid over a stretched surface in the regime of Newtonian heating. Recently, Qasim [12] studied the heat transfer in a micropolar fluid over a stretching sheet with Newtonian heating.

MHD flow past a flat surface has many important technological and industrial applications such as micro MHD pumps, micro mixing of physiological samples, biological transportation and drug delivery ([13-14]). Pal and Mondal [15] studied the flow and heat transfer effects of an electrically conducting fluids such as liquid metals; water mixed with a little acid and other equivalent substances in the presence of a magnetic field. Herdrich et al. [16] pointed out possible applications of magnetically controlled plasmas in space technology. The similarity solutions for MHD flow with heat transfer over a wedge considering variable viscosity and thermal conductivities effects were investigated by Seddeek et al. [17]. Alam et al. [18] discussed the effects of suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over an inclined radiative plate. Rahman and Salauddin [19] dealt with the effects of variable electric conductivity and viscosity on hydromagnetic heat and mass transfer flow. Hendi *and Hussain [20] presented solution of MHD Falkner-Skan flow over a porous surface by homotopy analysis method. Uddin et al. [21] studied the MHD free convective boundary layer flow of a nanofluid past a flat vertical plate with newtonian heating. Later, Uddin el [22] investigated the MHD forced convective laminar boundary layer flow from a convectively heated moving vertical plate with radiation and transpiration effect. Srinivasacharya and Upendar [23] analyze the free convection in MHD micropolar fluid with soret and dufour effects.*

The heat source/sink effects in thermal convection, are significant where there may exist a high temperature difference between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reactions. Singh et al. [24], investigated the effect of volumetric heat generation/absorption on mixed convection stagnation point flow on an isothermal vertical plate in porous media. Das and his co-workers [25] analyzed the effect of mass transfer on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Pal and Talukdar [26] studied the unsteady MHD heat and mass transfer along with heat source past a vertical permeable plate using a perturbation analysis, where the unsteadiness is caused by the time dependent surface temperature and concentration. Ibrahim et al.[27] have examined the effects of unsteady MHD micropolar fluids over a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. Rehman and sattar [28] have analyzed the effect of magnetohydrodynamic convective flow of a micropolar fluid past a continuously moving porous plate in the presence of heat generation/ absorption.

The simultaneous effects of heat and mass transfer with chemical reaction are of great importance to engineers and scientists because of its occurrence in many branches of science and engineering. Rajagopal et al. [29] investigated the linear stability of Hagen-Poiseuille flow in a chemically reacting fluid. Effects of chemical reaction and thermal radiation on heat and mass transfer flow of MHD micropolar fluid in rotation frame of reference is investigated by Das [30].

The present study investigates a steady, two dimensional flow of an incompressible, electrically conducting and chemically reacting micropolar fluid over a stretching sheet with Newtonian heating in the presence of a uniform transverse magnetic field and heat source/sink. The similarity solutions were obtained using suitable transformations. The similarity ordinary differential equations were then solved by MATLAB bvp4c solver. The numerical results for the velocity, microrotation, temperature and concentration functions are carried out for the wide range of important parameters namely, material parameter, magnetic parameter, heat source/sink parameter and chemical reaction parameter. The skin friction, the couple wall stress, the rate of heat transfer and the rate of mass transfer have also been computed.

2. MATHEMATICAL FORMULATION

A steady boundary layer flow of an incompressible and chemically reacting micropolar fluid induced by a stretching surface is considered. The sheet is stretched with a velocity $u_w(x) = cx$ *(where c is a real number).*

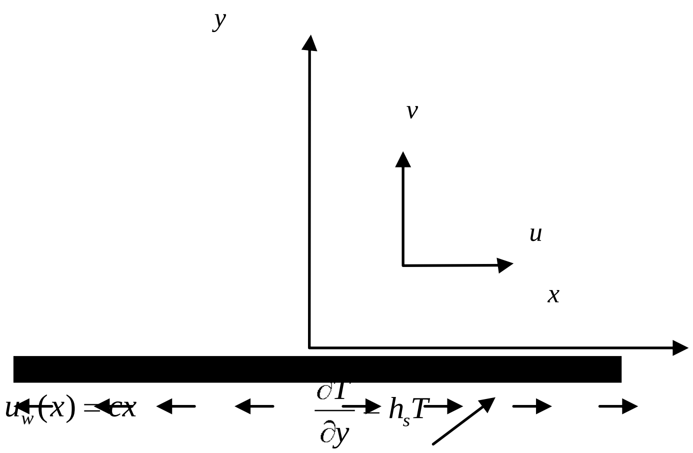

Figure A: Physical model and coordinate system.

The physical model and coordinate system is shown in figure A. A uniform magnetic field of strength B_0 is assumed to be applied in the positive y-direction normal to the plate. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is negligible. It is further assumed that the fluid properties are taken to be constant except for the density variation with the temperature and concentration in the buoyancy terms (Boussinesq approximation). Under the usual boundary layer approximation, the governing equations are

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2.1)$$

Linear momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\nu+\frac{\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u+\frac{\kappa}{\rho}\frac{\partial N}{\partial y} \qquad (2.2)$$

Angular momentum equation

$$\rho j\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=\gamma^*\frac{\partial^2 N}{\partial y^2}-\kappa\left(2N+\frac{\partial u}{\partial y}\right) \qquad (2.3)$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+Q_0(T-T_\infty) \qquad (2.4)$$

Species equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_0(C-C_\infty) \qquad (2.5)$$

The boundary conditions for the velocity, temperature and concentration fields are

$$u=u_w(x)=cx, v=0, N=-n\frac{\partial u}{\partial y}, \frac{\partial T}{\partial y}=h_s T, C=C_w \quad at \quad y=0$$

$$u\to 0, N\to 0, T\to T_\infty, C\to C_\infty \quad as \quad y\to\infty \qquad (2.6)$$

where u and v are the velocity components parallel to the x and y axes, respectively, ρ the fluid density, ν the kinematic viscosity, T is temperature, C is concentration, N the microrotation or angular velocity, T_∞ is the ambient temperature, C_∞ is the ambient concentration, c_p the specific heat, k the thermal conductivity of the fluid, Q_0 is the heat generation or absorption rate constant, k_0 is the chemical reaction rate constant, $j=(\nu/c)$ is microinertia per unit mass, $\gamma^*=\mu+\kappa/2\ j$ and κ are the spin gradient viscosity and vortex viscosity, respectively. Here $\kappa=0$ corresponds to situation of viscous fluid and the boundary parameter n varies in the range $0\le n\le 1$. Here $n=0$ corresponds to the situation when microelements at the stretching sheet are unable to rotate and denotes weak concentrations of the microelements at sheet. The case $n=1/2$ corresponds to the vanishing of anti-symmetric part of the stress tensor and it shows weak concentration of microelements and the case $n=1$ is for turbulent boundary layer flows, h_s is the heat transfer coefficient.

The continuity equation (2.1) is satisfied by the Cauchy Riemann equations

$$u=\frac{\partial\psi}{\partial y} \quad and \quad v=-\frac{\partial\psi}{\partial x} \qquad (2.7)$$

where $\psi(x,y)$ is the stream function.

In order to transform equations (2.4), (2.5) (2.6) and (2.7) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced.

$$\eta=\sqrt{\frac{c}{\nu}}y, N=\sqrt{\frac{c}{\nu}}cxg(\eta), u=cxf'(\eta), v=-\sqrt{c\nu}f(\eta), \theta(\eta)=\frac{T-T_\infty}{T_\infty}$$
$$\phi(\eta)=\frac{C-C_\infty}{C_\infty}, K=\frac{\kappa}{\mu}, M=\frac{\sigma B_0^2}{\rho c}, \Pr=\frac{\mu c_p}{\rho}, Sc=\frac{\nu}{D}, \beta=\frac{Q_0}{\rho c_p c}, Kr=\frac{k_0}{c} \tag{2.8}$$

where $f(\eta)$ is the dimensionless stream function, θ is the dimensionless temperature, ϕ is the dimensionless concentration, η is the similarity variable, M is the magnetic parameter, β is the heat source/sink parameter, Pr is the Prandtl number, Kr is the chemical reaction parameter and Sc is the Schmidt number.

In view of the equations (2.7) and (2.8), the equations (2.2), (2.3), (2.4), (2.5) and (2.6) transform into

$$(1+K)f'''+ff''-(f')^2+Kh'-Mf'=0 \tag{2.9}$$
$$(1+K/2)h''+fh'-f'h-K(2h+f'')=0 \tag{2.10}$$
$$\frac{1}{\Pr}\theta''+f\theta'+\beta\theta=0 \tag{2.11}$$
$$\frac{1}{Sc}\phi''+f\phi'-Kr\phi=0 \tag{2.12}$$

The corresponding boundary conditions are

$$f(0)=0, f'(0)=1, h(0)=-nf''(0), \theta'(0)=-\gamma(1+\theta(0)), \phi(0)=1$$
$$f'=h=\theta=\phi=0 \quad as \quad \eta\to\infty \tag{2.13}$$

where the primes denote differentiation with respect to η and $\gamma=h_s\sqrt{\nu/c}$ his the conjugate parameter for Newtonian heating.

For the type of problem under consideration, the physical quantities of interest are the skin friction coefficient C_{fx}, the local couple wall stress M_{wx}, the local Nusselt number Nu_x and Sherwood number Sh_x which are defined as

$$C_{fx}=\frac{1}{\rho u_w^2}\left[(\mu+\kappa)\left(\frac{\partial u}{\partial y}\right)_{y=0}+\kappa(N)_{y=0}\right]=(1+(1-n)K)\operatorname{Re}_x^{-1/2}f''(0) \tag{2.14}$$
$$M_{wx}=\gamma^*\left(\frac{\partial N}{\partial y}\right)_{y=0}=\frac{\gamma^* u_w}{\nu}h'(0) \tag{2.15}$$
$$Nu_x=-\frac{x}{T_w-T_\infty}\left(\frac{\partial T}{\partial y}\right)_{y=0}=\gamma\left[1+\frac{1}{\theta(0)}\right]\operatorname{Re}_x^{2} \tag{2.16}$$
$$Sh_x=-\frac{x}{C_w-C_\infty}\left(\frac{\partial C}{\partial y}\right)_{y=0}=-\operatorname{Re}_x^{2}\phi'(0) \tag{2.17}$$

where $\operatorname{Re}_x=(cx^2/\nu)$ is the local Reynolds number.

Our main aim is to investigate how the values of f ''(0), $h'(0)$, $-\theta'(0)$ and $-\phi'(0)$ vary in terms of the various parameters.

3 SOLUTION OF THE PROBLEM

The set of equations (2.9) to (2.12) were reduced to a system of first-order differential equations and solved using a MATLAB boundary value problem solver called bvp4c. This program solves boundary value problems for ordinary differential equations of the form $y'=f(x,y,p), a\le x\le b$, by implementing a collocation method subject to general nonlinear, two-point boundary conditions $g(y(a),y(b),p)$. Here p is a vector of unknown parameters. Boundary value problems (BVPs) arise in most diverse forms. Just about any BVP can be formulated for solution with bvp4c. The first step is to write the ODEs as a system of first order ordinary differential equations. The details of the solution method are presented in Shampine and Kierzenka[31].

4. RESULTS AND DISCUSSION

The governing equations (2.9) - (2.12) subject to the boundary conditions (2.13) are integrated as described in section 3. In order to get a clear insight of the physical problem, the velocity, temperature and concentration have been discussed by assigning numerical values to the parameters encountered in the problem. The effects of various parameters on velocity profiles in the boundary layer are depicted in Figs. 1-3. The effects of various parameters on Angular velocity profiles in the boundary layer are depicted in Figs.4-6. The effects of various parameters on temperature profiles in the boundary layer are depicted in Figs. 7-12. The effects of various parameters on concentration profiles in the boundary layer are depicted in Figs. 13-17.

Fig. 1 shows the dimensionless velocity profiles for different values of magnetic parameter (M). It is seen that, as expected, the velocity increases with an increase of magnetic parameter. The magnetic parameter is found to retard the velocity at all points of the flow field. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness increases with an increase in the magnetic parameter. The effect of material parameter (K) on the velocity is illustrated in Fig.2. It is noticed that the velocity increases with increasing values of the material parameter. Fig. 3 shows the variation of the velocity with the boundary parameter (n). It is noticed that the velocity thickness decreases with an increase in the boundary parameter.

Fig.4 illustrates the effect of magnetic parameter on the angular velocity. It is noticed that as the magnetic parameter increases, the angular velocity increases. Fig. 5 depicts the angular velocity with the material parameter. It is noticed that the angular velocity thickness increases with an increase in the material parameter. The effect of the boundary parameter on the angular velocity is illustrated in Fig.6. It is observed that as the boundary parameter increases, the angular velocity thickness increases.

The effect of the magnetic parameter on the temperature is illustrated in Fig.7. It is observed that as the magnetic parameter increases, the temperature increases. Fig. 8 depicts the thermal boundary-layer with the material parameter. It is noticed that the thermal boundary layer thickness decreases with an increase in the material parameter.

Fig.9 illustrates the effect of the boundary parameter on the temperature. It is noticed that as the boundary parameter increases, the temperature increases. Fig. 10 shows the variation of the thermal boundary-layer with the heat source/sink parameter (β). It is observed that the thermal boundary layer thickness increases with an increase in the heat source/sink parameter. Fig. 11 shows the variation of the thermal boundary-layer with the Prandtl number (Pr). It is noticed that the thermal boundary layer thickness decreases with an increase in the Prandtl number. Fig. 12 depicts the thermal boundary-layer with the conjugate parameter (γ). It is noticed that the thermal boundary layer thickness increases with an increase in the conjugate parameter.

The effect of magnetic parameter on the concentration field is illustrated Fig.13. As the magnetic parameter increases the concentration is found to be increasing. The influence of the Schmidt number (Sc) on the concentration field is shown in Fig.14. It is noticed that the concentration decreases monotonically with the increase of the Schmidt number. The influence of the chemical reaction parameter (Kr) on the concentration field is shown in Fig.15. It is noticed that the concentration decreases monotonically with the increase of the chemical reaction parameter.

Fig. 16 shows the variation of the skin friction with for different values of magnetic parameter and material parameter. It is observed that the skin friction decreases with an increase in the magnetic parameter and increases with increasing the material parameter. Fig. 17 depicts the variation of the couple wall stress with for different values of magnetic parameter and material parameter. It is observed that the couple wall stress decreases with an increase in the magnetic parameter and increases with increasing the material parameter. Fig. 18 depicts the variation of the Nusselt number with for different values of magnetic parameter and material parameter. It is observed that the Nusselt number increases with an increase in the magnetic parameter and decreases with increasing the material parameter. Fig. 19 depicts the variation of the Nusselt number with for different values of heat source/sink parameter and conjugate parameter. It is noticed that the heat transfer rate increases with an increase in the heat source/sink parameter and conjugate parameter. Fig. 20 shows the variation of the mass transfer rate with for different values of magnetic parameter and material parameter. It is observed that the mass transfer rate increases with an increase in the material parameter and decrease with an increase in the magnetic parameter.

For validation of the numerical method used in this study, results for heat transfer rate $-\theta'\ 0$ *of a Newtonian fluid* $\Delta=0$ *and γ=1, were compared with those of Salleh et al. [11] and*

Qasim et al. [12] for various values of Pr in the absence of magnetic field (M=0), heat generation (β=0) and chemical reaction (Kr=0). The quantitative comparison is shown in Table 1 and it is found to be in excellent agreement.

5. CONCLUSIONS

In the present chapter, steady, two dimensional flow of an incompressible, electrically conducting micropolar fluid over a stretching sheet with Newtonian heating by taking transverse magnetic field, heat source/sink and chemical reaction effects into account, is analyzed. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It has been found that

1. *The velocity decreases as well as the angular velocity, temperature and concentration increases with an increase in the magnetic parameter.*
2. *The velocity and the angular velocity increases as well as, the temperature decreases with an increase in the material parameter.*
3. *The heat source/sink enhances the temperature.*
4. *The chemical reaction reduces the concentration.*
5. *The skin friction reduces the magnetic parameter and increases the material parameter.*
6. *The heat source/sink reduces the heat transfer rate.*
7. *The chemical reaction enhances the mass transfer rate.*

6. GRAPHS

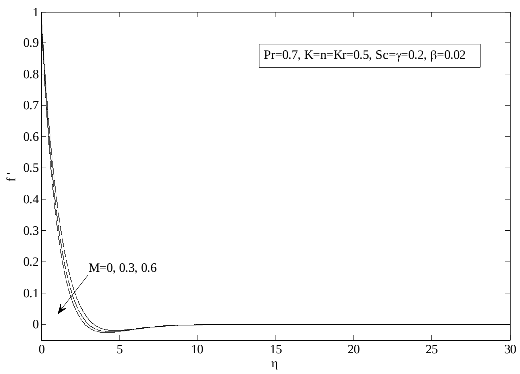

Fig.1 Velocity profiles for different values of M

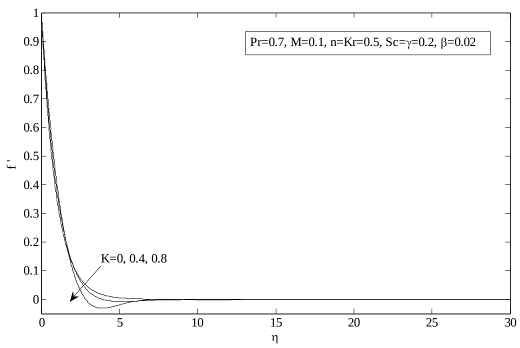

Fig.2 Velocity profiles for different values of K

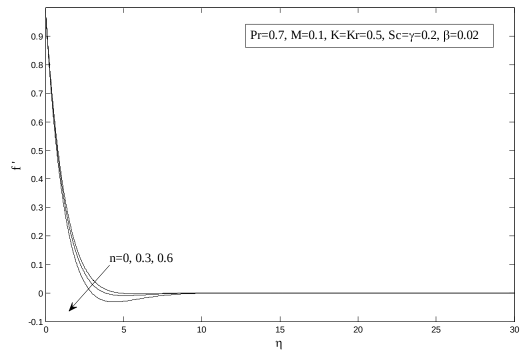

Fig.3 Velocity profiles for different values of n

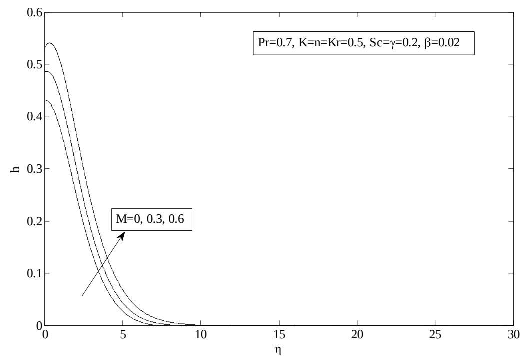

Fig.4 Angular velocity profiles for different values of M

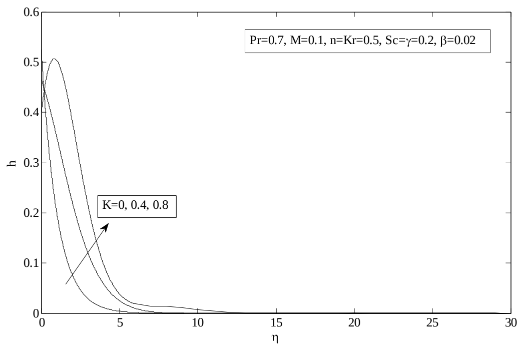

Fig.5 Angular velocity profiles for different values of K

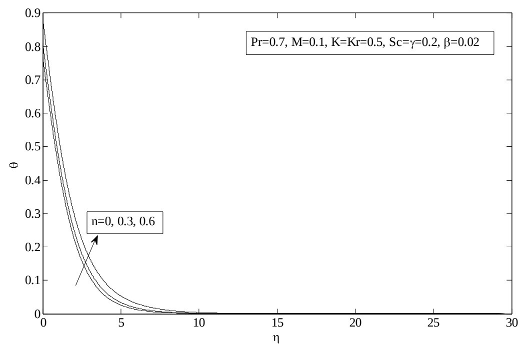

Fig.9 Temperature profiles for different values of n

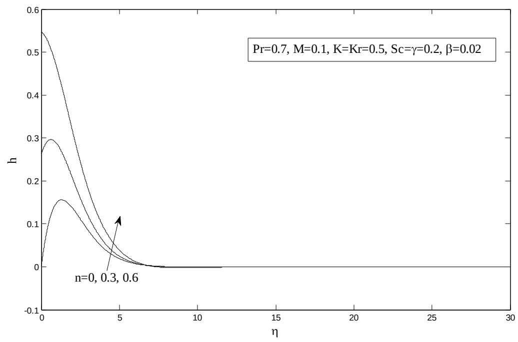

Fig.6 Angular velocity profiles for different values of n

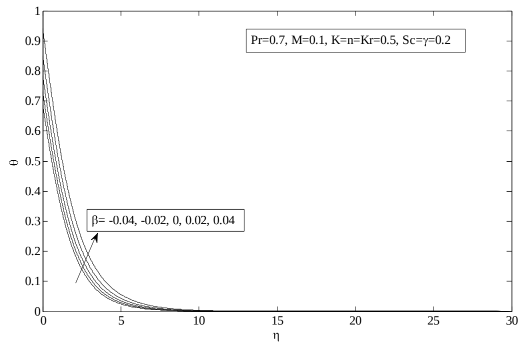

Fig.10 Temperature profiles for different values of β

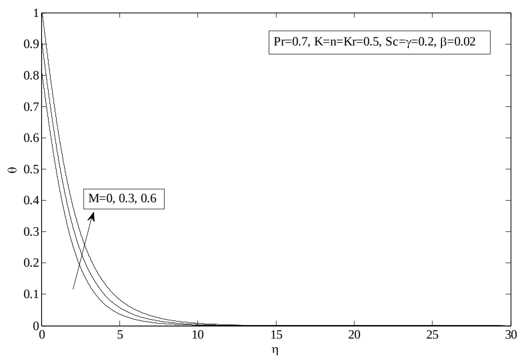

Fig.7 Temperature profiles for different values of M

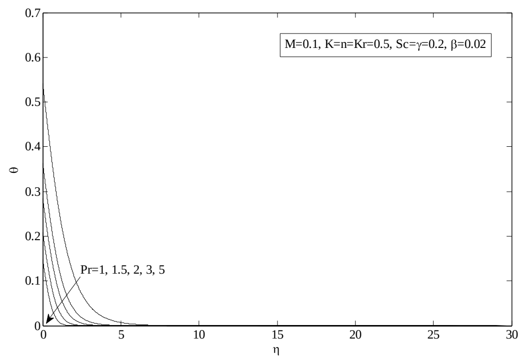

Fig.11 Temperature profiles for different values of Pr

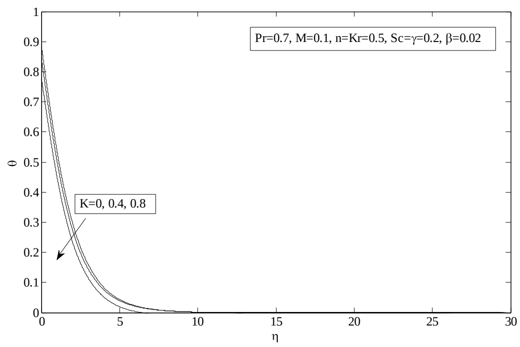

Fig.8 Temperature profiles for different values of K

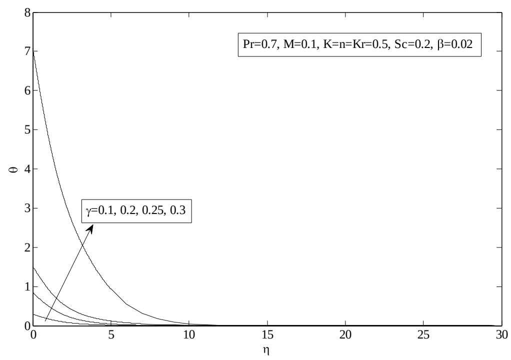

Fig.12 Temperature profiles for different values of γ

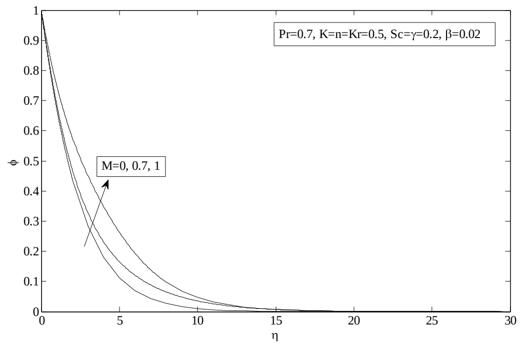

Fig.13 Concentration profiles for different values of M

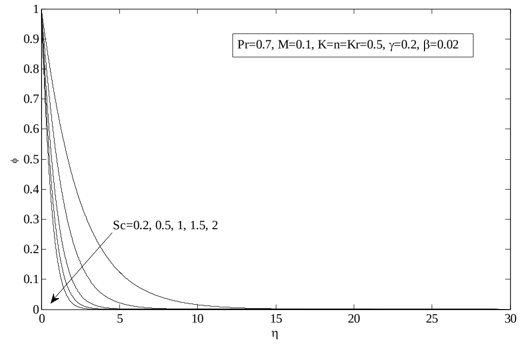

Fig.14 Concentration profiles for different values of Sc

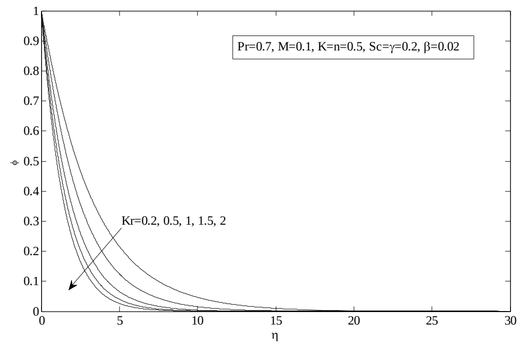

Fig.15 Concentration profiles for different values of Kr

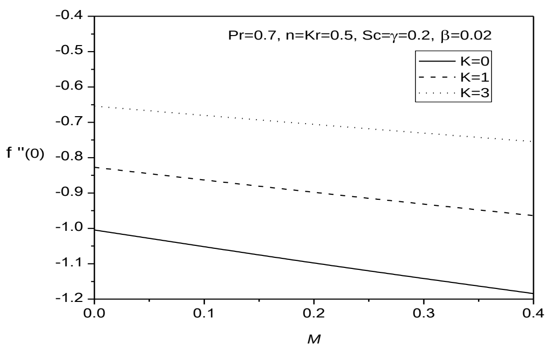

Fig.16 Skin friction profiles for different values of M and K

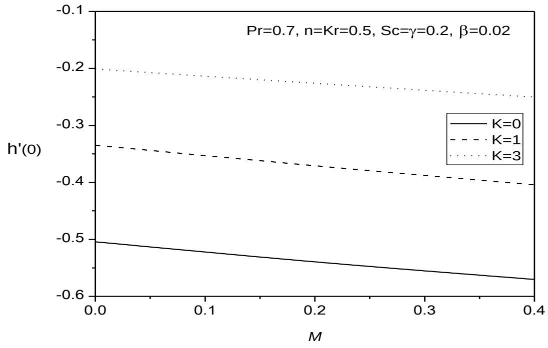

Fig.17 Profile of Couple wall stress for different values of M and K

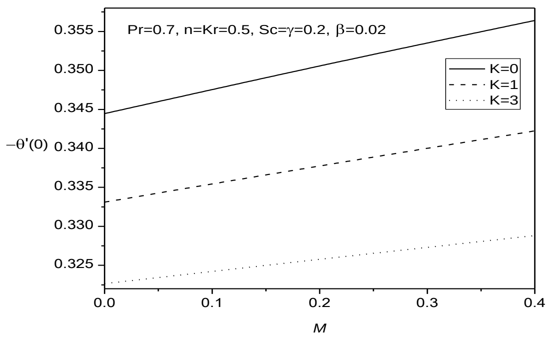

Fig.18 Profile of Nusselt number for different values of M and K

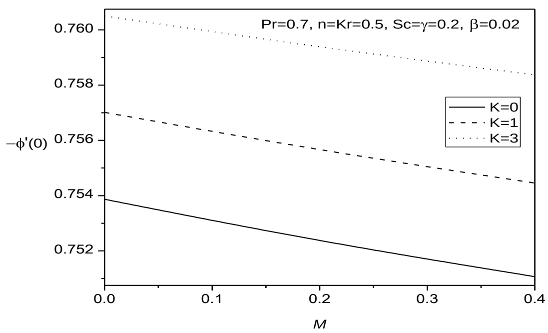

Fig.19 Profile of Sherwood number for different values of M and K

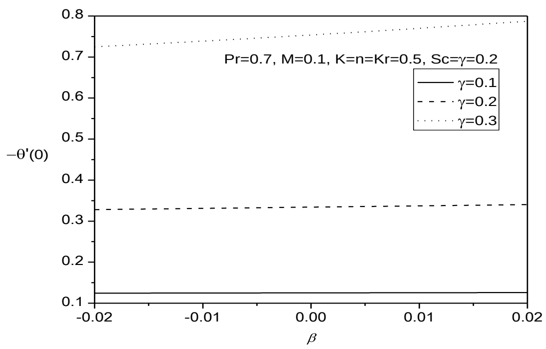

Fig.20 Profile of Nusselt number for different values of β and γ

Table 1 Comparison of $-\theta'(0)$ *for different values of Pr when K=Q=Sc=Kr=0, γ=1.*

Pr	$-\theta'(0)$		
	Present results	*Qasim et al.[12]*	*Salleh et al.[11]*
3	*7.051619*	*7.05168*	*7.02577*
5	*2.760395*	*2.76039*	*2.76594*
7	*2.116815*	*2.11682*	*2.13511*
10	*1.764524*	*1.76452*	*1.76531*
100	*1.147805*	*1.14780*	*1.16115*

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