Formation of adequate Investment Portfolio for Stochasticity of Profit Possibilities

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***Abstract**: The article focuses on the development problems of modern investment theories considering stochastic nature of investment profitableness. This is to be realised by offering investment portfolio adequate to the description of stochasticity of profit possibility, and motivating the scheme of its formation and use. Formation of adequate model and its practical use is based on computerised imitative technologies. Here are presented concrete examples of adequate model and the use of imitative technologies have been presented there. The work consists of 3 chapters: 1. "Formation of modern investment theory and development premises" focuses on the most reasonable results of this theory and actual problems that need to be solved. 2. "Opportunities and limitation of portfolio as the mean of investment management" indicates the importance of stochasticity of profit possibilities stressing on some investment issues incompletely solved by modern (classical) portfolio 3. "Formation and use of portfolio to evaluate adequate stochasticity of profit possibilities" presents the adequate concept and possibilities of practical adjustment. Portfolio description favourable for the stochasticity of profit possibilities is named after adequate model and geometrical picture of the portfolio geometrical model.*

***Keywords:** Financial Mathematics, stochastic, linear algebra, quantitative models, investment portfolio*

1. Formation of modern investment theories and assumption of development premises

The middle of 20^{th} century is considered to be the beginning of modern investment theory. Before this date it was possible to change the whole complex of former investment criteria, regulations and rules by one proposal – to buy cheaper and sell more expensive. According to modern investment theory each investment is measured by its profitability and risk, and the possibilities of each investment being inter correlated are analysed as their interaction – portfolio. These circumstances caused the necessity to realise the interrelation of all assets involved in portfolio determining the possibility of investment management as a whole. H. Markowitz's work about the formation and optimisation of investment portfolio is considered to be a start point of modern investment theory formation [1]. This work of investment switched one – sided standpoint of investment profitability to two aspects - investment profitability and its instability (risk) analysing their interdependence and separate investment in their interaction. It speeded up the formation stages of modern investment and portfolio theories. Right after H.Markowitz's "Portfolio selection" there were a great avalanche of publications on investing theory and practise. Of course, in its own the object of investment theory is of great importance while analysing equivalency of generation interaction. The right investment strategy influences by wellbeing of separate countries and mankind future. But the portfolio consideration has an indisputable impact on the processes mentioned. Portfolio theory as one of ways searching for investment decision overgrew the concept of investment portfolio and became constructive concept of systemic analysis in most spheres of research. The works of F. Modigliani M.H. Miller [4] and Rubinstein [5] attempt to revaluate the decisions of corporation finances therefore we select them according to portfolio ideology or modern investment theory.

On the way to modern investment theory there were some fundamental results that speeded up the formation of modern theory and determined the discretion for its further development, such as the results of Fisher's interest rate analysis [6] and J.B. William's theoretical fundamentals of investment value. It is essential to mention T.H.Knight's work, which stressed on non-deterministic processes developing the grounds of utility theory [8]. Further abundance of research and multi-aspectability limits the possibility to allot the results in consecutive order. Anyway, J. Tobin's works [9] spread the ideas of H. Markowitz and adapted them to macroeconomics while N.F.Sharp [10], [11], Lintner [11], J. Mossin [13] predetermine Markowitz's idea to be the main investment theory. At the same abundance of research and importance of results caused some doubts concerning the theory. Need to possess more common analysis of investment decisions as well as laws of social sciences imply more exceptions while this causes more doubts concerning some results of portfolio theory, focusing on adequacy of capital asset pricing model – CAPM. R.Roll [14] expressed his doubts concerning some results of portfolio theory and possibilities of CAPM verification. He suggested arbitrage-pricing model – APM [15] that refers to prerequisite that relation of profit and risk shouldn't let achieve the constant utility only from the arbitrage transactions. S. Ross and R. Roll, the supporters of arbitrage theory assume [16] that it is possible to check APM empirically. The main tools of arbitrage model are based on – the existence of efficient market which possibilities were actively analysed during the 7th decade [17], [15], [16]. Acknowledgement of efficient market hypothesis could be interpreted under the idea Black – School model [18], which refers to the possibility of non-risk transaction precondition when an option transaction is employed beside the basic assets. F.Black, M.Shloes and Merton's [19] started analysing the possibilities of portfolio profitability as the function of distributions of asset profit possibilities. There are lots of other doubts concerning the newest assumptions of modern investment theory as well as the whole finance theory, f.e., "chaos theory" which supporters asserts that the laws of modern finance (economics) theory are just the exceptions not the rules [20], or that there are inconsistencies between. It's the logic of the theory and practical issues. The reasons of objections should be found among the premises of point estimation, even where processes are described by probability theory. Tempting to realize, the origin of objection for the main assumptions of modern investment theory, or inadequacy of its conclusions and real facts, it's essential to determine the conditionality of premises about efficiency line. On the one hand, the existence of the efficiency line from the standpoint of intercourse between traditional portfolio risk and average profitability is mathematical – logical expression of any investment collection. But real existence of average portfolio profitability is equal to zero, in case any of portfolio asset profit possibility distribution isn't discrete. Thus, constructive analysis of the situation could be performed just leaning on the expression of efficiency line possibility distribution - efficiency zone that determines all possibilities of portfolio profitability maximum in their distributions achieved for every level of portfolio standard deviation (see fig. 2).

2. The main opportunities and limitations of portfolios as the means of investment management.

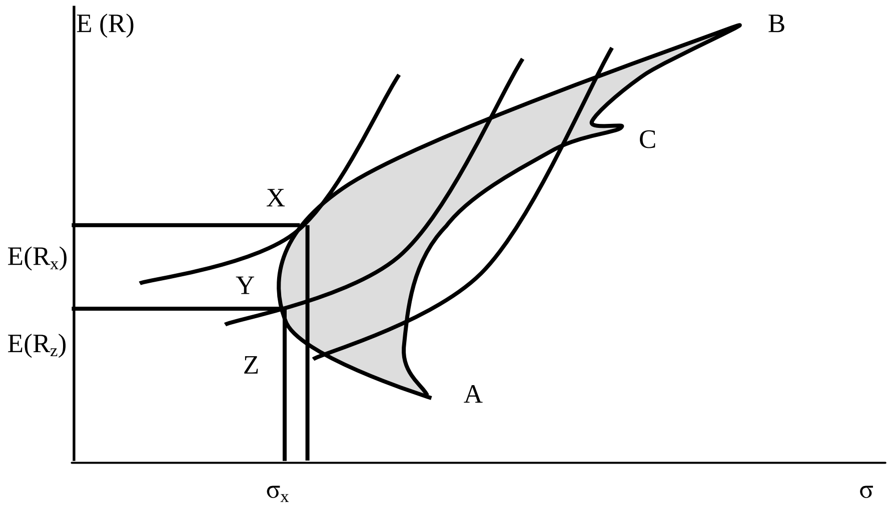
Portfolio is a set of various kinds of assets belonging to any of institutions or individual, which principles of formation are redirected to the employment of various kinds of assets and collection proportions seeking for the utility by the owner of portfolio. Financial portfolios are the collection of financial assets. The content of portfolio is considered to be of great variety. Besides assets portfolio, liabilities portfolio is possible or it can include both.

It is usual to say that, if the portfolio consists of A_1, A_2……A_n assets, we assume that the structure of portfolio is w_1, w_2….w_n (w_i>0, $w_1+w_2+\ldots w_n=1$) and its value $v=w_1a_1+w_2a_2+w_na_n$, when a_i is the value of asset i.

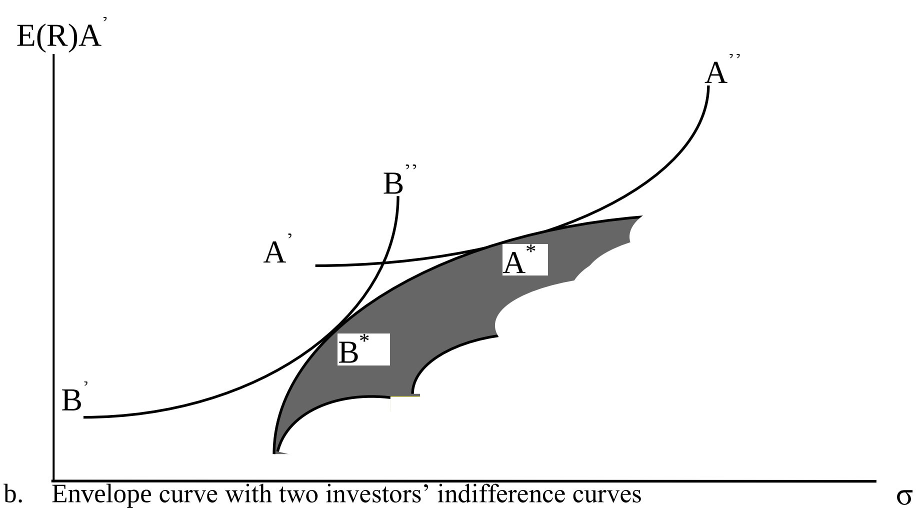
Theory of securities portfolio is system of knowledge where an investor could gain the highest expected profit from risk and non-risk collections of securities. The main issues by the theory of portfolio are determination of the whole complex of available portfolios, establishment of efficient portfolio line, and selection of optimal portfolio for each investor.

To understand the solution of arisen problems more easily it is essential to concentrate on the geometry surface of those problems reflecting their criteria of decisions in order to find better solution. It is usual to fix the average meaning of portfolio profitability on the ordinate axis, while the instability (risk) measure of the same profitability i.e., average standard deviation – on the abscissa. Thus, the average and deviation of the same probability distribution are set on separate co-ordinates. Having chosen the set of assets while knowing the meanings of their profitability and standard deviation, and assuming that each of assets could take the part altering from 0 to 1; we'll achieve the set of possible portfolios (see fig. 1). This is the way to identify the whole complex of available portfolios or the set of investor's choices.

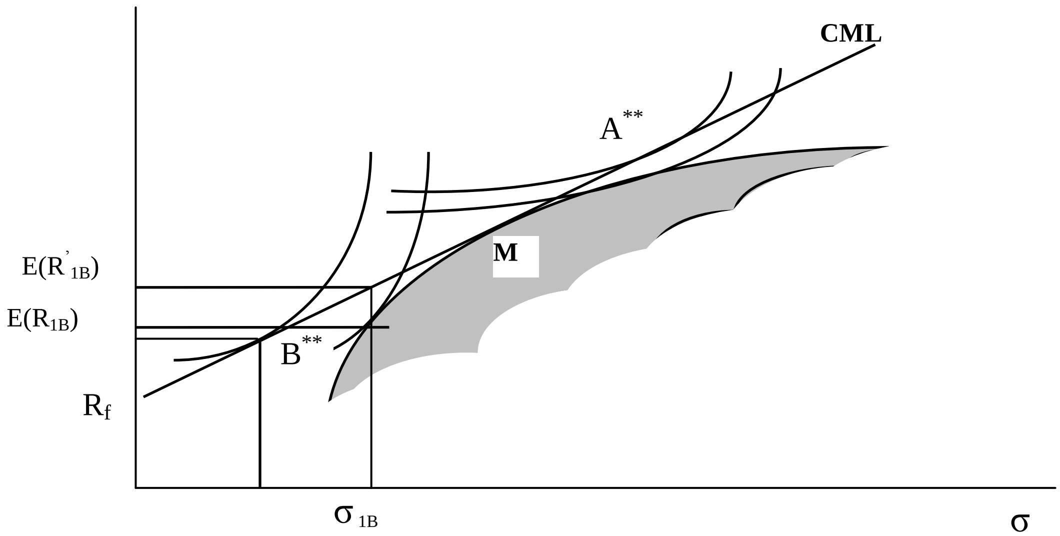
Mathematical random variables and characteristics of their weighted sums influence such form of available portfolios. Line YB is called efficiency line and is the part of convex curve AB. Those lines are considered to be of reasonable meaning while analysing separate characteristics of portfolio. Efficiency line is the highest profit average line of available portfolios possessing the proper level of risks.



a. Selection of the best alternative from available variants



b. Envelope curve with two investors' indifference curves



c. Capital market line (CML)

Fig. 1. Variants of the search for portfolio decision

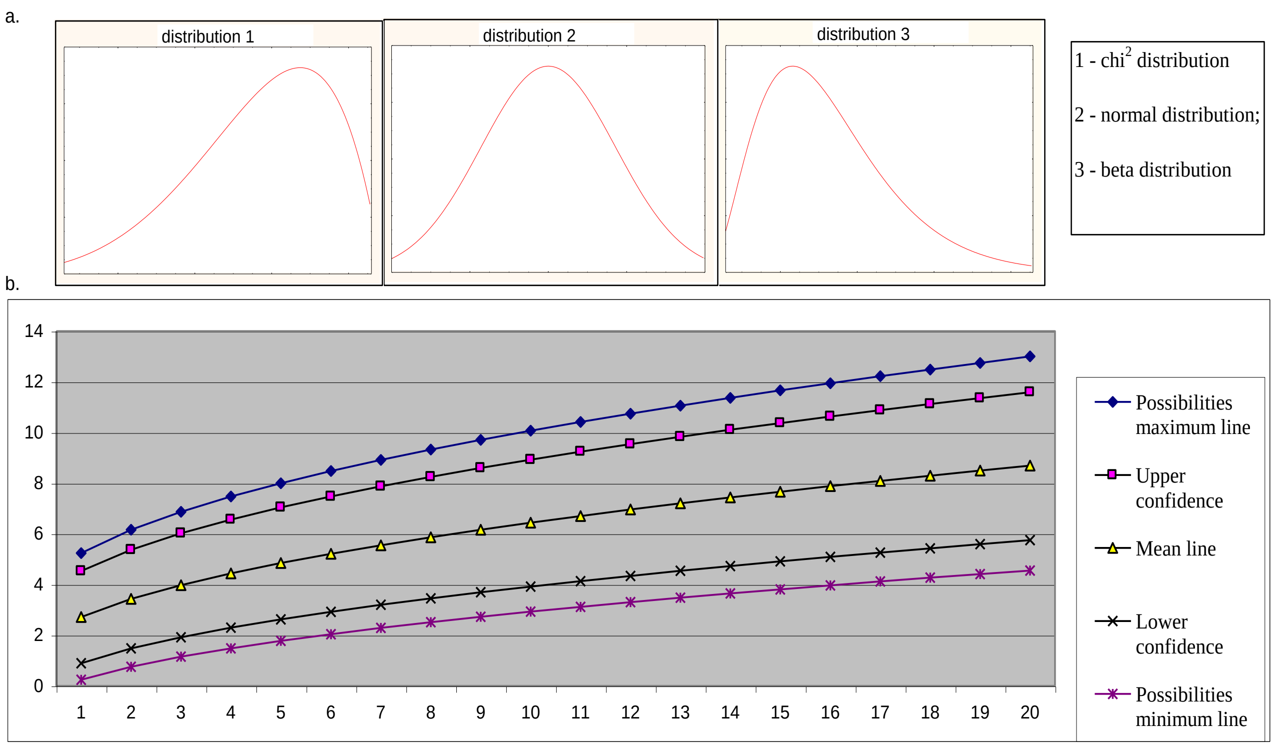


Figure 6. Common sight and main properties of the efficiency zone:
a. distributions of the highest profitability of possible portfolio set at given level of risk;
b. minimal and maximal boundaries, upper and lower confidencies and mean line.

Mission of the portfolio as the instrument of investment theory - the realization of investment set structure - $w_1, w_2,\ldots,w_n$ allowing to maximize the portfolio profitability at present risk level or minimize the risk at chosen profit level. For further analysis it is essential to refer to the allocation of general risks: systemic and non-systemic ones. If the portfolio possesses reasonable amount of

investments, it experiences systemic risk - that part which is experienced by the whole (country, region, and the world) economic system. The further analysis stresses emphasize the systemic part of general risks.
The last issue is the selection of optimal portfolio for the separate investor from the set of possible portfolios- the selection of portfolio on the efficiency line. It is important to note that in the portfolio theory the investments (their set) are analyzed regarding the investor's utility. That is the distinguishing characteristic of portfolio maximization criteria. The modern portfolio theory uses the simplified change of utility function - indifference curves. This concept came from the consumption theory where it determines the combination of two goods equally useful for the consumer. In the portfolio theory it expresses the equal reasonable combination of profit and risk for the investor. Figure 1 show the way the investor should select the most useful portfolio regarding the set of available portfolios and his function of utility (indifference). Figure 1b. Illustrates the way the investors possessing different indifference curves A'A" and B'B" choose separate optimal (maximising their utility) portfolios at the same maximising general utility of investment formation.

It is obvious that such logic of portfolio optimisation justifies in case the portfolio involves only risky investments. In this case, its decision belongs to the efficiency line and simultaneously in the network of indifference map. That is the point X in fig. 1 and the points A* and B* in fig. 2a. Therefore, the premises of risky investments do not show the investor's possibilities in the real world of investments involving non-risk securities, such as government bonds. In this case, the investor is trying to achieve the higher utility, the one in the points A* and B*. Those are the points A** and B* indicated in the investors' indifference maps (see ex. [22], [24], [26], [27]). It is illustrated in fig. 1c.

Rate of non-risk short-term government securities is indicated at the point R_f . Under certain circumstances, any investor could allocate his means as the risk investments at the point M on the efficiency line, and non- - risk securities - at point R_f. There is a rectilinear expression:

$$\omega_m M + (1-\omega_m)B$$

Where M - parameters of risk investments
B - Parameters of non-risk investments
W_f - part of risk investments.

All points achieved as rectilinear combination of risk and non-risk securities lay on the straight line connecting the points R_f and M. If the investor prefers the lower risk, this position shifts closer to R_f; if his priority is given to the higher risk, the position approaches above the point M. Let's go back to the investor B in order to realise his choice. If there is no possibility to invest in non-risk assets, the investor chooses portfolio B* with risk σ_{1B} and higher profit E (R_{1B}). In case there is a possibility to acquire non-risk asset, the investor B can combine his choice between M and B so that he could gain higher profit E (R'_{1B}) while experiencing the same risk. Actually, the investor B while leaning upon his indifference map, should prefer the point B*, where the profit decreases but the risk is reasonably lower. Assuming the latter assumption, the investor prefers this position (a little bit lower profit but reasonably lower risk) because indifference curve crossing line R_f M at the point B** is indifference curve of higher level than the one crossing the point B*. The same could be seen by graphical view of investor's indifference curves. Let's go back to investor A. He takes the higher growth of risks. His indifference curve shifts his choice to the right of point M. he doesn't allocate his money between M and B. Instead, he borrows for non-risk price R_1 , and invests the disposed means in the way he could reach the point A**.

As far as we know, the straight line is named as capital market line (CML). This separation of risk and non-risk capital causes the result known as J. Tobin's separation theorem [9].

Let's analyse the portfolio M. the investor A as well as investor B uses the portfolio of risky asset. Realising the advantages of this portfolio, any of investors will include this portfolio into the combination of his investments. It's the only equilibrium portfolio experienced by any of risk adverse investor. That means that in case of two investments of the same profit but different risks, the priority will be given to the lower risk. Thus, the portfolio M becomes the market portfolio and, in case of equilibrium, it should involve all available risk investments in the way of proportional ones.

3. Formation and Use of Portfolio Adequate for the Evaluation of Stochasticity of Profit Possibilities

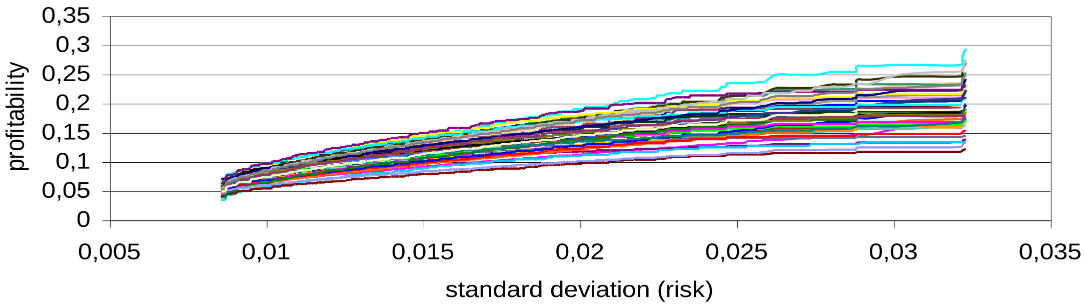
3.1. Development Opportunities and limitations of Traditional Portfolio Analysis Outline

Classical scheme of portfolio analysis, management or other use is convenient for practical utilisation. Functional expression of efficiency line and envelope curve one should be used in practical portfolio application, but it is not obvious in general case. Formation and management of portfolio requires effective evaluation of various portfolio conditions existing on the efficiency curve, description of their interaction, or analysis of other portfolio characteristics. Portfolio decisions should be achieved when it is impossible to describe the profit possibilities of portfolio as point estimated but as their distribution of probability. It is essential to turn to geometric model of portfolio (geometric picture). In case of traditional portfolio, expectation (averages) of investment profit as random quantity is set on ordinate axes while their standard deviations are set on the abscissa ones. That is an obvious geometric illustration of the main portfolio analysis results. Geometric evidence wouldn't disappear if we used rectilinear functions instead of expectations or standard deviations, f. e. we shifted them along the abscissa and/or ordinate axes. Considering the stochasticity of portfolio investment profitability, average value of portfolio profit is not the most appropriate indicator. The expected or average profitability is the generalised condition of profit possibilities for the real period. However, this is only one of the set of possibilities, usually without paying such a big attention as, let's say, quintile of a certain (95%) level etc. In every concrete case, the concrete profit will be one of appropriate profit possibilities described by the distribution of their probability. The necessity of interpreting portfolio profitability as the random quantity is confirmed by the circumstance that the price of separate investments (bonds, securities, projects, etc.) and portfolio is the random quantity in the market. Thus, the whole picture of portfolio profit possibilities is possible requesting logic of random quantity as the most adequate financial-mathematical model of the profit.

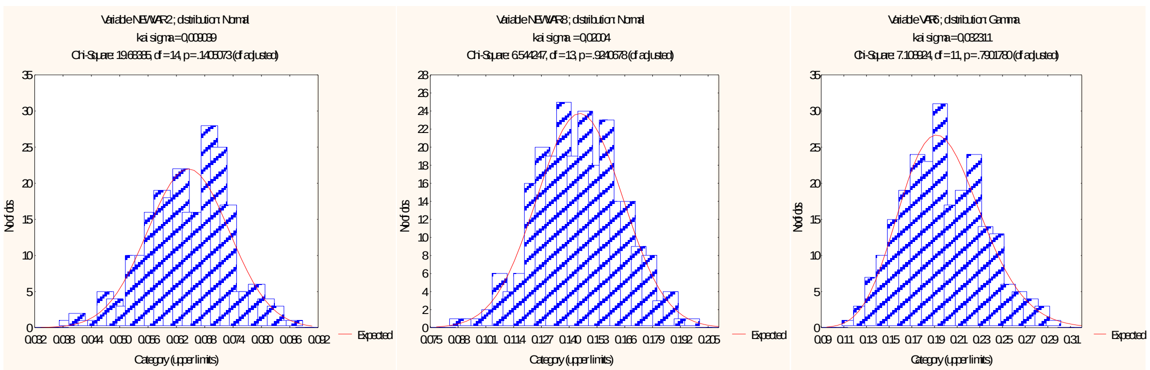
Interpreting investment portfolio in classical way and leaning on the concept of average profit, the problem of profit adequacy management arises. Considering the profitability average in the future, it is possible to foresee and describe it under the categories of random quantity. Such description of portfolio profitability explains the interaction of risk as the instability of profit possibilities and investor's function of utility that is essential striving for systemic risk evaluation and formation of its adequate management model. System of co-ordinates, the one geometric model of portfolio is analysed in, is considered to be the determined one: on the ordinate axis there are averages of random quantities (processes) or other rectilinear functions of available values while the abscissa axis implies average standard deviations or their rectilinear functions. Thus, the facts analogic to classical (traditional) portfolio such as the set of available portfolios, efficiency lines, existence of envelope curve and their characteristics are true for every possibility of portfolio profit. Considering the possibilities of portfolio profit, it is essential to analyse the entire efficiency zone instead of efficiency line (see fig. 2). Thus, the further analysis of portfolio should be switched from the co-ordinate system of portfolio profit standard deviation and the expected values (averages) of this profit to the more complex and adequate one where the abscissa axis implies average standard deviations of portfolio profit, and the ordinate axis indicates the efficiency zone (fig. 2b. or fig. 3a.), distributions of all available portfolio maximum profitability (fig. 2a. or fig. 3b.), utility (response) functions of available investors (fig. 3c.), and the values of created utility (fig. 3d.). For more adequate comprehension complex schemes of investment portfolio risk analysis (fig. 3) should be compared to the outlet of modern portfolio analysis. According to classical theory of modern portfolio, the investor should be interested in the portfolios located on the efficiency line. The efficiency line could be realised as the whole complex of maximum expected profit (averages) acquired for concrete standard deviation of portfolio set. In classical scheme the set of available portfolios could be formed joining present investments to portfolio in all possible proportions, in this way evaluating profit average (expected value) and standard deviation for each of formed portfolios. Actually, the investments are observed and realised by some possible values defined by investment market and price. That's why investors should face the entire set of portfolio profit possibilities. He is interested in the whole efficiency zone, which is realised as the whole complex of efficiency lines. In this way the analysis of efficiency line that involves portfolios possessing maximum average switches to the analysis of efficiency zone. In its turn investors' indifference curves should be changed (enlarged) by utility functions. This creates the complex picture of portfolio risk analysis (fig. 3) where the set of possible values of investment portfolio risk is connected to distributions of portfolio profit distributions (fig. 2a. or fig. 3b.), functions of portfolio owner (recipient of portfolio risk) response (fig. 3c.) and possibilities of portfolio utility for the region or country (fig. 3c.).

Summarising this stage, it is essential to underline the main differences determined by use of classical (modern) portfolio theory and adequate portfolio theory. Let's present them by table:

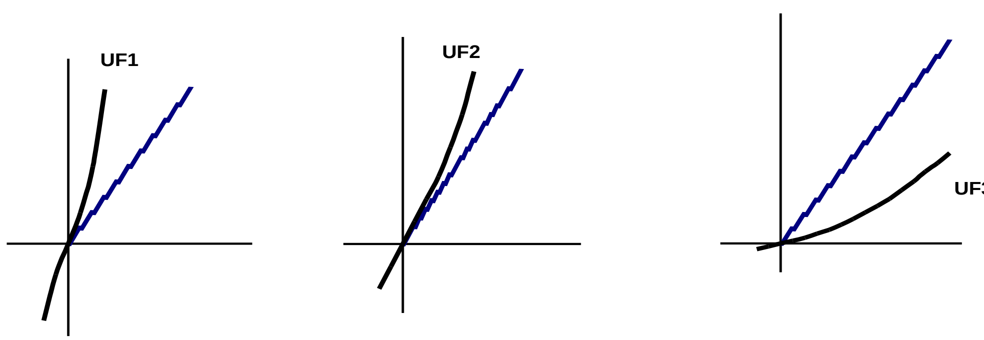
Classical portfolio theory	Adequate portfolio theory
• Determines the efficiency line where the existing portfolios possess maximum expected (average) profitability among the given portfolios of riskiness • Each investor's indifference curve allows the choice of portfolio where the investor is able to gain the maximum of average profitability	• Determines the efficiency zone where each level of possible portfolio risk possesses the distribution of maximum possibility probability • Each investor's utility function could experience such level of risk and distribution of the highest possibilities that maximise the investor's utility.



a. Efficiency zone



b. distributions of the highest profitability of possible portfolio set at given level of risk



c. functions of possible investors' utility

U1=u(UF1;p1) **U2=u(UF2;p2)** **U3=u(UF3;p3)**

d. utility created by investments

Figure 7. Complex picture of portfolio risk analysis:
a. efficiency zone;
b. distributions of the highest profitability of possible portfolio set at given level of risk;
c. functions of possible investors' utility;
d. utility created by investments.

- **Imitative picture of functional-numerical portfolio analysis**

Realisation of complex analysis of investment portfolio risk requires multiple analytic methods including multiple logic operations and optimisation algorithms. This refers to the determination of efficiency zone. The complexity is also illustrated by the problems faced describing efficiency line. When the possibilities of separate asset profitability submit to normal distributions and it is possible to interpret them as independent ones, the determination of efficiency line causes the selection of gravity weights $w_1, w_2, \ldots, w_n$ (when $w_1, w_2, \ldots, w_n = 1$) maximising rectilinear form $w_1a_1 + w_2a_2 + \ldots w_na_n$ where $a_1, a_2, \ldots a_n$ - averages of corresponding asset profits.

This simplicity shows up for the average of normally distributed quantity and standard deviation is not interdependent quantities.

Actually, integral I

$$I = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x-a)^2}{2\sigma^2}} dx = 1,$$

is equal to one in the presence of any value of a and σ, and average of asset as the random quantity $M\xi = a$ and variance $D\xi = \sigma^2$.

Anyway, this simplicity should disappear when the distributions of portfolio asset profitability possibilities become more complicated. F. i., when those distributions are lognormal - distribution (density) function possesses the form:

$$p(x) = \begin{cases} \frac{1}{xS\sqrt{2\pi}} \exp\left(-\frac{\ln x - m}{2S^2}\right) & \text{when } x>0 \\ 0 & \text{when } x\leq 0 \end{cases}$$

then

$$M\xi = e^{\frac{S^2}{2}+m},$$

$$D\xi = e^{S^2+2m}(e^{S^2} - 1)$$

or

$$D\xi = (M\xi)^2 \left(e^{S^2} - 1\right).$$

There is non-rectilinear dependence between the average and variance. When this dependence is particularly complicated there are some distributions, and the solutions of discussed objects are multiple ones. Turning to the variety of dependence of separate assets, it becomes obvious that the discovery of efficiency line is the complex issue as well. The complexity doesn't decrease while determining efficiency zone.

In most cases the discovery of portfolio analysis and management decisions is inefficient if the possibilities of imitative technologies are not used for the search of solutions ([28], [29]. Figure 4 presents general system of portfolio set analysis and management using imitative technologies. This system - computerised functional quantitative and imitative numerical models devoted to the analysis of investments, their individual characteristics and their interaction; characteristics of investment portfolios; the main characteristics of possible investment portfolio set - efficiency zone; investment portfolio risk, possessing the characteristics of real investment portfolios and permitting to fulfil all provided logic operations while selecting portfolios with the required characteristics.

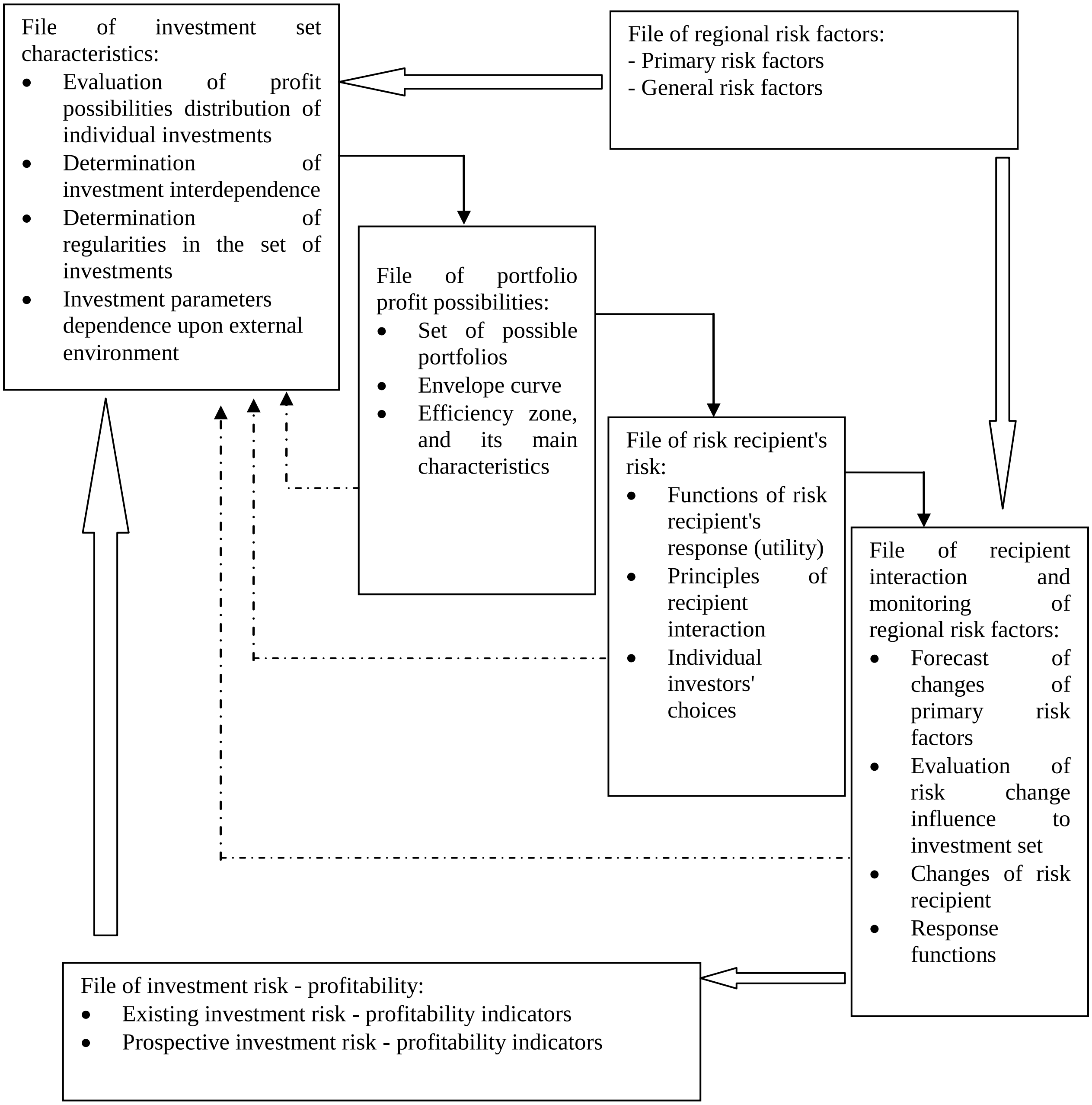


Fig 4 General picture of investment portfolio analysis and generation of management information

Figure 2 shows that efficiency zone - set of distributions changing upon the alteration of portfolio risk and describing the distributions of possibilities of the highest profit at given variance. While most activities (stock exchange, conclusion of transactions, etc.) require instant acceptance of decisions, it is obvious that in all cases it should involve computerised decision making systems. There is no other alternative except the use of imitative technologies in order to solve the objects of portfolio evaluation and management Scheme of figure 4 describes this methodise.

Summarising the goals and possibilities of investment portfolio set analysis (fig.4) and complex investment portfolio risk analysis, it is to state that the result of the application of those pictures could be the following:
Ranking the possible investment portfolio set by the interaction of profit and risk;
Selection of the best portfolios considering individual characteristics of risk recipients;
Evaluation of primary risk change influence to the portfolio risk**.**

3.2. Evaluation of the influence of primary risk factors to the portfolio risk

The concept of primary risk factors was presented analysing the necessity and problems of formation of regional business risk monitoring [30]. The attempt was to distinguish and systemise the primary factors of regional business risk. The primary (basic) factors of risk do not depend on the other factors of risk, determine other factors of risk to the proper group of objects, and are stipulated by general conditions in the analysed region. Analysing financial and business risk, primary factors of risk included the risks of exchange rate, market and capital expenditures, and probably the most important - interdependence of separate investment profitability.

Later statistical dependence of financial events, factors and processes is described as primary factor of the risk. It is important in four aspects. The first one – level of statistical dependence of two parts of the system (sub-system) impacts the stability (risk) of the whole system behaviour. The second aspect is that statistical dependence named after the factor of risk corresponds to the image of primary risk factor. The third one – management of most financial processes is based on the management of separate process dependence. And the fourth aspect – dependence of separate assets or collections should be realised as non- – determined measure. While analysing finances and investment probability theory and mathematical statistics presents unlimited possibilities of dependence and its influence to other processes.

Thus, analysing the influence of primary risk factors to the riskiness of possible investment portfolios, we should stress the influence of separate investment dependence level change to the riskiness of investment portfolio. In this case, figure 4 – general picture of investment portfolio analysis and generation of management information using computerised imitative technologies explains general logic and sequence of portfolio analysis and management run but doesn’t lead to an understanding of concrete analytic decisions and the way imitative (numerical) technologies make the decisions of complex analytic issues (goals). The power of imitative (numerical) technologies as well as the sequence of decision was presented at fig. 5 by schematic picture of search system of imitative investment portfolio decisions. Some concrete questions of imitative technology use are analysed at works [28] and [29].

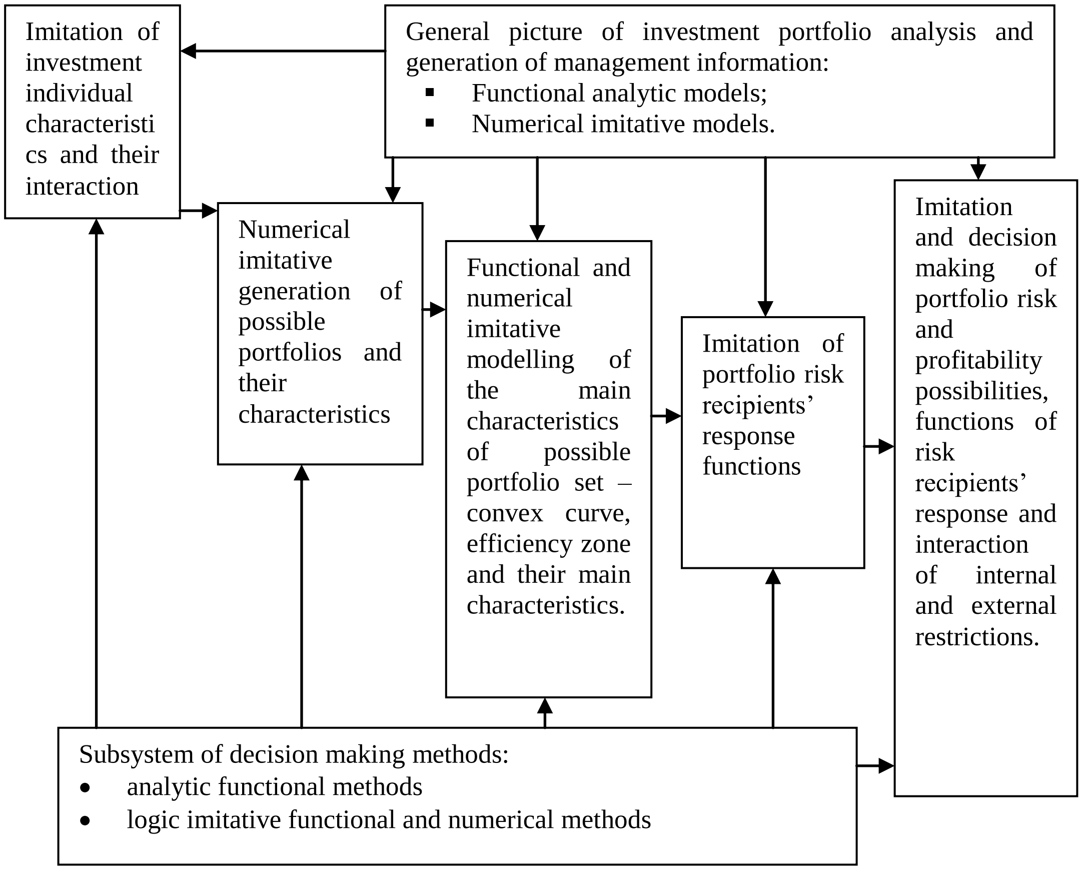


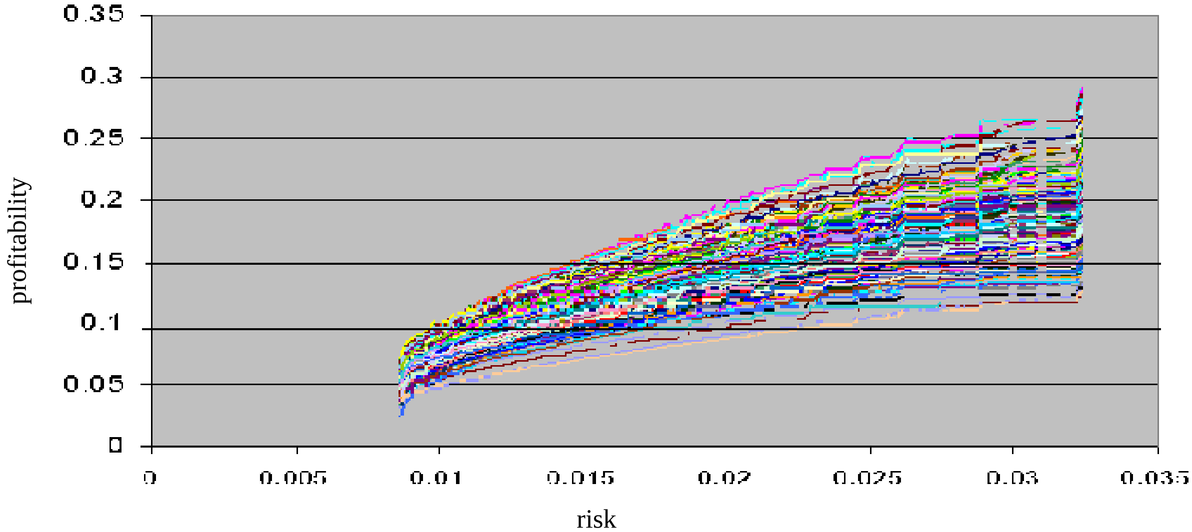
Fig. 5 Imitative system of investment portfolio decision search

- **Concise analysis of adequate model practical utilisation**

Practical application of suggested adequate portfolio model and created imitative technology of its numerical decision involved the assets of Lithuanian security exchange, the data about the profitability and variability (risk) of separate shares. The whole complex of shares was split in order to form interdependent groups of assets. It was impossible to split the whole complex of shares. Having refused the part of shares, the rest one was split into 4 statistically independent groups. Their correlation matrix C_{ij} became close to diagonal one:

$$C_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Those independent collections of general shares required the evaluation of profitability and risk (instability) and were accepted as the assets constructing the possible set of portfolios. Analysing imitative technologies it was easier to determine efficiency zone (fig. 6a.) and its main characteristics: probability distributions (fig. 7a.) of the highest portfolio possibilities at the given risk of possible portfolio set; the highest and lowest limits of the highest profitability possibilities, a.



b.

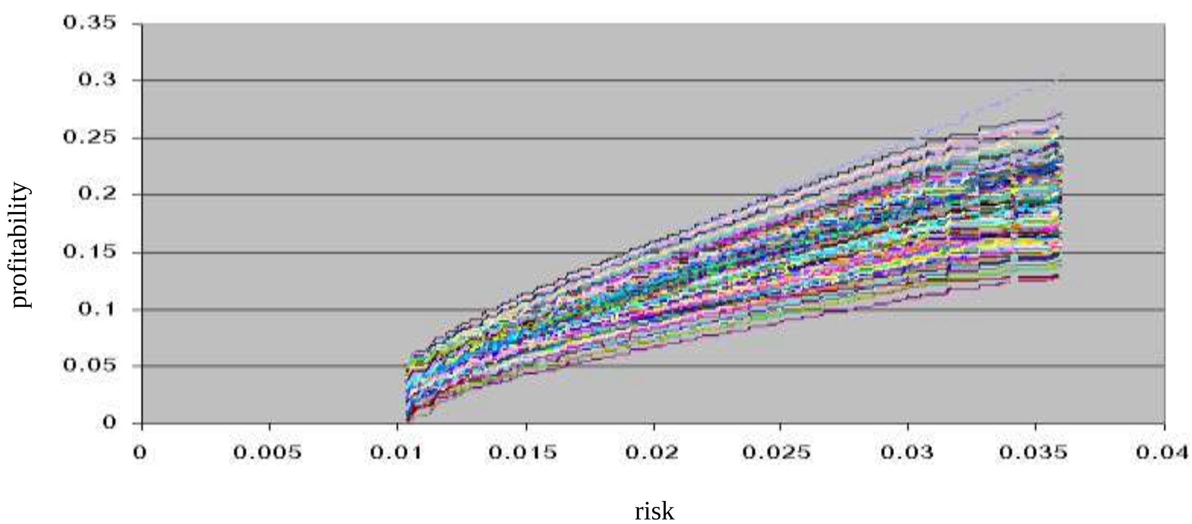


Fig. 6. Efficiency zones:
a. when investments are independent - $c_{ii} = 1$; $c_{ij} = 0$;
b. when investments are depended - $c_{ii} = 1$; $c_{ij} = 0{,}95$.

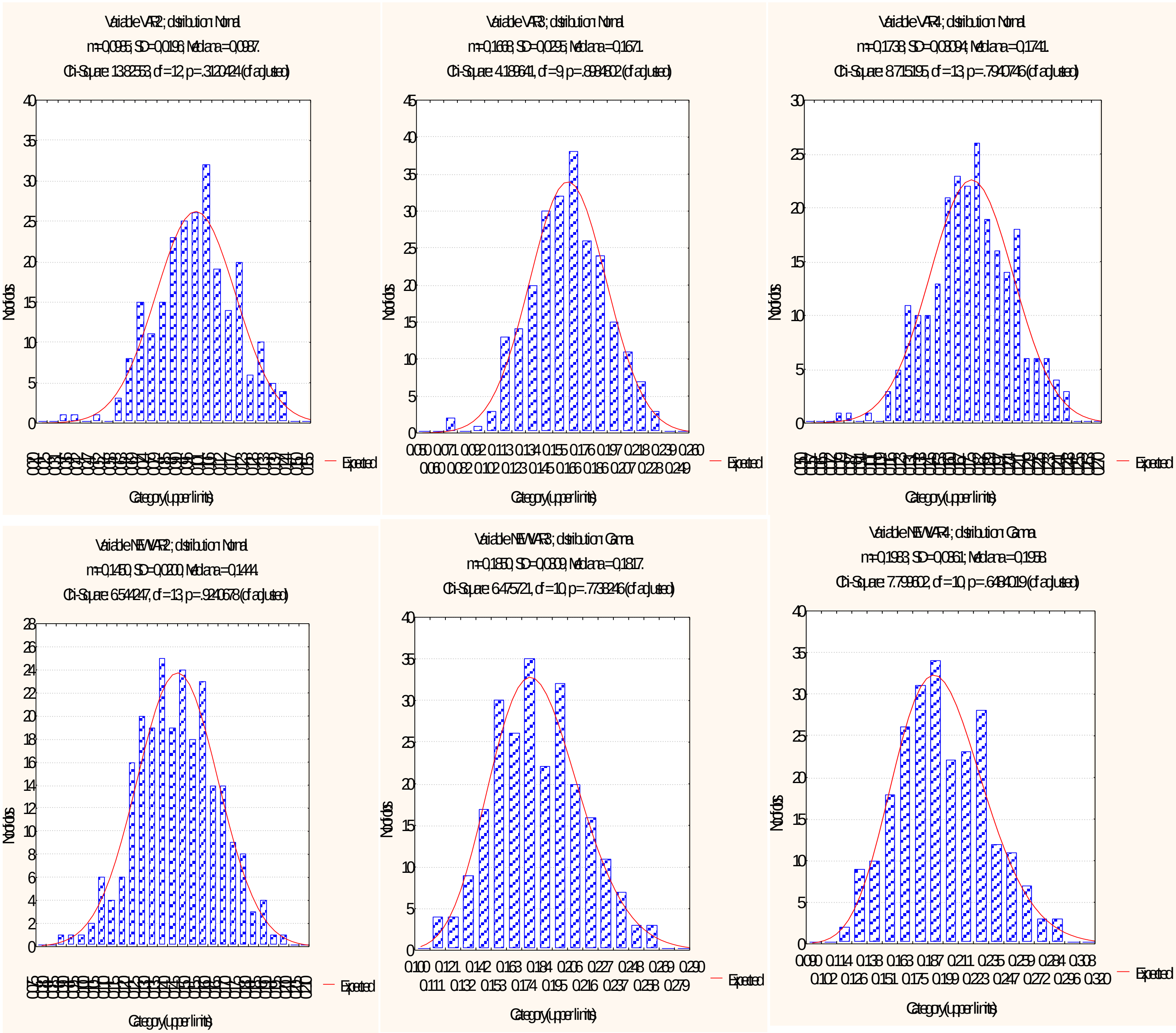
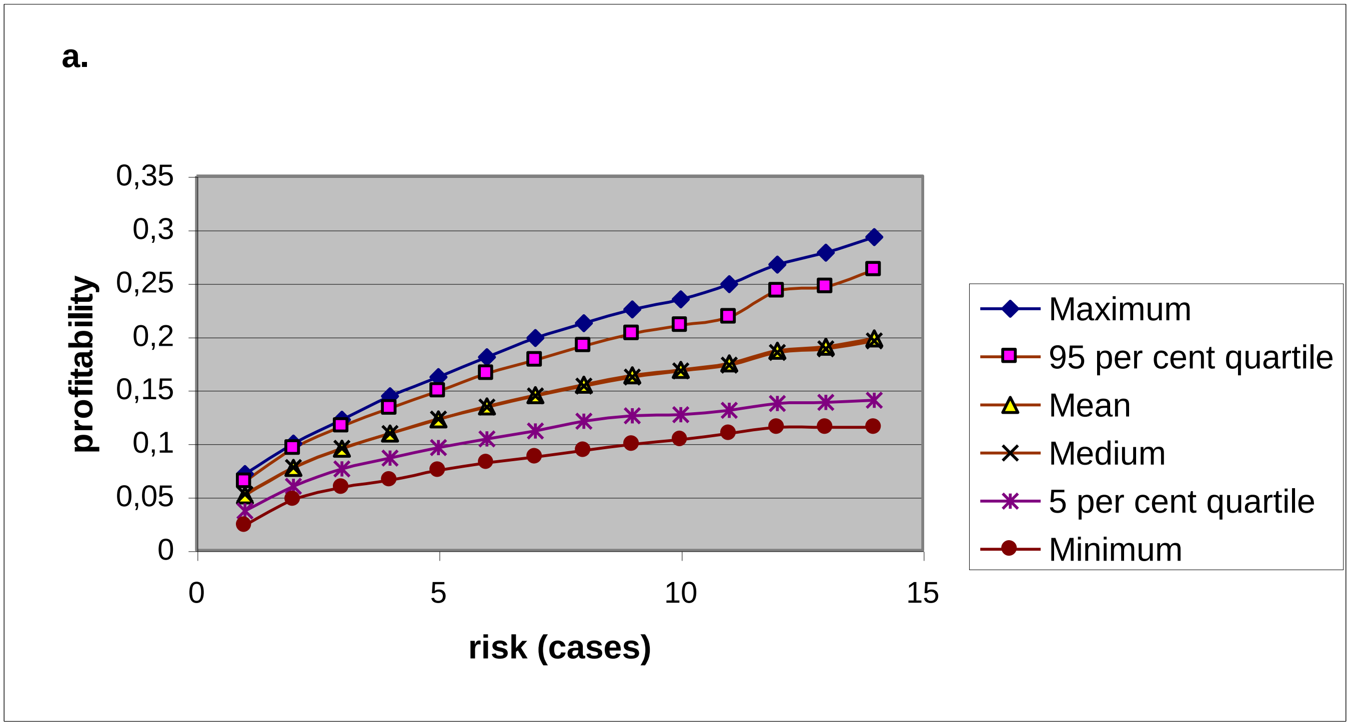


Fig. 7. Distribution of the highest profit possibilities of portfolio set at the given level of set risk



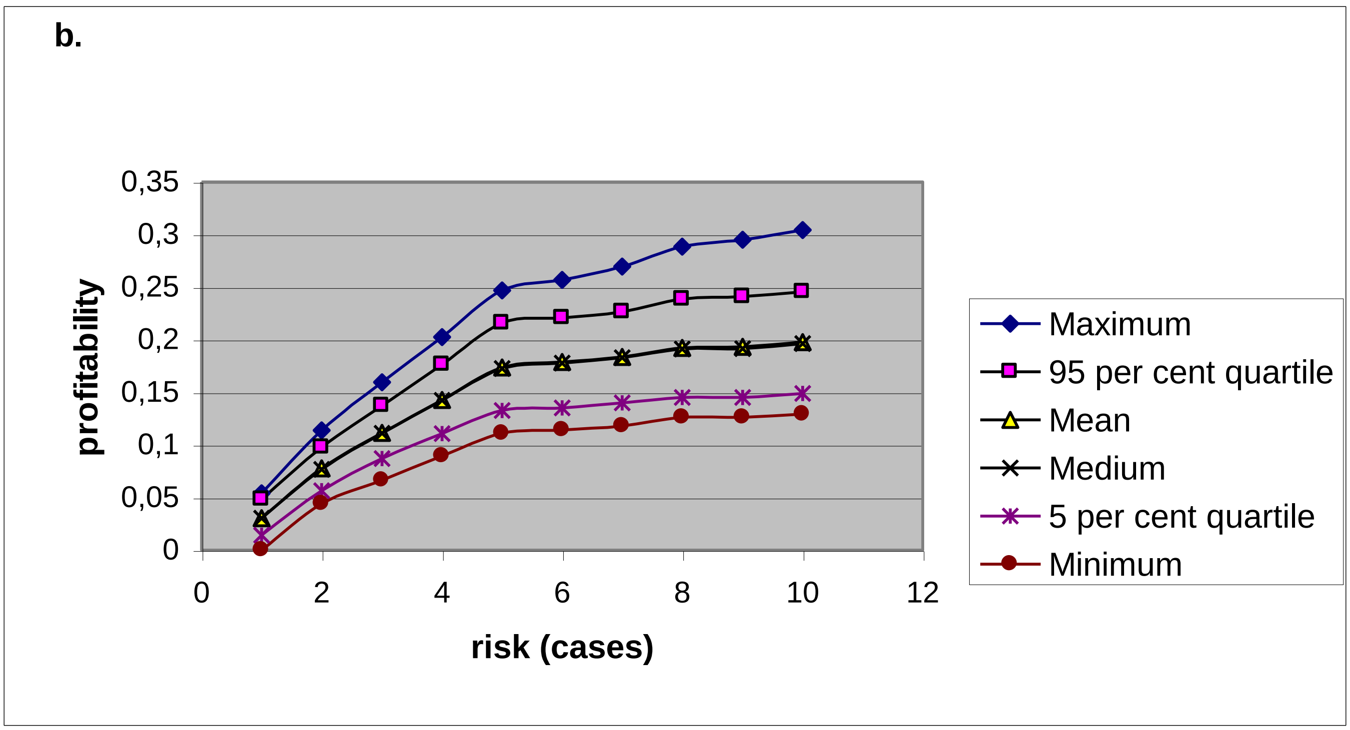


Fig. 8 The main characteristics of efficiency zone:

a. when investments are independent – $c_{ii} = 1$; $c_{ij} = 0$;

b. when investments are dependent - $c_{ii} = 1$; $c_{ij} = 0{,}95$.

5% and 95% reliance lines, curves of average and medium values (fig. 8a.).

For the evaluation of the influence of dependence increase (or decrease), it was assumed that there is statistical interdependence among the distinguished groups. The data was transformed in the way the profitability averages and medium quadratic deviations should remain the same changing the former correlation matrix as follows:

$$C_{ij} = \begin{pmatrix} 1 & 0,95 & 0,95 & 0,95 \\ 0,95 & 1 & 0,95 & 0,95 \\ 0,95 & 0,95 & 1 & 0,95 \\ 0,95 & 0,95 & 0,95 & 1 \end{pmatrix}.$$

Actually, it was possible to distinguish the groups possessing such correlation matrix and select separate assets possessing the same interdependence. Nevertheless, efficiency zone and its main characteristics were evaluated, and the above mentioned matrix appeared among the groups defining their interdependence.

In case of independent groups, we determine the efficiency zone and its appropriate characteristics. The results are presented in fig. 6, 7 and 8b.

Before starting quantitative analysis of the results, we should assume that we didn't dispose powerful computers necessary for the solution of such tasks in high precision. The obtained results are entirely satisfied and on their ground there are some assumptions:

- Efficiency zone while appearing the dependency among the distinguished groups shifted to the "south-east" direction - descended down and shifted to the right (compare fig. 6a and b.)
- Distributions of portfolio profitability possibilities:

a. Held (the normal were left) a stable type in the case of independent assets but transformed from the normal form to Gamma while increasing the risk of portfolio set (compare fig. 7a. and b.)
b. Standard deviation grew increasing the variance of possible portfolios in the case of independent and dependent assets although in latter case the growth of standard deviation was slower.
c. The dimensions of efficiency zone (the difference between the highest and lowest value) and average standard deviations were larger in the case of dependent assets. It is understandable with regard to general probability theory. But the obtained solution confirms the comprehensible truth and allows the accomplishment of quantitative evaluations.

- **More on interest rates:**

A. "Yield" for a bond

Let's represent a bond as a cash flow sequence, (c_i, t_i) for i = 1, …, n. (One of these pairs represents the final payment, and the rest represent coupons.)

The price of the bond is necessarily

$$V = \sum_{i=1}^{n} c_i P(t_i) \qquad (0.1)$$

where P is the present value function. (This must be true by the no-arbitrage assumption, even if you got P from some other risk-free instrument like STRIPS. If 1.1 didn't hold, somebody would find a way to buy STRIPS and sell bonds or vice versa, and make a guaranteed profit.)

The *yield* on the bond, which we will call $\bar{r}$, is the unique number that satisfies the equation

$$V = \sum_{i=1}^{n} c_i e^{-\bar{r}t_i} \qquad (0.2)$$

(In case it's hard to read --- that exponent is $-\bar{r}t_i$.) That is, if there were a constant interest rate, it would have to be $\bar{r}$ in order to be consistent with the price of the bond.

How do we know that 1.2 has a unique solution for $\bar{r}$? It's because the right side is strictly decreasing in $\bar{r}$ and ranges from (when $\bar{r} \to -\infty$) to $-\infty$ (when $\bar{r} \to +\infty$). So by continuity, the right side must be equal to V for some value of $\bar{r}$, and by monotonicity, it can only happen once. (In theory $\bar{r}$ might have to be negative, but in practice V is never large enough to make that necessary.)

How do we compute $\bar{r}$? In general there's no better method than trial and error. (Well, if you have time to write the computer code, Newton's method works pretty well, too.)

NOTE 1: The yields you compute this way probably won't match the yields you see in newspapers, for both good and bad reasons. For one thing, financial reporters are sometimes taking into account the exact days on which interest is to be paid. For another, they often convert the continuously-compounded rate $\bar{r}$ into an annual or semi-annual compounded rate. TreasuryDirect does both of these things, with a semi-annual rate. I can't tell what ZionsBank is doing.

NOTE 2: The yield for a bond depends on V as well as on the cash flows. That is, it isn't just about the bond, it's about the price the bond is selling for. If the price changes, so does the yield.

NOTE 3: Equation 1.2 allows you to convert V to $\bar{r}$ and back again, for a particular bond. Financial people get good at converting both ways. They often talk about $\bar{r}$ when they care about V or vice versa. V is closer to reality, but $\bar{r}$ is often easier to understand and compare to other bonds.

NOTE 4: **Yields for STRIPS.** It's easier to compute the yield for a STRIP, because there is just one payment. Write (c, t) in place of (c_1, t_1); then 1.2 becomes

$$V = ce^{-\bar{r}t} \tag{0.3}$$

so

$$\bar{r} = \frac{1}{t}\left(-\ln(V/c)\right). \tag{0.4}$$

Put another way:

$$\bar{r} = \frac{1}{t}\left(-\ln(P(t))\right). \tag{0.5}$$

B. The "pure" yield curve

You often see yield curves in the newspaper, where somebody has found a class of bonds with different maturities, and plotted their yields as a function of the maturity (time). These curves are helpful for those who understand them, and they prove beyond doubt that interest rates (even in the absence of uncertainty) aren't constant. But in a way, they are misleading, since the bonds aren't usually pure payments at the maturity date --- they have coupons scattered over the entire period between 0 and T.

Also, newspaper yield curves use various notions of compounded rates.

We'll build a pure risk-free yield curve based on market prices for guaranteed payments (e.g., STRIPS), and we'll use only continuously compounded rates. We define a function $\bar{r}$, where

$\bar{r}$ (t) = yield for a guaranteed payment at time t

Equation 1.5 applies, and we can relate $\bar{r}$ (t) to P (t) as follows:

$$\boxed{\bar{r}(t) = \frac{1}{t}\left(-\ln(P(t))\right)} \tag{0.6}$$

for every t satisfying $t \geq 0$.

In this class we don't really care about $\bar{r}$ (t), but it's good to know what the rest of the world uses.

C. Instantaneous Forward Interest Rates.

The function $\bar{r}$ still mixes time periods. Specifically, $\bar{r}$ (t) is about what happens to interest rates over the entire time period from 0 to t. What is happening exactly at time t?

Define the **forward interest rate at time t** as follows:

r(t) = the rate we would agree to for a very-short-term loan at time t, if we made the deal now.

That is, r (t) is something like a prediction of the future. (We're not admitting to any uncertainty here, or acknowledging that anything would ever change. r(t) is just the market price, now, of certain agreements about future transactions.)

We have already found a relationship between D(t) and r(t):

$$r(t) = \frac{D'(t)}{D(t)}. \tag{0.7}$$

Since P(t) = 1/D(t), it's not too hard to convert this to

$$r(t) = -\frac{P'(t)}{P(t)}. \tag{0.8}$$

Hey, these were supposed to be very sketchy notes, and there has to be something left to think about in class. So, I'll get sketchy here.

By the way, we DO care about r(t) in economics, because that function is the basis of most good economic theories for how interest rates change. It's true that P(t) represents reality, but the theories all apply to r(t), so we have to be good at translating back and forth. (Alas, we won't have time for these theories in MATH 53. Peek at chapter 11 of the text for the first hints; then when you're ready to get serious, read the original paper by Heath, Jarrow, Morton.)

D. Converting between r, $\bar{r}$, P(t), D(t)

D is the easy part, since D(t) = 1/P(t) and P(t) = 1/D(t). We won't mention D(t) again.

From P to r and $\bar{r}$:

r(t) = -P'(t) / P(t) = - d/dt (ln P(t))

$\bar{r}$ (t) = (1/t) (-ln P(t))

From r to $\bar{r}$:

$$\bar{r}(t) = \left(\frac{1}{t}\right)\int_0^t r(s)ds$$ (that is, $\bar{r}$ (t) is the average value of r(s) on the interval [0, t])

r(t) = $\bar{r}$ (t) + t$\bar{r}$ '(t) (this is a differential equation, so we need to add the initial condition $\bar{r}$ (0) = r(0))

From r and $\bar{r}$ to P:

$$P(t) = \exp\left(-\int_0^t r(s)ds\right)$$

(that is, P(t) gets hit with the cumulative effect of r(s) while s runs from 0 to t)

$$P(t) = \exp\left(-t\bar{r}(t)\right)$$

Conclusions -

- Modern (classical) investment theory remains the main idea of mobilising investment theory and practical development.
- Present needs of investment process management influenced by stochasticity of future possibilities require the formation and utilisation of adequate investment portfolio model for the realisation of those needs.
- The suggested model of investment process analysis and management orientated to the necessity of the reflection of future possibility.
- The suggested computerised imitative (numerical) technologies of model realisation permits practical use of adequate investment model which decision is inefficient while using usual methods of programming and other numerical decisions.
- The suggested adequate model of investment process analysis and management altogether with innovative possibilities of imitative technologies allows the assumption that adequate model enlarges possibilities of modern investment theory and supplements them with new quality.

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